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CALIBRATED CONFIDENCE INTERVALS FOR INTENSITIES OF A TWO STAGE OPEN QUEUEING NETWORK WITH FEEDBACK

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Abstract- The aim of this paper is to provide an approximate 100(1-α)% calibrated CAN, Exact t, Standard Bootstrap, Bootstrap-t, Variance-stabilized Bootstrap-t, Bayesian Bootstrap, Percentile Bootstrap and Bias-corrected and accelerated bootstrap confidence intervals for intensity parameters of a two stage open queueing network with feedback with distribution-free interval and service times. Numerical simulation study is conducted to demonstrate performances of the confidence intervals by using calibration technique. We consider a measure, named relative coverage, to evaluate performances of the said intervals.

Keywords- Calibration, calibrated confidence intervals, Coverage percentage, Relative coverage

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Introduction

Calibration technique is used for improving the coverage accuracy of any system of approximate confidence intervals. The general theory of calibration is reviewed in Efron and Tibshirani [6], following ideas of Loh [16], Beran [1], Hall [11] and Hall and Martin [12]. The bootstrap calibration technique was introduced by Loh [16,17]. The idea of the bootstrap calibration is to first use bootstrap to estimate the true coverage of confidence intervals and the intervals is then adjusted by comparing with the target nominal level. As we aware that in literature, no work regarding the calibration technique in queueing networks is found. So it is tempting to use calibration technique to construct new confidence intervals called calibrated confidence interval for intensity parameters of a two stage open queueing network whose true coverage probabilities come closer to desired value.

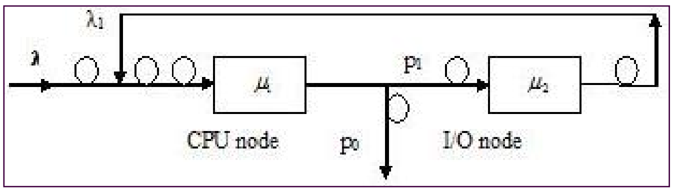

Fig. 1- Two stage open queueing network with feedback

Consider a network model of a computer system with feedback in which a job may return to previously visited nodes. The system consists of two nodes CPU node and I/O node with respective service rates μ_1 and μ_2. The external arrival rate is λ. After service completion at CPU node, the job proceeds to the I/O node with probability p_1, and departs from the system with probability p_0 where $p_0=1-p_1$. Jobs leaving the I/O node are always feed back to the CPU node [Fig-1]. The successive service time at both nodes are assumed to be mutually independent and independent of the state of the system.

The traffic intensities at the CPU node and I/O node are respectively given by

$$\rho_1 = \frac{\lambda}{p_0\mu_1}, \rho_2 = \frac{p_1\lambda}{p_0\mu_2} \qquad (1)$$

where ρ_1 and ρ_2 can be interpreted as expected number of arrivals per mean service time. The condition for stability of the system is both ρ_1 and ρ_2 are less than unity.

Burke [2] has shown that the output of an M/M/1 queue is also Poisson with rate λ. Jackson [14] showed that the product form solution also applies to open network of Markovian queues with feedback, also Jackson's theorem states that each node behaves like an independent queue. Disney [3] introduces basic properties of queueing networks. Thiruvaiyaru, Basawa and Bhat [23] established maximum likelihood estimators of the parameters of an open Jackson network. Thiruvaiyaru and Basawa [22] considered the problem of estimation for the parameters in a Jackson's type queueing network.

Efron [7-9] the greatest statistician in the field of nonparametric resampling approach, originally developed and proposed the bootstrap, which is a resampling technique that can be effectively applied to estimate the sampling distribution of any statistic. For necessary background on bootstrap technique, we refer to Efron and Gong [4], Efron and Tibshirani [5], Guntur [10], Mooney and Duval [19], Young [24], Rubin [21], Miller [18]. Ke and Chu [15] constructed various confidence intervals for intensity parameter of a queueing system.

Nonparametric Statistical Inference of Intensities

Let (X_i, Y_i, i = 1,2) be nonnegative random variables representing the inter-arrival and service times of CPU and I/O node respectively. Once a job complete CPU node burst, it will proceed to I/O node for further service with probability p_1 and departs from the system with probability p_0 where p_0 = 1-p_1. Then the intensities are defined as follows:

$$\rho_1 = \frac{p_0 \mu_{Y_1}}{\mu_{X_1}} \; and \; \rho_2 = \frac{p_0 \mu_{Y_2}}{p_1 \mu_{X_2}}$$

Where μ_{X_1} *and* μ_{X_2} denote the mean inter-arrival times and μ_{Y_1} *and* μ_{Y_2} denote the mean service times of CPU node and I/O node respectively.

Assume that (X_{1j}, p_0Y_{1j}, j= 1,2...n) is a random sample drawn from (X_1, Y_1) and (p_1X_{2j}, p_0Y_{2j}, j= 1,2...n) is a random sample drawn from (X_2, Y_2). Define $(\bar{X}_i, \bar{Y}_i, i = 1,2)$ to be the sample means of (X_i, Y_i, i= 1,2) respectively. Thus according to the Strong Law of Large Numbers [20]; we know that $(\bar{X}_i, \bar{Y}_i, i = 1,2)$ are strongly consistent estimator of $(\mu_{X_i}, \mu_{Y_i}, i = 1,2)$ respectively. Thus strongly consistent estimators of intensities are given by $\hat{\rho}_i = \frac{\bar{Y}_i}{\bar{X}_i}, \; i = 1,2$. The true distributions of (X_i, Y_i, i= 1,2) are not often known in practice so the exact distributions of $\hat{\rho}_i$, i= 1,2 cannot be derived. But under the assumption that X_i and Y_i being independent, the asymptotical distributions of $\hat{\rho}_i$ i= 1,2 can be developed as the following procedures. By Central Limit Theorem and Slutsky's theorem [13], we have

$$\sqrt{n}(\hat{\rho}_i - \rho_i) \xrightarrow{D} N(0, \sigma_i^2), \; i = 1,2.$$

Where

$$\sigma_1^2 = (\mu_{X_1}^2 \sigma_{Y_1}^2 + p_0^2 \mu_{Y_1}^2 \sigma_{X_1}^2) / \mu_{X_1}^4 \quad and \; \sigma_2^2 = (p_1^2 \mu_{X_2}^2 \sigma_{Y_2}^2 + p_0^2 \mu_{Y_2}^2 \sigma_{X_2}^2) / (p_1^2 \mu_{X_2}^4)$$

Also, $\xrightarrow{D}$ denotes convergence in distribution.

