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Fuzzy approach to solve multi-objective capacitated transportation problem

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Abstract: The linear multi-objective capacitated transportation problem in which the supply and demand constraints are equality type, capacity restriction on each route are specified and the objectives are non commensurable and conflict in nature. The fuzzy programing technique (Linear, Hyperbolic and Exponential) is used to find optimal compromise solution of a multi-objective capacitated transportation problem has been presented in this paper. An example is illustrate the methodology. Also comparision is taken out, using same example.

Keyword: Multi-criteria Decision Making, Capacitated Transportation Problem, Linear Membership Function, Non-linear Membership Function.

1. Introduction

A transportation problem with capacity restriction is a linear programming problem. A basic solution to a capacitated transportation problem may contain more than m+n-2 positive values due to the capacity constraints which are additional to the m+n-2 independent equations. Fuzzy linear programming occurs when fuzzy set theory is applied to linear multicriteria decision making problem. In fuzzy set theory, an element x has a degree of membership in a set A, denoted by a membership function (X). The range of the membership function is [0, 1]. Degree of the membership function for each objective represents its satisfaction level. If the membership function of an objective is one or zero then objective is fully achieved or not at all achieved, respectively. If the membership function of the objective lies in (0, 1) then the objective is partially achieved. Zadeh [13] introduced the concept of fuzzy set theory. Zimmermann [14] first applied the fuzzy set theory concept with some suitable membership function to solve Multi-objective linear programming problems. He showed that solutions obtained by fuzzy linear programming efficient. Ringuest and Rinks [11] have mentioned the existing solution procedures for Multi-objective transportation problem. Bit [1,2] have shown the application of fuzzy programming with linear membership function to the multicriteria decision making solid transportation problem and classical transportation problem . Leberling [10] has developed algorithms for obtaining compromise solution in multicriteria problems using the min-operator. In this paper, we present fuzzy programming with linear, hyperbolic and exponential membership function for solving multi-objective capacitated transportation problem.

2. Multi-objective capacitated transportation problem

Consider m origins (i =1, 2,…,m) and n destinations (j =1,2,…,n) at each origin O_i, let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j to satisfy the demand for b_j units of the product there. A penalty c_{ij}^p is associated with transportation of a unit of the product from source i to destination j for the p-th criterion. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under used capacity. A variable X_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. Let r_{ij} be the capacity restrictions on route i, j for capacitated transportation problem.

A multi-objective capacitated transportation problem can be represented as:

$$\text{Minimize } Z_p = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^p x_{ij} \qquad p=1,2,\ldots,P \tag{1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i=1,2,\ldots,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j=1,2,\ldots,n \tag{3}$$

$$0 \le x_{ij} \le r_{ij} \qquad \text{for all } i, j \tag{4}$$

Where the subscript on Z_p and superscript on c_{ij}^p denote p-th penalty criterion; $a_i > 0$ for all i $b_j > 0$ for all j, $r_{ij} \ge 0$ for all i, j

And $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ as balanced condition.

This balanced condition is necessary condition for the problem to have a feasible solution, however, this is not sufficient because of the condition (4).

For p=1, problem become to a single objective capacitated transportation problem. It may be considered as a special case of linear programming problem.

3. Fuzzy programming technique for the multi-objective capacitated transportation problem

Step 1:
Solve the multi-objective capacitated transportation problem as a single objective capacitated transportation problem P times, by taking one of the objectives at a time.

Step 2:
From the results of step 1, calculate the values of all the P objective functions . Then a pay off matrix is formed. The diagonal of the matrix constitutes individual optimum minimum values for the p objectives.

