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FUZZY EOQ MODEL FOR DETERIORATING ITEMS WITH HYPERBOLIC INVERSE MEMBERSHIP FUNCTION

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Abstract- A multi item EOQ model for deteriorating items is developed in fuzzy environment. Inventory costs, profit, deteriorating rate and investment constraints are considered to be vague and imprecise in nature. The vagueness of holding cost and set up cost is represented by Hyperbolic Inverse membership function and other fuzzy parameters are represented by linear membership function. The model has been solved by fuzzy non linear programming method. Results have been presented along with crisp model.

Keywords- Fuzzy inventory, Crisp model, Hyperbolic Inverse membership function.

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Introduction

In inventory control theory, demand and deterioration of an item are important parameters, which cannot be ignored to develop inventory models. Certain models have been developed by considering the demand rate to be constant, time-dependent, ramp type or selling price dependent. However, in the present competitive market, stock of an item plays an important role to increase its demand. Deterioration of an item is a natural phenomenon. Some items like food grains, vegetables, milk, eggs etc. deteriorate during their storage time. Giri B.C., Pal S., Goswami A. and Chudhari K.S. [3] have developed an EOQ model with stock-dependent demand for deteriorating items.

Due to the vagueness and uncertainty of inventory parameters, the fuzzy set theory is more appropriate to formulate inventory model. In fuzzy decision, making process first Bellman and Zadeh [1] introduced the concept of fuzzy set theory. Tanaka, et.al. [6] used the concept of fuzzy sets to solve the decision problems and Zimmerman [9] showed the classical algorithms can be used to solved fuzzy linear programming problems. Generally both linear and non-linear membership functions are proposed to represent the fuzzy objective and fuzzy constraints of an inventory model. To reflect the decision makers' performances regarding the relative importance of each objective, fuzzy weights are used. In this area Mandal and Maiti [5] and Mandal, Roy and Maiti [4] solved the classical inventory model in a fuzzy environment with fuzzy objective, fuzzy inventory cost with constraints by nonlinear programming method. Particularly in case of non-linear membership function, Umap [7,8] used Exponential and Hyperbolic membership functions to develop fuzzy Inventory models for deteriorating.

In this paper, a multi item inventory model for deteriorating items is developed in fuzzy environment. Here inventory costs such as holding cost and setup cost are represented by Hyperbolic Inverse membership function and profit, deteriorating rate and total investment constraint are represented by linear membership function. The model has been solved by fuzzy non-linear programming (FNLP) method. Results are presented along with those of corresponding crisp model and a sensitivity analysis.

Assumptions

i. Selling price is Known and constant.
ii. Demand rate is stock dependent.
iii. Shortages are not allowed.
iv. Deteriorating rate is age specific failure rate.

Notations

T_i : Time period for each cycle for the ith item.
Ri : Demand rate per unit time of ith item; [Ri=ai+biqi]
θ_i : Deterioration rate of ith item
Qi(t) : Inventory level at time t of ith item.
CH : Total Holding cost.
C1i : Holding cost per unit of ith item.
C3i : Set up cost for ith item.
Sdi : Total deteriorating units of ith item.
Pi : Selling price per unit of ith item.
Qi : Initial stock level of ith item.
PF(Qi) :Total profit if ith item.
n : Number of items.
(wavy bar (~) represents the fuzzification of the parameters)

Formulation of model

Crisp Model:

As Qi(t) is the inventory level at time t of the i th item, then the differential equation describing the state of inventory is given by

$$\frac{d}{dt}Q_i(t)+\theta_i Q_i(t)=-(a_i+b_i Q_i(t)) \qquad 0\le t\le T_i$$

solving the above differential equation using boundary condition Qi(t)=Qi at t=o, we get

$$Q_i(t)=-\frac{a_i}{(\theta_i+b_i)}+\left[Q_i+\frac{a_i}{(\theta_i+b_i)}\right]e^{-(\theta_i+b_i)t} \qquad [1]$$

and using boundary condition Qi(t)=0 at t=Ti

$$\therefore \quad T_i=\frac{1}{(\theta_i+b_i)}\log\left\{1+\frac{(\theta_i+b_i)Q_i}{a_i}\right\} \qquad \text{--- [2]}$$