Now set $\hat{\sigma}_1^2 = \bar{X}_1^2 S_{Y_1}^2 + p_0^2 \bar{Y}_1^2 S_{X_1}^2 / \bar{X}_1^4 \; and \; \hat{\sigma}_2^2 = p_1^2 \bar{X}_2^2 S_{Y_2}^2 + p_0^2 \bar{Y}_2^2 S_{X_2}^2 / p_1^2 \bar{X}_2^4$

where

$$S_{X_1}^2 = \frac{1}{n}\sum_{j=1}^{n}(X_{1j} - \bar{X}_1)^2, \; S_{X_2}^2 = \frac{1}{n}\sum_{j=1}^{n}(p_1 X_{2j} - \bar{X}_2)^2, \; S_{Y_1}^2 = \frac{1}{n}\sum_{j=1}^{n}(p_0 Y_{1j} - \bar{Y}_1)^2 \; and \; S_{Y_2}^2 = \frac{1}{n}\sum_{j=1}^{n}(p_0 Y_{2j} - \bar{Y}_2)^2$$

Then $\hat{\sigma}_i^2$, i= 1,2 is strongly consistent estimator of σ_i^2, i= 1,2. Again applying the Slutsky's theorem we have $\frac{\sqrt{n}(\hat{\rho}_i - \rho_i)}{\hat{\sigma}_i} \xrightarrow{D} N(0,1), \; i = 1,2.$

Thus $\hat{\rho}_i$, i= 1,2 is strongly consistent and asymptotically normal (CAN) estimator with approximate variances $\hat{\sigma}_i^2/n$, i= 1,2.

Calibration Technique

Let a confidence limit $\hat{\rho}_i[\alpha]$ is supposed to have probability α of covering the true value ρ_i, that is, $P_{F_i}\{\rho_i \le \hat{\rho}_i[\alpha]\} = \alpha \; , i = 1,2$ where F_i is unknown continuous probability distribution. Thus ρ_i is supposed to be less than $\hat{\rho}_i[0.95]$, 95% of the time and $\hat{\rho}_i[0.05]$, 5% of the time. For an approximate confidence limit there is true probability β_i that ρ_i is less than $\hat{\rho}_i[\alpha]$ say, $\beta_i(\alpha) = P_{F_i}\{\rho_i \le \hat{\rho}_i[\alpha]\}$.

The actual coverage of a confidence procedure is rarely equal to the desired coverage and often it is substantially different. If we knew the function $\beta_i(\alpha)$ then we could calibrate an approximate confidence interval to give exact coverage. Suppose we know that $\beta_i(0.03)$=0.05 and $\beta_i(0.94)$=0.95. Then instead of $(\hat{\rho}_i[0.05], \hat{\rho}_i[0.95])$ we would use $(\hat{\rho}_i[0.03], \hat{\rho}_i[0.94])$ to get a central 90% interval with correct coverage probabilities.

In practice we usually don't know the calibration function $\beta_i(\alpha)$. However we can use the bootstrap to estimate $\beta_i(\alpha)$. The bootstrap estimate of $\beta_i(\alpha)$ is $\hat{\beta}_i(\alpha) = P_{\hat{F}_i}\{\hat{\rho}_i \le \hat{\rho}_i[\alpha]^*\}$ where $\hat{F}_i$ and $\hat{\rho}_i$ are fixed, nonrandom quantities and $\hat{\rho}_i[\alpha]^*$ is the α^{th} confidence limit based on bootstrap dataset from $\hat{F}_i$. The estimate $\hat{\beta}_i(\alpha)$ is obtained by taking B bootstrap data sets and seeing what proportion of them have $\hat{\rho}_i \le \hat{\rho}_i[\alpha]^*$.

CAN Calibrated Confidence Interval

Using the CAN estimators $\hat{\rho}_i$, i= 1,2 and its associated approximate variances $\hat{\sigma}_i^2/n \;, i = 1,2$, we construct calibrated confidence intervals for intensities ρ_i, i= 1,2 of a two stage open queueing network with feedback. Let Z_α be the upper α^{th} quantile of the standard normal distribution.

Compute $\hat{\beta}(\alpha) = P\{\rho_i \le (\hat{\rho}_i - z_{\alpha/2}\hat{\sigma}_i/\sqrt{n})\} \; and \; \hat{\beta}(1-\alpha) = P\{\rho_i \le (\hat{\rho}_i + z_{\alpha/2}\hat{\sigma}_i/\sqrt{n})\}$

By the asymptotic distribution of $\frac{\sqrt{n}(\hat{\rho}_i - \rho_i)}{\hat{\sigma}_i}, \; i = 1,2$ an approximate 100(1-α)% calibrated confidence intervals for ρ_i, i= 1,2 are given as

$$\left(\hat{\rho}_i' - z_{\hat{\beta}(\alpha)/2}\hat{\sigma}_i'/\sqrt{n} \; , \; \hat{\rho}_i' + z_{(1-\hat{\beta}(1-\alpha))/2}\hat{\sigma}_i'/\sqrt{n}\right) \; i = 1,2 \quad (2)$$

Exact- t Calibrated Confidence Interval

Let t_α be the upper α^{th} quantile of the Student's t-distribution.

Compute $\hat{\beta}(\alpha) = P\{\rho_i \le (\hat{\rho}_i - t_{(n-1),\alpha/2}\hat{\sigma}_i/\sqrt{n}\} \; and \; \hat{\beta}(1-\alpha) = P\{\rho_i \le (\hat{\rho}_i + t_{(n-1),\alpha/2}\hat{\sigma}_i/\sqrt{n}\}$

Then an approximate 100(1-α)% exact-t calibrated confidence intervals for ρ_i, i= 1,2 are given as

$$\left(\hat{\rho}_i' - t_{(n-1),\hat{\beta}(\alpha)/2}\hat{\sigma}_i'/\sqrt{n} \; , \; \hat{\rho}_i' + t_{(n-1),(1-\hat{\beta}(1-\alpha))/2}\hat{\sigma}_i'/\sqrt{n}\right) \; i = 1,2 \quad (3)$$

Standard Bootstrap Calibrated Confidence Interval

Using bootstrap procedure, a simple random samples $(X_{ij}^*, \; p_0 Y_{ij}^*, i = 1; j = 1,2,....n)$ and $(p_1 X_{ij}^*, p_0 Y_{ij}^*, i = 2; j = 1,2,....n)$ are taken from the empirical distribution functions of (X_{ij},p_0Y_{ij},i=1; j= 1,2...n) and (p_1X_{ij},p_0Y_{ij},i=2; j= 1,2...n). Bootstrap estimate of ρ_i, i= 1,2 is calculated as $\hat{\rho}_i^* = \frac{\bar{y}_i^*}{\bar{x}_i^*}, i = 1,2$. The above resampling process is repeated N times and $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i = 1,2$ are computed from the bootstrap re-samples. Averaging the N bootstrap estimates we get bootstrap estimate of ρ_i, i= 1,2 as

$\hat{\rho}_N(i) = \frac{1}{N}\sum_{j=1}^{N}\hat{\rho}_{ij}^*, i = 1,2$ and standard deviation of $\hat{\rho}_i$, i= 1,2 is

$$sd(\hat{\rho}_N(i)) = \left\{\frac{1}{N-1}\sum_{j=1}^{N}(\hat{\rho}_{ij}^* - \hat{\rho}_N(i))^2\right\}^{1/2}, i = 1,2.$$

Then by central limit theorem, the distribution of $\hat{\rho}_i, i = 1,2$ is approximately normal. Compute

$$\hat{\beta}(\alpha) = P\{\rho_i \le (\hat{\rho}_i - z_{\alpha/2} sd(\hat{\rho}_N(i)))\} \; and \; \hat{\beta}(1-\alpha) = P\{\rho_i \le (\hat{\rho}_i + z_{\alpha/2} sd(\hat{\rho}_N(i)))\}$$