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \\ . \\ . \\ X^{(P)} \end{array} \begin{array}{c} \begin{array}{cccc} Z_1(X) & Z_2(X) & \dots & Z_p(X) \end{array} \\ \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1p} \\ Z_{21} & Z_{22} & \dots & Z_{2p} \\ \dots & \dots & \dots & \dots \\ Z_{p1} & Z_{p2} & & Z_{pp} \end{bmatrix} \end{array} \quad (5)$$

Step 3:
From step 2, we find for each objective, the lower bound (Lp) and upper bound (Up) corresponding to the sets of p solutions, where,

$$U_p = \max(Z_{1p}, Z_{2p}, \dots, Z_{pp}) \quad \text{and} \quad L_p = Z_{pp} \quad p = 1,2,\dots,P$$

An initial fuzzy model of the problem (1)-(4) can be stated as: -
can be stated as

$$\text{Find } X_{ij} \quad i = 1,2,\dots,m \quad j = 1,2,\dots,n \quad (6)$$

$$\text{so as to satisfy } Z_p \tilde{<} L_p \quad p = 1,2,\dots,P$$

$$\text{Subject to (2)-(4)}$$

Step 4: **Case (i) Define Hyperbolic membership function**

$$\mu^H Z_p(x) = \begin{cases} 1 & \text{if } Z_p \le L_p \\ \dfrac{1}{2}\,\dfrac{e^{\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p} - e^{-\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p}}{e^{\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p} + e^{-\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p}} + \dfrac{1}{2} & \text{if } L_p < Z_p < U_p \\ 0 & \text{if } Z_p \ge U_p \end{cases}$$

(7)

Case (ii) Define Linear membership function for the p^{th} objective function as follows:

$$\mu_p(X) = \begin{cases} 1 & \text{if } Z_p(X) \le L_p \\ \dfrac{U_p - Z_p(X)}{U_p - L_p} & \text{if } L_p < Z_p < U_p \\ 0 & \text{if } Z_p \ge U_p \end{cases} \quad (8)$$

Step 5:
Find an equivalent crisp model by using a linear membership function for the initial fuzzy model

$$\text{Maximize } \lambda$$

$$\lambda \le \frac{U_p - Z_p(X)}{U_p - L_p} \quad (9)$$

$$\text{subject to (2)-(4)}$$

Step 6: Solve the crisp model by an appropriate mathematical programming algorithm.

$$\text{Maximize } \lambda$$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{p} X_{ij} + \lambda(U_p - L_p) \le U_p \quad p = 1,2,\dots,P \quad (10)$$

Subject to (2)-(4)

$$\sum_{j=1}^{n} X_{ij} = a_i \qquad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} X_{ij} = b_j \qquad j = 1,2,\dots n$$

$$0 \le x_{ij} \le r_{ij} \qquad \text{for all } i,j$$

The foregoing linear programming problem that can be solved by linear programming algorithm to find an optimal compromise solution .

Case iii) Now, by using exponential membership function for the p th objective function and is defined as

$$\mu^E Z_p(x) = \begin{cases} 1, & \text{if } Z_p \le L_p \\ \dfrac{e^{-S\Psi_p(X)} - e^{-S}}{1 - e^{-S}}, & \text{if } L_p < Z_p < U_p \\ 0, & \text{if } Z_p \ge U_p \end{cases} \quad (11)$$

Where, $\Psi_p(X) = \dfrac{Z_p - L_P}{U_p - L_p} \qquad p = 1,2,\dots,P$

S is a non zero parameter, prescribed by the decision maker

Then an equivalent crisp model for fuzzy model can be formulated as

$$\text{Maximize } \lambda$$

Subject to

$$\lambda \le \frac{e^{-s\psi_p(x)} - e^{-s}}{1 - e^{-s}} \qquad p = 1, 2, \text{---}, P$$

subject to (2)-(4)

6. Numerical Example:

$$\text{Minimize } Z_1 = 5X_{11} + 3X_{12} + 2X_{13} + 6X_{21} + 4X_{22} + 7X_{23} + 2X_{31} + 8X_{32} + 6X_{33}$$

$$\text{Minimize } Z_2 = 4X_{11} + 6X_{12} + 5X_{13} + 7X_{21} + 8X_{22} + 6X_{23} + 5X_{31} + 2X_{32} + 3X_{33}$$