The holding cost of i^{th} item in each cycle is

CH = C1i Gi (Pi , Qi) ---[3]

$$G_i(P_i,Q_i)=\int_0^{Q_i}\frac{q_i\,dq_i}{a_i+(\theta_i+b_i)q_i}$$

where,

$$=\frac{a_i}{(\theta_i+b_i)^2}\log\left\{1+\frac{(\theta_i+b_i)Q_i}{a_i}\right\}$$

By neglecting the higher power terms, we get

$$G_i(P_i,Q_i)=\frac{Q_i^2}{2a_i}\left\{1-2\frac{(\theta_i+b_i)Q_i}{3a_i}\right\}$$

The total number of deteriorating units of the i^{th} item is

$$S_{di}(Q_i)=\theta_i G_i(Q_i)$$

The net revenue for the i^{th} item is

$$N(Q_i)=(P_i-C_i)Q_i-P_i.S_{di}(Q_i)$$

$$N(Q_i)=(P_i-C_i)Q_i-P_i\theta_i.G_i(Q_i) \qquad \text{--- [4]}$$

The profit of i^{th} item is

$$PF(Q_i)=N(Q_i)-C_{1i}G_i(P_i,Q_i)-C_{3i} \quad , \quad i=1,2,\text{---}n.$$

$$PF(Q_i)=(P_i-C_i)Q_i-P_i\theta_i.G_i(Q_i)-C_{1i}G_i(P_i,Q_i)-C_{3i} \quad , \quad i=1,2,\text{---}n.$$

$$PF(Q_i)=(P_i-C_i)Q_i-(C_{1i}+P_i\theta_i)G_i(P_i,Q_i)-C_{3i} \quad , i=1,2,\text{---}n. \qquad \text{--- [5]}$$

Hence the problem is

$$\text{Max } PF=\sum_{i=1}^{n}\left[(P_i-C_i)Q_i-(C_{1i}+P_i\theta_i)G_i(P_i,Q_i)-C_{3i}\right]$$

subject to

$$\sum_{i=1}^{n}C_iQ_i \le B$$

$$Q_i \ge 0, \qquad i=1,2,3,\text{---},n \qquad \text{--- [6]}$$

Fuzzy Model:

When above profit, costs, deteriorating rate and total budget become fuzzy, the said crisp model is transform to

$$\tilde{\text{M}}\text{ax } PF=\sum_{i=1}^{n}\left[(P_i-C_i)Q_i-(\tilde{C}_{1i}+P_i\tilde{\theta}_i)G_i(P_i,Q_i)-\tilde{C}_{3i}\right]$$

subject to

$$\sum_{i=1}^{n}C_iQ_i \le \tilde{B}$$

$$Q_i \ge 0, \qquad i=1,2,3,\text{---},n \qquad \text{--- [7]}$$

Mathematical Analysis:

A crisp non-linear programming problem may be defined as follows

$$\text{Max } g_0(x,c_0)$$

subject to

$$g_r(x,c_r)\le b_r \quad , \quad r=1,2,\text{---},m.$$

$$x\ge 0$$

Where $x=(x_1,x_2,\text{---},x_n)^T$ is a variable vector g_0,gi's are algebraic expressions in x with coefficients $C_0=(C_{01},C_{02},\text{---},C_{0n})^T$ and $C_r=(C_{r1},C_{r2},\text{---},C_{rl})^T$ respectively.