Then 100(1-α)% SB calibrated confidence interval for ρ_i as,

$$\left(\hat{\rho}_i' - z_{\hat{\beta}(\alpha)/2} sd'(\hat{\rho}_N(i)) \; , \; \hat{\rho}_i' + z_{(1-\hat{\beta}(1-\alpha))/2} sd'(\hat{\rho}_N(i))\right) \; i = 1,2 \quad (4)$$

Bootstrap-t Calibrated Confidence Interval

Consider N bootstrap estimates $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i = 1,2$ computed from the bootstrap resample. We compute

$Z_{ij}^* = \frac{(\hat{\rho}_{ij}^* - \hat{\rho}_N(i))}{\text{sd}(\hat{\rho}_N(i))}, i = 1,2, j = 1,2....N$ and $Z_{ij}^*, i = 1,2. \; j = 1,2,....N$

follow an approximate t distribution. Also compute

$$\hat{\beta}(\alpha) = P\{\rho_i \le (\hat{\rho}_i - t_{\alpha/2} sd(\hat{\rho}_N(i)))\} \; and \; \hat{\beta}(1-\alpha) = P\{\rho_i \le (\hat{\rho}_i + t_{\alpha/2} sd(\hat{\rho}_N(i)))\}$$

Then 100(1-α)% Bootstrap-t calibrated confidence interval for ρ_i is

$$\left(\hat{\rho}_i' - \hat{t}_{\hat{\beta}(\alpha)/2} sd'(\hat{\rho}_N(i)) \; , \; \hat{\rho}_i' + \hat{t}_{(1-\hat{\beta}(1-\alpha))/2} sd'(\hat{\rho}_N(i))\right) \; i = 1,2 \quad (5)$$

Where $\hat{t}_{\hat{\beta}(\alpha)/2}$ and $\hat{t}_{(1-\hat{\beta}(1-\alpha))/2}$ equals the $\alpha/2$ and $(1-\alpha/2)$ percentile of the random sample $Z_{i1}^*, Z_{i2}^*, \cdots, Z_{iN}^*, i=1,2$

Variance-stabilized Bootstrap-t Calibrated Confidence Interval

Let $\hat{\rho}_i$, i= 1,2 be a strongly consistent and asymptotically normal estimator with approximate variances $\hat{\sigma}_i^2/n, i=1,2$. Let $\hat{\sigma}_i = \phi(\hat{\rho}_i)$

To find a transformation $f(\hat{\rho}_i)$ such that $Var(f(\hat{\rho}_i)) \approx$ constant, we use the first order Taylor series expansion:

$$f(\hat{\rho}_i) \approx f(\rho_i) + (\hat{\rho}_i - \rho_i) f'(\rho_i) \Rightarrow [f(\hat{\rho}_i) - f(\rho_i)]^2 \approx (\hat{\rho}_i - \rho_i)^2 (f'(\rho_i))^2$$

Taking expectations on both sides, we get:

$$\Rightarrow Var[f(\hat{\rho}_i)] \approx Var(\hat{\rho}_i)(f'(\rho_i))^2 = (\phi(\rho_i))^2 (f'(\rho_i))^2, i=1,2.$$

Now consider $f(\hat{\rho}_i) = \sqrt{n}\log(\phi(\hat{\rho}_i)), i=1,2$ is the variance-stabilizing transformation. Then we have,

$$V[f(\hat{\rho}_i)] \approx \left(\frac{\sqrt{n}}{\phi(\hat{\rho}_i)}\right)^2 Var[\hat{\rho}_i] = \left(\frac{\sqrt{n}}{\hat{\sigma}_i}\right)^2 Var[\hat{\rho}_i] = \frac{n}{\hat{\sigma}_i^2}\frac{\hat{\sigma}_i^2}{n} = 1, i=1,2.$$

Here we consider N bootstrap estimates $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i=1,2$ computed from the bootstrap resample.

We obtain $\theta_{ij}^* = (\sqrt{n}\log(\hat{\rho}_{ij}^*) - \sqrt{n}\log(\hat{\rho}_i)), i=1,2, j=1,2....N$

Also compute

$$\hat{\beta}(\alpha) = P\{\rho_i \le e^{\log(\hat{\rho}_i) - \frac{1}{\sqrt{n}} v_i \hat{t}_{1-\alpha/2}}\} \; and \; \hat{\beta}(1-\alpha) = P\{\rho_i \le e^{\log(\hat{\rho}_i) - \frac{1}{\sqrt{n}} v_i \hat{t}_{\alpha/2}}\}$$

A 100(1-α)% Variance- stabilized Bootstrap-t (VST) calibrated confidence interval for ρ_i, i=1,2 is

$$\left(e^{\log(\hat{\rho}_i) - \frac{1}{\sqrt{n}} v_i \hat{t}_{\hat{\beta}(\alpha)/2}}, e^{\log(\hat{\rho}_i) - \frac{1}{\sqrt{n}} v_i \hat{t}_{\hat{\beta}(1-\alpha)/2}} \right) \quad (6)$$

Where $v_i \hat{t}_{\hat{\beta}(\alpha)/2}$ and $v_i \hat{t}_{\hat{\beta}(1-\alpha)/2}$ are $\alpha/2$ and (1-$\alpha/2$) percentile of the random sample $\theta_{i1}^*, \theta_{i2}^*, \cdots, \theta_{iN}^*, i=1,2$

Bayesian Bootstrap Calibrated Confidence Interval

Here each BB replication generates a posterior probability for each X_{ij}, i=1, j=1,2...n and p_1X_{ij}, i=1, j=1,2...n. One BB replication is generated by drawing n-1 uniform (0, 1) random numbers $r_1, r_2, ... r_{n-1}$, ordering them, and calculating the gaps $w_j = r_{(j)} - r_{(j-1)}$, j=1,2...n where $r_{(0)}=0$ and $r_{(n)}=1$. Then $w_i=(w_{i1}, w_{i2}, \ldots, w_{in})$, i=1,2 is the vector of probabilities attached to the inter-arrival data values (X_{1j}, p_1X_{2j}, j=1,2...n) respectively. Next considering all BB replications gives the BB distribution of the distribution of X_i and thus of any parameter of this distribution we calculate $\mu_{x_i}, i=1,2$ (the mean of X_i), in each BB replication. Let w_{ij} be the probability that $X_i = x_{ij}$ then we calculate $\bar{X}_1^{**} = \sum_{j=1}^{n} w_{1j} x_{1j}$ and $\bar{X}_2^{**} = p_1 \sum_{j=1}^{n} w_{2j} x_{2j}$ and the distribution of the values of $\bar{X}_i^{**}$ overall BB replications is the BB distribution of μ_{X_i}.

Now generating a vector of probabilities $v_i=(v_{i1}, v_{i2}, \ldots, v_{in})$, i=1,2 attached to the service time data values p_0Y_{ij}, i=1,2. j= 1,2...n in a BB replication.

We calculate $\bar{Y}_i^{**} = p_0 \sum_{j=1}^{n} v_{ij} y_{ij}$ for μ_{X_i}.