(12)

$$\text{Minimize } Z_3 = 9X_{11} + 9X_{12} + 7X_{13} + 3X_{21} + 9X_{22} + 3X_{23} + 7X_{31} + 9X_{32} + 10X_{33}$$

$$\sum_{j=1}^{3} X_{1j}=120 \quad ; \quad \sum_{j=1}^{3} X_{2j}=145 \quad ; \quad \sum_{j=1}^{3} X_{3j}=95$$
$$\sum_{i=1}^{3} X_{i1}=80 \quad ; \quad \sum_{i=1}^{3} X_{i2}=100 \quad ; \quad \sum_{i=1}^{3} X_{i1}=180$$
$$X_{ij} \geq 0 \quad i=1,2,3. \;, \quad j=1,2,3.$$
(13)

Capacity restrictions of the routes are given as:

$$0 \le x_{11} \le 45,\ \ 0 \le x_{12} \le 60,\ \ 0 \le x_{13} \le 100$$
$$0 \le x_{21} \le 90,\ \ 0 \le x_{22} \le 100,\ 0 \le x_{23} \le 80S$$
$$0 \le x_{31} \le 125,\ 0 \le x_{32} \le 85,\ \ 0 \le x_{33} \le 130$$

(14)

Step1 and step 2 . Optimal solutions for minimizing the first objective Z_1

Subject to constraints (2) and (4) are as follows

$$x11 = 20, x12 = 60, x_{13} = 40, x_{21} = 25,$$
$$x22 = 40,\ x_{23} = 80,\ x_{31} = 35,\ x_{33} = 60$$

and other decision variable are zero

and $Z_1 = 1660$

Optimal solutions for minimizing the second objective Z_2

Subject to constraints (2) and (4) are as follows

$$x11 = 45, x12 = 35, x_{13} = 40, x_{21} = 35,$$
$$x22 = 30,\ x_{23} = 80,\ x_{32} = 35,\ x_{33} = 60$$

and other decision variable are zero

and $Z_2 = 1805$

Optimal solutions for minimizing the third objective Z_3

Subject to constraints (2) and (4) are as follows

$$x11 = 20, x12 = 60, x_{13} = 40, x_{21} = 60,$$
$$x22 = 5,\ x_{23} = 80,\ x_{32} = 35,\ x_{33} = 60$$

and other decision variable are zero

and $Z_3 = 2380$

Now for $X^{(2)}$ we can find out

Z_1, $\quad Z_1(X^{(2)})=1935$

Now for $X^{(3)}$ we can find out

Z_1, $\quad Z_1(X^{(3)})=1940$

Now for $X^{(1)}$ we can find out

Z_2, $\quad Z_2(X^{(1)})=1570$

Now for $X^{(3)}$ we can find out

Z_2, $\quad Z_2(X^{(3)})=2190$

Now for $X^{(1)}$ we can find out

Z_3, $\quad Z_3(X^{(1)})=2670$

Now for $X^{(2)}$ we can find out

Z_3, $\quad Z_3(X^{(2)})=2530$

The pay off matrix is

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \\ X^{(3)} \end{array}\begin{array}{c} \begin{matrix} Z_1 & Z_2 & Z_3 \end{matrix} \\ \begin{bmatrix} 1660 & 1570 & 2520 \\ 1935 & 1805 & 2530 \\ 1940 & 2190 & 2380 \end{bmatrix} \end{array}$$

$$U_1 = 1940, \quad U_2 = 2190, \quad U_3 = 2530$$
$$L_1 = 1660, \quad L_2 = 1805, \quad L_3 = 2380$$

Find $\left\{x_{ij},\ i = 1, 2, 3;\ j = 1, 2, 3\right\}$ so as satisfy

$Z_1 \lesssim 1660,\ Z_2 \lesssim 1805,\ Z_3 \lesssim 2380$ and constraints (1), (2)