Introducing fuzziness in the crisp parameters, the above problem in fuzzy environment becomes

$$\tilde{\text{M}}\text{ax } g_0(x,\tilde{c}_0)$$

subject to

$$g_r(x,\tilde{c}_r)\le \tilde{b}_r \quad , \quad r=1,2,\text{---},m.$$

$$x\ge 0$$

In fuzzy set theory the fuzzy objective goal, coefficients and resources are defined by their membership functions, which may be linear and / or non-linear. According to Bellman and Zadeh [1] and following Carlsson and Korhnen [2] above problem is transform to

$$\text{Max } \alpha$$

subject to

$$g_0(x,\mu_{C_0}^{-1}(\alpha)) \le \mu_r^{-1}(\alpha)$$

$$g_i(x,\mu_{C_r}^{-1}(\alpha)) \ge \mu_r^{-1}(\alpha) \text{ ,r=1,2,---m}$$

$$x\ge 0, \alpha\in[0,1]$$

where

$$\mu_{C_r}^{-1}=\left\{\mu_{C_1}^{-1}\mu_{C_2}^{-1}\text{---}\mu_{C_k}^{-1}\right\}$$

Hyperbolic Inverse Membership Function

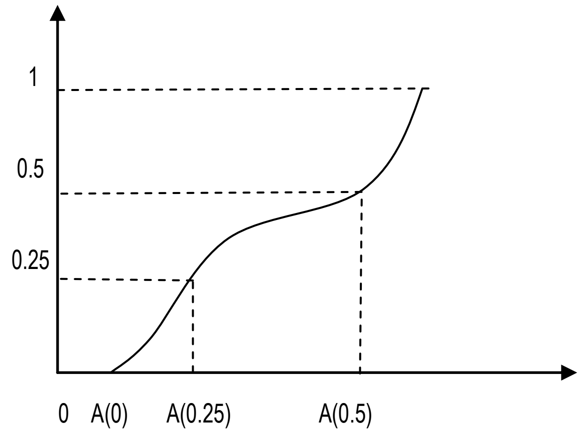
For each fuzzy parameter $\tilde{A}$ the corresponding hyperbolic membership function is defined by

$$\mu_A(u)=a\tanh^{-1}((A(u)-b)c)+\frac{1}{2} \qquad \text{where } a>0,\ c>0.$$

The hyperbolic inverse membership function can be determined by asking the decision maker to specify the three points A(0), A(0.25) and A(0.5) within A_{max} and A_{min}.

Following figure gives the pictorial representation of the hyperbolic membership function

Fig: Hyperbolic inverse membership function



$$\mu_{ci}^{-1}(\alpha)=b_{ci}+\frac{1}{C_{ci}}\tanh\left[\frac{\alpha-\frac{1}{2}}{a_{ci}}\right]$$

Fuzzy Model: Fuzzy costs are represented by Hyperbolic inverse membership function and fuzzy goal, deteriorating rate and total budget are represented by linear membership function:

Let

$$\mu^{-1}_{C_{1i}}(\alpha)=b_{C_{1i}}+\frac{1}{C_{C_{1i}}}\tanh\left[\frac{\alpha-1}{a_{C_{1i}}}\right]$$

$$\mu^{-1}_{C_{3i}}(\alpha)=b_{C_{3i}}+\frac{1}{C_{C_{3i}}}\tanh\left[\frac{\alpha-1}{a_{C_{3i}}}\right]$$

$$\mu^{-1}_{\theta i}(\alpha)=\theta ci+P_{\theta i}(1-\alpha)$$

$$\mu^{-1}_{PF}(\alpha)=PF_o-P_{PF}(1-\alpha)$$

$$\mu^{-1}_{B}(\alpha)=B+P_B(1-\alpha)$$

Here P_{PF}, $P_{\theta i}$ are the maximum acceptable violation of the aspiration levels PF_o and B

Then the fuzzy model reduces to crisp model as

$$\text{Max } \alpha$$
$$\text{subject to}$$
$$PF(\theta_i,\alpha)\geq PF_o-P_{PF}(1-\alpha)$$
$$\sum_{i=1}^{n}C_iQ_i\leq B+P_B(1-\alpha)$$
$$Q_i\geq 0,\ \alpha\in[0,1] \qquad \text{--- [8]}$$

Where Qi >0 (i=1,2,---n) are decision variables and

$$PF(Q_i,\alpha)=\sum_{i=1}^{n}[(P_i-C_i)Q_i-[(b_{C_{1i}}+\frac{1}{C_{C_{1i}}}\tanh\left[\frac{\alpha-1}{a_{C_{1i}}}\right])$$
$$+P_i(\theta_i+P_{\theta i}(1-\alpha))\left\{\frac{Q_i^2}{2a_i}\left[1-\frac{2[(\theta_i+P_{\theta i}(1-\alpha))+b_i]}{3a_i}\right]\right\}$$
$$-(b_{C_{3i}}+\frac{1}{C_{C_{3i}}}\tanh\left[\frac{\alpha-1}{a_{C_{3i}}}\right])]$$