Thus estimate of Intensity ρ_i be calculated from BB replications as

$$\hat{\rho}_i^{**} = \frac{\bar{Y}_i^{**}}{\bar{X}_i^{**}}, i=1,2.$$

The above BB process can be repeated N times. The N BB estimates $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i=1,2$ are computed from the BB replications. Averaging the N BB estimates, we obtain that

$\hat{\rho}_{BB}(i) = \frac{1}{N}\sum_{j=1}^{N} \hat{\rho}_{ij}^{**}, i=1,2$ is the BB estimate of ρ_i, i=1,2 and the standard deviation of $\hat{\rho}_i$ can be estimated by

$$sd(\hat{\rho}_{BB}(i)) = \left\{\frac{1}{N-1}\sum_{j=1}^{N}(\hat{\rho}_{ij}^{**} - \hat{\rho}_{BB}(i))^2\right\}^{\frac{1}{2}}, i=1,2.$$

Find $\hat{\beta}(\alpha) = P\{\rho_i \le (\hat{\rho}_i - z_{\alpha/2} sd(\hat{\rho}_{BB}(i)))\}$ and $\hat{\beta}(1-\alpha) = P\{\rho_i \le (\hat{\rho}_i + z_{\alpha/2} sd(\hat{\rho}_{BB}(i)))\}$

Applying the asymptotical normality of $\hat{\rho}_i$, i= 1,2, The 100(1-α)% BB calibrated confidence interval for ρ_i, i= 1,2 is

$$\left(\hat{\rho}_i' - z_{\hat{\beta}(\alpha)/2} sd'(\hat{\rho}_{BB}(i)) \;,\; \hat{\rho}_i' + z_{(1-\hat{\beta}(1-\alpha))/2} sd'(\hat{\rho}_{BB}(i))\right) \quad i=1,2 \quad (7)$$

Percentile Bootstrap Calibrated Confidence Interval

Now call $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i=1,2$ the bootstrap distribution of $\hat{\rho}_i$, i = 1,2. Let $\hat{\rho}_i^*(1), \hat{\rho}_i^*(2), \cdots, \hat{\rho}_i^*(N), i=1,2$ be the order statistics of $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*$, i= 1,2.

Compute $\hat{\beta}(\alpha) = P\{\rho_i \le \hat{\rho}_i^*([N(\frac{\alpha}{2})])\}$ and $\hat{\beta}(1-\alpha) = P\{\rho_i \le \hat{\rho}_i^*([N(1-\frac{\alpha}{2})])\}$

Then utilizing the 100(α/2)th and 100(1-α/2)th percentage points of the bootstrap distribution, a 100(1-α)% PB calibrated confidence interval for ρ_i, i= 1,2 are obtained as

$$\left(\hat{\rho}_i^*([N(\frac{\hat{\beta}(\alpha)}{2})]),\; \hat{\rho}_i^*([N(1-\frac{(1-\hat{\beta}(1-\alpha))}{2})])\right) \quad i=1,2 \quad (8)$$

where [x] denotes the greatest integer less than or equal to x.

Bias-corrected and Accelerated Bootstrap Calibrated Confidence Interval

The bootstrap distribution $\hat{\rho}_{i1}^*, \hat{\rho}_{i2}^*, \cdots, \hat{\rho}_{iN}^*, i=1,2$ may be biased, consequently the Percentile Bootstrap confidence interval of intensity method is designed to correct this potential bias of the bootstrap designed.

Set $p_i = \frac{1}{N}\sum_{j=1}^{N} I(\hat{\rho}_{ij}^* < \hat{\rho}_i), i=1,2$ where I(·) is the indicator function.

Define $\hat{z}_i = \phi^{-1}(p_i), i=1,2$, where Φ^{-1} denotes the inverse function of the standard normal distribution Φ. Except for correcting the potential bias of the bootstrap distribution, we can accelerate convergence of bootstrap distribution. Let $(\tilde{X}_i(k), p_0\tilde{Y}_i(k)), i=1, k=1,2...n)$ and $(p_1\tilde{X}_i(k), p_0\tilde{Y}_i(k)),\; i=2, k=1,2,..., n)$ denote the original samples with the k^{th} observation deleted, also $\hat{\rho}_{ik}, i=1,2$ be the estimator of ρ_i, i= 1,2 calculated as

$$\tilde{\rho}_i = \frac{1}{n}\sum_{k=1}^{n}\hat{\rho}_{ik}, i=1,2 \text{ and } \hat{a}_i = \frac{\sum_{k=1}^{n}(\tilde{\rho}_i - \hat{\rho}_{ik})^3}{\left\{6(\sum_{k=1}^{n}(\tilde{\rho}_i - \hat{\rho}_{ik})^2)^{(\frac{3}{2})}\right\}}, i=1,2$$

Where $\hat{z}_i$ and $\hat{a}_i, i=1,2$ are named bias-correction and acceleration respectively.

Also compute

$$\hat{\beta}(\alpha) = P\{\rho_i \le \hat{\rho}_i^*([N\alpha_{i1}])\} \; and \; \hat{\beta}(1-\alpha) = P\{\rho_i \le \hat{\rho}_i^*([N\alpha_{i2}])\}$$

where $\alpha_{i1} = \phi\left\{\hat{z}_i + \frac{(\hat{z}_i - z_{\alpha/2})}{1-\hat{a}_i(\hat{z}_i - z_{\alpha/2})}\right\}$, $\alpha_{i2} = \phi\left\{\hat{z}_i + \frac{(\hat{z}_i + z_{\alpha/2})}{1-\hat{a}_i(\hat{z}_i + z_{\alpha/2})}\right\}, i=1,2$

Thus a 100(1-α)% Bias-corrected and accelerated bootstrap (BCaB) calibrated confidence interval of intensities ρ_i, i= 1,2 are constructed by

$$(\hat{\rho}_i^*([N\alpha'_{i1}]) \;,\; \hat{\rho}_i^*([N\alpha'_{i2}])), i=1,2. \quad (9)$$

where

$$\alpha'_{i1} = \phi\left\{\hat{z}_i' + \frac{(\hat{z}_i' - z_{\hat{\beta}(\alpha)/2})}{1-\hat{a}_i'(\hat{z}_i' - z_{\hat{\beta}(\alpha)/2})}\right\} \quad \alpha'_{i2} = \phi\left\{\hat{z}_i' + \frac{(\hat{z}_i' + z_{(1-\hat{\beta}(1-\alpha))/2})}{1-\hat{a}_i'(\hat{z}_i' + z_{(1-\hat{\beta}(1-\alpha))/2})}\right\}, \quad i=1,2$$

Simulation Study

To evaluate performances of calibrated confidence intervals, numerical simulation study was undertaken. Relative coverage is defined as the ratio of coverage percentage to average length of confidence interval. Larger relative coverage implies the better performances of the corresponding confidence intervals. Here we set a continuous distribution with mean $1/\lambda$ on inter-arrival time of X_1 and X_2 and a continuous distribution with mean $1/\mu_1$ on the service time Y_1 at CPU node and that of $1/\mu_2$ on Y_2 at I/O node. We have considered the values ρ_i <1, i=1, 2 for simulation study from [Table-1]. The intensity parameters ρ_i, i=1, 2 are calculated using [Eq-1]. The different values of λ, μ_1, μ_2, p_o and p_1 are considered as shown in [Table-1].