Step4. With

$$\alpha_p = \frac{6}{U_p - L_p}, \alpha_1 = \frac{6}{U_1 - L_1} = \frac{6}{280}, \alpha_2 = \frac{6}{U_2 - L_2} = \frac{6}{385}$$

$$\alpha_3 = \frac{6}{U_3 - L_3} = \frac{6}{150}, \frac{U_1 + L_1}{2} = 1800,$$

$$\frac{U_2 + L_2}{2} = 1997.50, \frac{U_3 + L_3}{2} = 2455$$

We get the membership functions $\mu_1^H(Z_1), \mu_2^H(Z_2), \mu_3^H(Z_3)$ for the objectives Z_1, Z_2 and Z_3 respectively, are as follows:

Case (i): Hyperbolic membership function

$$\mu_1^H(Z_1) = \begin{cases} 1, & \text{if } Z_1(x) \le 1660 \\ \frac{1}{2}\tanh[(1800 - Z_1(x))\frac{6}{280}] + \frac{1}{2}, & \text{if } 1660 \le Z_1(x) \le 1940 \\ 0, & \text{if } Z_1(x) \ge 1940 \end{cases}$$

$$\mu_2^H(Z_2) = \begin{cases} 1, & \text{if } Z_2(x) \le 1805 \\ \frac{1}{2}\tanh[(1997.5 - Z_2(x))\frac{6}{385}] + \frac{1}{2}, & \text{if } 1805 \le Z_2(x) \le 2190 \\ 0, & \text{if } Z_2(x) \ge 2190 \end{cases}$$

$$\mu_3^H(Z_3) = \begin{cases} 1, & \text{if } Z_3(x) \le 2380 \\ \frac{1}{2}\tanh[(2455 - Z_3(x))\frac{6}{150}] + \frac{1}{2}, & \text{if } 2380 \le Z_3(x) \le 2530 \\ 0, & \text{if } Z_3(x) \ge 2530 \end{cases}$$

$$\text{Maximize } X_{3x3+1}$$

Subject to

$$\alpha_1 Z_1(X)+X_{mn+1} \le \alpha_1\left(\frac{U_1+L_1}{2}\right)$$

$$\frac{6}{280}(5X_{11}+3X_{12}+2X_{13}+6X_{21}+4X_{22}+7X_{23}+2X_{31}+8X_{32}+6X_{33})+X_{mn+1} \le \frac{6}{280}(1800)$$

$$30X_{11}+18X_{12}+12X_{13}+36X_{21}+24X_{22}+42X_{23}+12X_{31}+48X_{32}+36X_{33}+280X_{mn+1} \le 10800$$

Now,

$$\alpha_2 Z_2(X)+X_{mn+1} \le \alpha_2\left(\frac{U_2+L_2}{2}\right)$$

$$\frac{6}{385}(4X_{11}+6X_{12}+5X_{13}+7X_{21}+8X_{22}+6X_{23}+5X_{31}+2X_{32}+3X_{33})+X_{mn+1} \le \frac{6}{385}(1997.5)$$

$$24X_{11}+36X_{12}+30X_{13}+42X_{21}+48X_{22}+36X_{23}+30X_{31}+12X_{32}+18X_{33}+385X_{mn+1} \le 11985$$

And

$$\alpha_3 Z_3(X)+X_{mn+1} \le \alpha_3\left(\frac{U_3+L_3}{2}\right)$$

$$\frac{6}{150}(9X_{11}+9X_{12}+7X_{13}+3X_{21}+9X_{22}+3X_{23}+7X_{31}+9X_{32}+10X_{33})+X_{mn+1} \le \frac{6}{150}(2455)$$

$$54X_{11}+54X_{12}+42X_{13}+18X_{21}+54X_{22}+18X_{23}+42X_{31}+54X_{32}+60X_{33}+150X_{mn+1} \le 14730$$