Numerical Example: For n=2

P1=10, C1=7, a1=110, b1=0.5, θ1=0.025, P2=10, C2=6.75 a2=100, b2=0.5, θ2=0.03, B=1800, PFo=500, PPF=50 PB=100, Pθ1=0.005, Pθ2=0.005, aC11=1.4, aC12=2.1, aC31=34, aC32=42, bC11=1.6, bC12=2.7, bC31=36, bC32=47,CC11=1.8, CC12=2.8, CC31=38, CC32=48.

The optimum results are:

α= 0.7785115, Q1=114.3249, Q2=170.312, PF=259.0751,

From the tables, the following observations are made:

i) In [Table-I], the optimum values are given for the fuzzy model with Hyperbolic inverse membership function along with the crisp model. Here the profit varies from Rs. 193 to Rs. 299. As our permissible profit range is (210-160), only three crisp model profits are in this range. Actually, if we make parametric study on the crisp model with the different values of parameters, some of those results will coincide with the optimum values of fuzzy model. This laborious and time-consuming parametric study can be avoided by using fuzzy analysis.

ii) In [Table-II] it is seen that α increases but it never becomes one as it is expected. The values of decision variables (Qi's) become invariant but profit and budget decreases with the increase in tolerance of objective.

Concluding Remark

Membership function of fuzzy parameter is always constructed on the basis of the earlier experience or historical data. In this paper, Hyperbolic Inverse membership function is used to represent the nature of fuzzy parameters. In Umap [7 & 8], Hyperbolic and Exponential membership functions are used respectively for that purpose. In all these three cases fuzzy models give better results as compared to crisp model. Membership function only depicts the nature of parameter. Hence, the superiority of any particular membership function cannot be proved. Researchers can use other types of membership functions such as piecewise linear, cubical parabolic, L-R fuzzy number, etc., to extend this model. The proposed methodology can be used to develop inventory models with two storage, multi-objective, etc.

Table I: *Comparison Table for crisp and fuzzy model*

Model	C_{11}	C_{12}	C_{31}	C_{32}	θ_1	θ_2	B	Q_1	Q_2	PF	α
Crisp	2.2	2.4	80	70	0.03	0.035	1000	58.67146	87.30367	205.8378*	
	2.1	2.3	80	80	0.035	0.035	1000	56.40799	89.65098	199.1940*	
	2.2	2.4	65	60	0.02	0.03	1000	60.4744	85.8339	233.0279	
	2.4	2.2	65	50	0.02	0.03	1000	45.9012	100.547	236.7420	
	2	2.4	65	50	0.03	0.025	1100	65.6204	94.9122	275.0873*	
	2	2.2	65	60	0.03	0.025	1100	51.88589	109.1554	271.7629*	
	2	2.2	70	70	0.03	0.03	900	52.47932	78.91033	193.8238*	
	2	2.2	60	40	0.03	0.02	1100	50.5353	110.5561	298.0897*	
	2	2.2	80	70	0.03	0.03	1000	60.6636	85.2377	213.0223	
Fuzzy	1.93	2.18	64	49	0.25	0.03	1000	71.11281	77.8235	256.6153	0.67082

* Not feasible with respect to the range considered for the objective goal (210 – 260)

Table II: *Effect of Variations in P_{PF}*

P_{PF}	α	Q_1	Q_2	PF	B
0	0.3390	73.6523	78.5596	260.0000	1066.0935
50	0.67082	71.1128	77.8235	256.6153	1027.1686
100	0.7853	70.27195	77.5255	250.7505	1021.9510
200	0.8682	69.6061	76.9163	246.7964	1013.1783
500	0.9400	69.06414	76.4141	243.5254	1005.9944
1000	0.9683	68.7076	76.3612	242.2800	1003.1409

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