Table 1- Different levels of intensity parameters considered in the simulation study

		λ=0.1, μ1=1, μ2=2		λ=0.5, μ1=1, μ2=2		λ=0.9, μ1=1, μ2=2	
po	p1	ρ1	ρ2	ρ1	ρ2	ρ1	ρ2
0.1	0.9	1	0.45	5	2.25	9	4.05
0.2	0.8	0.5	0.2	2.5	1	4.5	1.8
0.3	0.7	0.33	0.12	1.67	0.58	3	1.05
0.4	0.6	0.25	0.08	1.25	0.38	2.25	0.68
0.5	0.5	0.2	0.05	1	0.25	1.8	0.45
0.6	0.4	0.17	0.03	0.83	0.17	1.5	0.3
0.7	0.3	0.14	0.02	0.71	0.11	1.29	0.19
0.8	0.2	0.13	0.01	0.63	0.06	1.13	0.11
0.9	0.1	0.11	0.01	0.56	0.03	1	0.05
		λ=0.1, μ1=2, μ2=1		λ=0.5,μ1=2,μ2=1		λ=0.9, μ1=2, μ2=1	
po	p1	ρ1	ρ2	ρ1	ρ2	ρ1	ρ2
0.1	0.9	0.5	0.9	2.5	4.5	4.5	8.1
0.2	0.8	0.25	0.4	1.25	2	2.25	3.6
0.3	0.7	0.17	0.23	0.83	1.17	1.5	2.1
0.4	0.6	0.13	0.15	0.63	0.75	1.13	1.35
0.5	0.5	0.1	0.1	0.5	0.5	0.9	0.9
0.6	0.4	0.08	0.07	0.42	0.33	0.75	0.6
0.7	0.3	0.07	0.04	0.36	0.21	0.64	0.39
0.8	0.2	0.06	0.03	0.31	0.13	0.56	0.23
0.9	0.1	0.06	0.01	0.28	0.06	0.5	0.1

For each level of ρ_1 random samples of inter-arrival times and service times (X_{1j}, p_0Y_{1j}, j= 1,2...n) are drawn from (X_1,Y_1) respectively. Also for each level of ρ_2 random samples of inter-arrival times and service times (p_1X_{2i}, p_0Y_{2i}, j= 1,2...n) are drawn from (X_2,Y_2) respectively. Next N=1000 bootstrap resamples each of size n = 10 and 29 are drawn from the original samples, as well as N=1000 BB replications are simulated for the original samples. According to [Eq -2] to [Eq-9], we obtain 90% calibrated confidence intervals for intensities ρ_i, i=1, 2. The above simulation process is replicated N=1000 times and we compute coverage percentages, average lengths and relative coverage of the above mentioned calibrated confidence intervals. We utilize a PC Dual Core and apply Matlab®7.0.1 to accomplish all simulations.

Here C.V. represents coefficient of variation corresponding to the inter-arrival/service time distribution. M represents exponential distribution, E_4 a 4-stage Erlang distribution, H_4^{Pe} a 4-stage hyper-exponential distribution and H_4^{P0} a 4-stage hypo-exponential distribution. Simulated results of $\hat{\beta}(\alpha)$, $\hat{\beta}(1-\alpha)$, coverage percentage, average lengths and relative coverage for intensities ρ_i, i=1, 2 of a two stage open queueing network models (presented in [Table -2]) for 90% calibrated confidence intervals with short run are shown in [Table-3], [Table-4], [Table-5], [Table-6].

Table 2- Different queueing network models simulated for study

Queueing Networks type	Models simulated	C.V. of interarrival time for CPU node	C.V. of interarrival time for I/O node	C.V. of service time for CPU node	C.V. of service time for I/O node
M/G/1 to G/M/1	$M/E_4/1$ to $E_4/M/1$	1	½	½	1
	$M/H_4^{Pe}/1$ to $H_4^{Pe}/M/1$	1	>1	>1	1
G/G/1 to G/G/1	$E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$	½	>1	>1	½
	$E_4/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$	½	<1	<1	½

According to the simulation results shown in [Tables 3-6], we find that average lengths are decreasing but both coverage percentages and relative coverage's are increasing with sample size n. Also we observe that the coverage percentage can approaches to 90% when n increases to 29.

From [Table-7], we observe that under M/G/1 to G/M/1 model the calibrated confidence intervals with inter-arrival distribution and service time distribution of small CV (<1) have greater relative coverage than those of large CV (>1) for intensities ρ_1 and ρ_2. The estimation approaches Variance-Stabilized Bootstrap-t (VST), Bootstrap-t and Bias-corrected and accelerated bootstrap (BCaB) calibrated confidence interval has the greatest relative coverage. Also the calibrated confidence intervals of model $M/E_4/1$ to $E_4/M/1$shows the greatest relative coverage for ρ_1 and ρ_2. Similarly under G/G/1 to G/G/1 models the calibrated confidence interval with inter-arrival distribution and service time distribution of large CV(>1) have greatest relative coverage than those of small CV(<1) for intensities ρ_1 and ρ_2. The estimation approaches BCaB, VST, PB, Bootstrap-t and BB has the greatest relative coverage. Also the calibrated confidence intervals of model $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$ show the greatest relative coverage for ρ_1 and ρ_2. Further we observe that average lengths are decreasing and relative coverage increasing with n increases for ρ_1 and ρ_2. It is important to point out that, some poor coverage percentage of above confidence intervals with respect to the nominal level 90% may be due to small sample size n.

Conclusions

This paper provides the calibrated confidence intervals for intensities ρ_1 and ρ_2 of two stage open queueing network with feedback. The relative coverage is adopted to understand, compare and assess performance of the resulted confidence intervals. The simulation results imply that VST, Boot-t and BCaB method has the best performance for M/G/1 to G/M/1 and under G/G/1 to G/G/1 the estimation approach PB, VST, Boot-t, BCaB and BB out performs. The above mentioned approaches are easily applied to practical queueing network such as all types of open, closed, mixed queueing networks as well as cyclic, retrial queueing models.

Table 3- Queueing network model: $M/E_4/1$ to $E_4/M/1$.