The problem was solved by using the linear interactive and discrete optimization (LINDO) software, the optimal compromise solution is

$$X_{mn+1} = 0.1034$$

$$X^* = \left\{ \begin{array}{l} x_{11}=20, x12 = 60,\ x_{13}=40,\ x_{21}=41.896553,\ x_{22}=23.103449, \\ x_{23}=80,\ x_{31}=18.103449, x_{32}=16.896551\ x_{33}=60 \end{array} \right\}$$

$$Z_1^*=1789.3493\ ;\quad Z_2^*=1715.3103 \quad \text{and} \quad Z_3^*=2448.7931$$

$$\boxed{\lambda = 0.55}$$

ii) Linear Membership Function

$$\mu_1(X)=\begin{cases} 1, & \text{if} \quad Z_1(X) \le 1660 \\ \dfrac{1940-Z_1(X)}{1940-1660}, & \text{if} \quad 1660 < Z_1(X) < 1940 \\ 0, & \text{if} \quad Z_1(X) \ge 1940 \end{cases}$$

$$\mu_2(X)=\begin{cases} 1, & \text{if} \quad Z_2(X) \le 1805 \\ \dfrac{2190-Z_2(X)}{2190-1805}, & \text{if} \quad 1805 < Z_2(X) < 2190 \\ 0, & \text{if} \quad Z_2(X) \ge 2190 \end{cases}$$

$$\mu_3(X)=\begin{cases} 1, & \text{if} \quad Z_3(X) \le 2380 \\ \dfrac{2530-Z_3(X)}{2530-2380}, & \text{if} \quad 2380 < Z_3(X) < 2530 \\ 0, & \text{if} \quad Z_3(X) \ge 2530 \end{cases}$$

Find an equivalent crisp model

$$\text{Maximize } \lambda\ ,$$

$$Z_1(X)+280\lambda \le 1940$$

$$5X_{11}+3X_{12}+2X_{13}+6X_{21}+4X_{22}+7X_{23}+2X_{31}+8X_{32}+6X_{33}+280\lambda \le 1940$$

and

$$\text{Maximize } \lambda\ ,$$

$$Z_2(X)+385\lambda \le 2190$$

$$4X_{11}+6X_{12}+5X_{13}+7X_{21}+8X_{22}+6X_{23}+5X_{31}+2X_{32}+3X_{33}+385\lambda \le 2190$$

$$\text{Maximize } \lambda\ ,$$

$$9X_{11}+9X_{12}+7X_{13}+3X_{21}+9X_{22}+3X_{23}+7X_{31}+9X_{32}+10X_{33}+150\lambda \le 2530$$

$$Z_3(X)+150\lambda \le 2530$$

$$X^* = \left\{ \begin{array}{l} x_{11}=20, x12 = 60,\ x_{13}=40,\ x_{21}=41.896553,\ x_{22}=23.103449, \\ x_{23}=80,\ x_{31}=18.103449, x_{32}=16.896551\ x_{33}=60 \end{array} \right\}$$

$$Z_1^*=1789.3493\ ;\quad Z_2^*=1715.3103 \quad \text{and} \quad Z_3^*=2448.7931$$

$$\boxed{\lambda = 0.5172}$$

iii) Exponential Membership Function

$$\mu^E Z_1(x)=\begin{cases} 1, & \text{if } Z_1 \le 1660 \\ \dfrac{e^{-1\Psi_1(X)}-e^{-1}}{1-e^{-S}}, & \text{if } 1660 < Z_1 < 1940 \\ 0, & \text{if } Z_1 \ge 1940 \end{cases}$$

$$\mu^E Z_2(x)=\begin{cases} 1, & \text{if } Z_2 \le 1805 \\ \dfrac{e^{-1\Psi_2(X)}-e^{-1}}{1-e^{-S}}, & \text{if } 1805 < Z_2 < 2190 \\ 0, & \text{if } Z_2 \ge 2190 \end{cases}$$

$$\mu^E Z_3(x)=\begin{cases} 1, & \text{if } Z_3 \le 2380 \\ \dfrac{e^{-1\Psi_3(X)}-e^{-1}}{1-e^{-S}}, & \text{if } 2380 < Z_3 < 2530 \\ 0, & \text{if } Z_3 \ge 2530 \end{cases}$$