Intensity Parameters	Estimation Approaches	n=10		n=29		Coverage Percentages		Average Lengths		Relative Coverage	
		$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	n=10	n=29	n=10	n=29	n=10	n=29
po=0.2 p1=0.8 ρ1=0.5 and ρ2=0.2	CAN1	0.033	0.904	0.036	0.92	0.829	0.875	0.582	0.337	1.425	2.598
	CAN2	0.021	0.879	0.02	0.91	0.79	0.856	0.22	0.139	3.592	6.164
	tCAN1	0.012	0.946	0.023	0.933	0.919	0.91	1.031	0.405	0.892	2.248
	tCAN2	0.013	0.894	0.018	0.916	0.822	0.876	0.271	0.147	3.028	5.948
	Boott1	0.01	0.875	0.02	0.898	0.808	0.855	0.563	0.327	1.436	**2.613**
	Boott2	0.015	0.851	0.015	0.898	0.753	0.863	0.188	0.13	4.005	6.624
	VST1	0.075	0.919	0.054	0.931	0.774	0.847	0.513	0.324	**1.508**	2.611
	VST2	0.083	0.931	0.053	0.943	0.791	0.87	0.212	0.136	3.738	6.405
	SB1	0.021	0.929	0.032	0.931	0.891	0.901	0.803	0.372	1.109	2.424
	SB2	0.024	0.874	0.019	0.909	0.776	0.856	0.212	0.139	3.665	6.168
	BB1	0.037	0.907	0.035	0.919	0.831	0.878	0.595	0.338	1.397	2.6
	BB2	0.03	0.868	0.022	0.906	0.751	0.843	0.192	0.133	3.909	6.341
	PB1	0.109	0.957	0.077	0.95	0.765	0.845	0.676	0.342	1.131	2.471
	PB2	0.051	0.894	0.033	0.92	0.769	0.861	0.183	0.127	4.197	6.799
	BCaB1	0.097	0.949	0.073	0.951	0.773	0.843	0.625	0.34	1.237	2.478
	BCaB2	0.055	0.892	0.031	0.917	0.761	0.857	0.178	0.125	**4.273**	**6.85**
po=0.6 p1=0.4 ρ1=0.83 and ρ2=0.17	CAN1	0.046	0.902	0.048	0.931	0.833	0.872	0.911	0.55	0.914	1.586
	CAN2	0.022	0.867	0.026	0.892	0.777	0.834	0.179	0.108	4.335	7.687
	tCAN1	0.02	0.94	0.036	0.94	0.939	0.916	1.54	0.642	0.61	1.427
	tCAN2	0.016	0.888	0.022	0.899	0.809	0.855	0.218	0.116	3.717	7.371
	Boott1	0.02	0.88	0.027	0.918	0.816	0.875	0.905	0.56	0.902	1.564
	Boott2	0.018	0.846	0.019	0.87	0.737	0.81	0.153	0.101	4.811	8.052
	VST1	0.093	0.92	0.068	0.941	0.772	0.849	0.803	0.525	**0.962**	**1.618**
	VST2	0.07	0.928	0.053	0.94	0.785	0.859	0.187	0.113	4.197	7.634
	SB1	0.029	0.936	0.038	0.936	0.914	0.895	1.279	0.609	0.715	1.469
	SB2	0.022	0.864	0.026	0.888	0.773	0.832	0.176	0.107	4.381	7.755
	BB1	0.047	0.91	0.047	0.93	0.846	0.865	0.947	0.549	0.893	1.576
	BB2	0.028	0.853	0.028	0.884	0.743	0.816	0.158	0.103	4.698	7.892
	PB1	0.133	0.949	0.093	0.959	0.734	0.83	1.032	0.58	0.712	1.432
	PB2	0.041	0.888	0.036	0.907	0.772	0.841	0.154	0.101	4.997	**8.31**
	BCaB1	0.125	0.945	0.088	0.953	0.733	0.833	0.977	0.554	0.75	1.503
	BCaB2	0.039	0.887	0.037	0.911	0.762	0.836	0.152	0.101	**5.006**	**8.31**
po=0.1 p1=0.9 ρ1=0.5 and ρ2=0.9	CAN1	0.034	0.891	0.038	0.916	0.843	0.883	0.571	0.335	1.475	2.632
	CAN2	0.015	0.865	0.029	0.902	0.777	0.853	1.02	0.592	0.762	1.441
	tCAN1	0.017	0.946	0.03	0.933	0.938	0.923	0.991	0.395	0.947	2.337
	tCAN2	0.012	0.886	0.028	0.907	0.806	0.86	1.231	0.619	0.655	1.389
	Boott1	0.015	0.873	0.026	0.894	0.8	0.855	0.552	0.321	1.45	**2.667**
	Boott2	0.013	0.833	0.023	0.881	0.744	0.832	0.831	0.549	0.895	1.517
	VST1	0.069	0.914	0.055	0.929	0.783	0.859	0.524	0.326	**1.495**	2.639
	VST2	0.078	0.925	0.061	0.943	0.783	0.862	0.973	0.597	0.805	1.445
	SB1	0.025	0.922	0.032	0.922	0.906	0.903	0.782	0.369	1.159	2.449
	SB2	0.017	0.864	0.032	0.905	0.77	0.851	0.989	0.585	0.778	1.455
	BB1	0.033	0.901	0.041	0.916	0.848	0.878	0.605	0.331	1.403	2.657
	BB2	0.027	0.851	0.034	0.902	0.741	0.842	0.862	0.564	0.86	1.492
	PB1	0.116	0.957	0.081	0.947	0.746	0.83	0.681	0.339	1.096	2.447
	PB2	0.04	0.884	0.046	0.915	0.762	0.839	0.831	0.547	0.917	**1.535**
	BCaB1	0.111	0.955	0.08	0.946	0.74	0.827	0.643	0.334	1.151	2.479
	BCaB2	0.042	0.885	0.046	0.918	0.759	0.838	0.814	0.546	**0.932**	1.533
po=0.4 p1=0.6 ρ1=0.63 and ρ2=0.75	CAN1	0.038	0.9	0.043	0.947	0.863	0.874	0.706	0.441	1.223	1.983
	CAN2	0.023	0.871	0.03	0.905	0.78	0.839	0.795	0.489	0.981	1.716
	tCAN1	0.018	0.933	0.024	0.956	0.942	0.938	1.163	0.537	0.81	1.746
	tCAN2	0.013	0.891	0.026	0.91	0.811	0.844	1.004	0.52	0.808	1.623
	Boott1	0.017	0.878	0.018	0.925	0.842	0.883	0.684	0.447	1.231	1.974
	Boott2	0.013	0.835	0.024	0.893	0.732	0.831	0.683	0.463	1.071	1.795
	VST1	0.071	0.916	0.062	0.957	0.816	0.86	0.644	0.42	**1.267**	**2.046**
	VST2	0.093	0.931	0.059	0.941	0.781	0.854	0.75	0.495	1.041	1.726
	SB1	0.028	0.919	0.031	0.953	0.917	0.913	0.937	0.496	0.979	1.841
	SB2	0.021	0.869	0.031	0.905	0.78	0.84	0.795	0.485	0.981	1.731
	BB1	0.038	0.9	0.045	0.948	0.87	0.872	0.727	0.438	1.196	1.989
	BB2	0.032	0.853	0.033	0.901	0.741	0.825	0.688	0.467	1.078	1.767
	PB1	0.114	0.943	0.09	0.966	0.787	0.835	0.763	0.459	1.031	1.819
	PB2	0.05	0.895	0.042	0.921	0.764	0.838	0.683	0.464	1.119	1.806
	BCaB1	0.105	0.938	0.085	0.962	0.788	0.84	0.727	0.44	1.083	1.908
	BCaB2	0.046	0.889	0.046	0.918	0.756	0.825	0.668	0.452	**1.131**	**1.825**

Table 3- Continue…

Intensity Parameters	Estimation Approaches	*n*=10 $\hat{\beta}(\alpha)$	*n*=10 $\hat{\beta}(1-\alpha)$	*n*=29 $\hat{\beta}(\alpha)$	*n*=29 $\hat{\beta}(1-\alpha)$	Coverage Percentages *n*=10	Coverage Percentages *n*=29	Average Lengths *n*=10	Average Lengths *n*=29	Relative Coverage *n*=10	Relative Coverage *n*=29
ρo=0.9 ρ1=0.1 ρ1=0.5 and ρ2=0.1	CAN1	0.043	0.904	0.045	0.925	0.834	0.884	0.56	0.331	1.49	2.67
	CAN2	0.017	0.852	0.034	0.904	0.753	0.827	0.107	0.064	7.046	12.905
	tCAN1	0.016	0.943	0.032	0.938	0.948	0.922	0.97	0.393	0.977	2.343
	tCAN2	0.011	0.871	0.031	0.909	0.784	0.84	0.133	0.068	5.902	12.429
	Boott1	0.015	0.877	0.018	0.913	0.82	0.885	0.552	0.345	1.486	2.563
	Boott2	0.013	0.822	0.027	0.886	0.72	0.818	0.089	0.06	8.122	13.588
	VST1	0.093	0.919	0.066	0.937	0.774	0.855	0.485	0.317	**1.596**	**2.698**
	VST2	0.084	0.921	0.077	0.936	0.772	0.829	0.102	0.061	7.564	13.521
	SB1	0.029	0.93	0.039	0.936	0.914	0.908	0.763	0.368	1.197	2.469
	SB2	0.021	0.85	0.036	0.903	0.741	0.824	0.103	0.063	7.215	13.037
	BB1	0.044	0.907	0.046	0.924	0.842	0.883	0.578	0.329	1.458	2.687
	BB2	0.029	0.842	0.041	0.901	0.723	0.809	0.092	0.06	7.9	13.383
	PB1	0.133	0.957	0.095	0.954	0.777	0.831	0.659	0.341	1.179	2.435
	BCaB1	0.123	0.952	0.090	0.950	0.768	0.826	0.614	0.331	1.251	2.495
	BCaB2	0.047	0.874	0.057	0.914	0.738	0.819	0.085	0.058	**8.666**	**14.083**

Table 4- Queueing network model: $M/H_4^{Pe}/1$ to $H_4^{Pe}/M/1$.

Intensity Parameters	Estimation Approaches	*n*=10 $\hat{\beta}(\alpha)$	*n*=10 $\hat{\beta}(1-\alpha)$	*n*=29 $\hat{\beta}(\alpha)$	*n*=29 $\hat{\beta}(1-\alpha)$	Coverage Percentages *n*=10	Coverage Percentages *n*=29	Average Lengths *n*=10	Average Lengths *n*=29	Relative Coverage *n*=10	Relative Coverage *n*=29
ρo=0.2 ρ1=0.8 ρ1=0.5 and ρ2=0.2	CAN1	0.033	0.895	0.039	0.912	0.828	0.878	0.592	0.340	1.399	2.581
	CAN2	0.017	0.867	0.023	0.905	0.788	0.846	0.229	0.141	3.434	6.004
	tCAN1	0.014	0.937	0.024	0.927	0.929	0.912	1.011	0.413	0.919	2.210
	tCAN2	0.009	0.880	0.019	0.907	0.814	0.859	0.295	0.151	2.755	5.690
	Boott1	0.013	0.874	0.020	0.893	0.814	0.861	0.570	0.335	1.429	2.570
	Boott2	0.012	0.838	0.018	0.886	0.761	0.846	0.192	0.130	3.959	6.499
	VST1	0.081	0.921	0.064	0.931	0.784	0.864	0.526	0.327	**1.490**	**2.644**
	VST2	0.063	0.921	0.057	0.941	0.822	0.861	0.235	0.139	3.491	6.211
	SB1	0.020	0.921	0.030	0.923	0.890	0.903	0.814	0.383	1.093	2.359
	SB2	0.020	0.865	0.024	0.905	0.787	0.846	0.225	0.140	3.496	6.023
	BB1	0.035	0.900	0.038	0.911	0.833	0.877	0.613	0.341	1.360	2.571
	BB2	0.023	0.859	0.026	0.901	0.767	0.838	0.205	0.135	3.737	6.213
	PB1	0.122	0.953	0.077	0.950	0.779	0.859	0.669	0.355	1.164	2.418
	PB2	0.035	0.889	0.046	0.917	0.794	0.846	0.195	0.127	4.068	**6.684**
	BCaB1	0.113	0.947	0.077	0.943	0.771	0.855	0.627	0.340	1.229	2.512
	BCaB2	0.033	0.887	0.046	0.923	0.786	0.849	0.193	0.128	**4.081**	6.631
ρo=0.6 ρ1=0.4 ρ1=0.83 and ρ2=0.17	CAN1	0.030	0.884	0.041	0.918	0.829	0.855	0.964	0.564	0.860	**1.515**
	CAN2	0.022	0.858	0.020	0.906	0.753	0.844	0.181	0.120	4.154	7.018
	tCAN1	0.007	0.935	0.024	0.938	0.924	0.902	1.832	0.697	0.504	1.293
	tCAN2	0.014	0.879	0.020	0.917	0.791	0.869	0.228	0.128	3.472	6.809
	Boott1	0.009	0.858	0.018	0.895	0.818	0.841	0.910	0.561	0.899	1.499
	Boott2	0.018	0.829	0.017	0.888	0.697	0.836	0.151	0.110	4.603	7.606
	VST1	0.076	0.911	0.058	0.940	0.799	0.843	0.862	0.560	**0.927**	1.505
	VST2	0.067	0.925	0.059	0.945	0.774	0.849	0.191	0.116	4.044	7.333
	SB1	0.016	0.918	0.032	0.929	0.894	0.888	1.376	0.634	0.649	1.401
	SB2	0.020	0.858	0.022	0.904	0.759	0.842	0.184	0.118	4.124	7.109
	BB1	0.031	0.893	0.037	0.918	0.838	0.859	1.010	0.572	0.830	1.502
	BB2	0.028	0.846	0.027	0.900	0.728	0.825	0.162	0.112	4.492	7.382
	PB1	0.111	0.949	0.078	0.953	0.795	0.837	1.085	0.593	0.733	1.413
	PB2	0.043	0.887	0.045	0.929	0.758	0.833	0.158	0.110	**4.812**	7.595
	BCaB1	0.102	0.940	0.074	0.950	0.803	0.832	1.004	0.579	0.800	1.436
	BCaB2	0.042	0.884	0.047	0.929	0.741	0.830	0.155	0.109	4.786	**7.630**
ρo=0.1 ρ1=0.9 ρ1=0.5 and ρ2=0.9	CAN1	0.037	0.892	0.042	0.920	0.827	0.874	0.593	0.346	1.395	2.525
	CAN2	0.022	0.872	0.026	0.900	0.786	0.851	1.014	0.620	0.775	1.371
	tCAN1	0.012	0.929	0.030	0.940	0.923	0.914	1.040	0.417	0.888	2.189
	tCAN2	0.012	0.886	0.021	0.905	0.810	0.869	1.295	0.669	0.626	1.299
	Boott1	0.011	0.867	0.018	0.896	0.810	0.852	0.576	0.345	1.406	2.470
	Boott2	0.013	0.842	0.021	0.889	0.749	0.846	0.874	0.581	0.857	1.455
	VST1	0.083	0.917	0.068	0.939	0.777	0.843	0.532	0.331	**1.461**	**2.545**
	VST2	0.071	0.922	0.059	0.941	0.790	0.862	1.034	0.620	0.764	1.390
	SB1	0.022	0.910	0.037	0.933	0.890	0.897	0.808	0.385	1.102	2.331
	SB2	0.021	0.869	0.025	0.901	0.783	0.852	1.020	0.625	0.767	1.363

Table 4- Continue...