Then an equivalent crisp model for fuzzy model can be formulated as

$$\text{Maximize } \lambda$$

Subject to

$$\lambda \le \frac{e^{-1\psi_p(x)} - e^{-1}}{1 - e^{-1}}, \qquad p = 1, 2, \text{-----}, P \text{ and}$$

subject to (7)-(9)

$$\Psi_1(X) = \frac{Z_1 - L_1}{U_1 - L_1} = \frac{Z_1 - 1660}{1940 - 1660} = \frac{Z_1 - 1660}{280}$$

$$\Psi_2(X) = \frac{Z_2 - L_2}{U_2 - L_2} = \frac{Z_2 - 1805}{2190 - 1805} = \frac{Z_2 - 1805}{385}$$

$$\Psi_3(X) = \frac{Z_3 - L_3}{U_3 - L_3} = \frac{Z_3 - 2380}{2530 - 2380} = \frac{Z_3 - 2380}{150}$$

$\Psi_1(X) =$

$$(5X_{11}+3X_{12}+2X_{13}+6X_{21}+4X_{22}+7X_{23}+2X_{31}+8X_{32}+6X_{33}-1660)/280$$

$\Psi_2(X) =$

$$(4X_{11}+6X_{12}+5X_{13}+7X_{21}+8X_{22}+6X_{23}+5X_{31}+2X_{32}+3X_{33}-385)/385$$

$\Psi_3(X) =$

$$(9X_{11}+9X_{12}+7X_{13}+3X_{21}+9X_{22}+3X_{23}+7X_{31}+9X_{32}+10X_{33}-2380)/150$$

Then the problem can be simplified as

$\Rightarrow$ Maximize λ

$$e^{-\Psi_1(X)} - (1-e^{-1})\lambda \ge e^{-1} \Rightarrow e^{-\Psi_1(X)} - (1-0.368)\lambda \ge 0.368 \Rightarrow e^{-\Psi_1(X)} - (0.6321)\lambda \ge 0.368$$

$$e^{-\Psi_1(X)} - (1-e^{-1})\lambda \ge e^{-1} \Rightarrow e^{-\Psi_2(X)} - (1-0.368)\lambda \ge 0.368 \Rightarrow e^{-\Psi_2(X)} - (0.6321)\lambda \ge 0.368$$

$$e^{-\Psi_1(X)} - (1-e^{-1})\lambda \ge e^{-1} \Rightarrow e^{-\Psi_3(X)} - (1-0.368)\lambda \ge 0.368 \Rightarrow e^{-\Psi_3(X)} - (0.6321)\lambda \ge 0.368$$

The problem is solved by the (LINGO) software

$$X^* = \begin{Bmatrix} x_{12}=20,\ x_{13}=100,\ x_{21}=65,\ x_{23}=80,\ x_{31}=15,\ x_{32}=80. \\ \text{rest all } x_{ij} \text{ are zero's} \end{Bmatrix}$$

$$Z_1^* = 1880 \quad ; \quad Z_2^* = 1790 \quad \text{and} \quad Z_3^* = 2140$$

$$\boxed{\lambda = 0.8070}$$

And Ideal solution is {1660, 1805, 2380}
Also set of non-dominated solutions {1660, 1570, 2520}; {1935, 1805, 2530}; {1940, 2190, 2380}.

7. Conclusion

We have obtained same optimal compromise solution by our proposed algorithm and fuzzy algorithm with membership functions (Bit et al. [1]) for the multi-objective capacitated transportation problem. For a multi-objective capacitated transportation problem with p objective functions, the fuzzy programming with hyperbolic, linear and exponential membership function gives p non-dominated (efficient) solutions and an optimal compromise solution. The fuzzy programming algorithm with hyperbolic membership functions is applicable to multi-objective capacitated solid transportation problems and the vector minimum problems. This algorithm can be applied to the variants of multi-objective transportation problems similar linear multi-objective programming problems. This paper is to be seen as a first step to introduce non-linear membership functions to a multi-objective capacitated transportation problem. The value of membership function of an objective represents the satisfaction level of the objective.

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