Intensity Parameters	Estimation Approaches	n=10		n=29		Coverage Percentages		Average Lengths		Relative Coverage	
		$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	n=10	n=29	n=10	n=29	n=10	n=29
po=0.1 p1=0.9 ρ1=0.5 and ρ2=0.9	BB1	0.036	0.893	0.041	0.924	0.836	0.881	0.616	0.350	1.357	2.516
	BB2	0.027	0.857	0.028	0.898	0.756	0.839	0.907	0.598	0.834	1.402
	PB1	0.117	0.942	0.083	0.952	0.764	0.835	0.643	0.359	1.188	2.324
	PB2	0.045	0.895	0.055	0.914	0.782	0.842	0.878	0.553	0.891	**1.522**
	BCaB1	0.105	0.938	0.077	0.953	0.768	0.835	0.617	0.358	1.244	2.332
	BCaB2	0.045	0.893	0.052	0.918	0.785	0.851	0.862	0.560	**0.911**	1.519
po=0.4 p1=0.6 ρ1=0.63 and ρ2=0.75	CAN1	0.041	0.902	0.029	0.928	0.832	0.882	0.730	0.451	1.140	1.957
	CAN2	0.022	0.876	0.019	0.893	0.786	0.839	0.846	0.527	0.929	1.592
	tCAN1	0.017	0.942	0.022	0.939	0.929	0.920	1.254	0.525	0.741	1.753
	tCAN2	0.011	0.895	0.014	0.904	0.817	0.854	1.101	0.581	0.742	1.470
	Boott1	0.016	0.885	0.018	0.912	0.825	0.889	0.727	0.437	1.134	2.037
	Boott2	0.014	0.854	0.011	0.878	0.780	0.846	0.737	0.499	1.058	1.697
	VST1	0.087	0.922	0.056	0.940	0.775	0.877	0.648	0.420	**1.195**	**2.089**
	VST2	0.070	0.933	0.055	0.941	0.823	0.871	0.872	0.520	0.944	1.677
	SB1	0.025	0.928	0.025	0.933	0.894	0.907	1.013	0.490	0.883	1.850
	SB2	0.026	0.875	0.020	0.892	0.780	0.837	0.828	0.523	0.942	1.600
	BB1	0.042	0.907	0.031	0.925	0.834	0.875	0.755	0.442	1.104	1.978
	BB2	0.034	0.868	0.020	0.888	0.761	0.829	0.738	0.508	1.032	1.632
	PB1	0.127	0.953	0.075	0.958	0.755	0.877	0.837	0.454	0.902	1.932
	PB2	0.049	0.902	0.036	0.921	0.795	0.856	0.734	0.489	1.083	1.749
	BCaB1	0.119	0.951	0.074	0.954	0.754	0.879	0.799	0.439	0.943	2.000
	BCaB2	0.050	0.895	0.039	0.917	0.783	0.843	0.710	0.477	**1.103**	**1.769**
po=0.9 p1=0.1 ρ1=0.5 and ρ2=0.1	CAN1	0.039	0.895	0.038	0.910	0.842	0.872	0.582	0.339	1.446	2.576
	CAN2	0.019	0.860	0.023	0.890	0.781	0.838	0.114	0.069	6.861	12.124
	tCAN1	0.018	0.944	0.028	0.932	0.937	0.916	1.005	0.406	0.932	2.258
	tCAN2	0.010	0.885	0.019	0.899	0.824	0.856	0.149	0.075	5.536	11.439
	Boott1	0.011	0.857	0.023	0.900	0.796	0.864	0.552	0.336	1.442	2.573
	Boott2	0.016	0.836	0.015	0.879	0.758	0.837	0.095	0.066	7.998	12.743
	VST1	0.084	0.918	0.065	0.932	0.788	0.851	0.523	0.325	**1.506**	**2.621**
	VST2	0.067	0.931	0.059	0.936	0.821	0.856	0.119	0.069	6.924	12.459
	SB1	0.024	0.921	0.033	0.925	0.903	0.899	0.807	0.377	1.120	2.386
	SB2	0.020	0.860	0.026	0.889	0.778	0.834	0.114	0.068	6.853	12.272
	BB1	0.045	0.890	0.039	0.914	0.835	0.877	0.582	0.340	1.434	2.581
	BB2	0.022	0.853	0.026	0.888	0.755	0.827	0.104	0.066	7.269	12.475
	PB1	0.116	0.965	0.084	0.947	0.787	0.846	0.739	0.345	1.065	2.455
	PB2	0.042	0.893	0.047	0.909	0.797	0.835	0.098	0.062	8.104	**13.438**
	BCaB1	0.104	0.955	0.080	0.945	0.796	0.845	0.663	0.339	1.200	2.489
	BCaB2	0.043	0.892	0.045	0.906	0.790	0.825	0.096	0.061	**8.192**	13.437

Table 5- Queueing network model: $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/E_4/1$.

Intensity Parameters	Estimation Approaches	n=10		n=29		Coverage Percentages		Average Lengths		Relative Coverage	
		$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	n=10	n=29	n=10	n=29	n=10	n=29
po=0.2 p1=0.8 ρ1=0.5 and ρ2=0.2	CAN1	0.027	0.900	0.041	0.920	0.838	0.873	0.391	0.227	2.146	3.848
	CAN2	0.044	0.920	0.034	0.927	0.846	0.898	0.150	0.094	5.641	9.551
	tCAN1	0.021	0.915	0.034	0.933	0.888	0.897	0.468	0.247	1.899	3.628
	tCAN2	0.022	0.938	0.030	0.933	0.920	0.916	0.197	0.101	4.666	9.083
	Boott1	0.023	0.871	0.028	0.905	0.815	0.862	0.339	0.221	2.407	3.898
	Boott2	0.029	0.893	0.029	0.916	0.809	0.876	0.139	0.090	5.841	9.719
	VST1	0.070	0.925	0.075	0.944	0.826	0.853	0.354	0.215	2.335	**3.971**
	VST2	0.092	0.936	0.055	0.941	0.788	0.876	0.131	0.090	**6.009**	9.724
	SB1	0.032	0.898	0.040	0.921	0.832	0.870	0.378	0.227	2.200	3.829
	SB2	0.043	0.919	0.035	0.929	0.849	0.901	0.152	0.094	5.580	9.542
	BB1	0.040	0.882	0.045	0.913	0.794	0.851	0.334	0.215	2.377	3.954
	BB2	0.053	0.910	0.038	0.925	0.804	0.880	0.134	0.090	6.002	**9.797**
	PB1	0.068	0.920	0.066	0.942	0.822	0.857	0.340	0.220	**2.418**	3.894
	PB2	0.104	0.941	0.059	0.942	0.792	0.875	0.139	0.089	5.700	9.795
	BCaB1	0.065	0.920	0.068	0.946	0.814	0.857	0.341	0.222	2.390	3.860
	BCaB2	0.101	0.938	0.058	0.941	0.794	0.870	0.137	0.089	5.816	9.766
po=0.6 p1=0.4 ρ1=0.83 and ρ2=0.17	CAN1	0.027	0.900	0.041	0.920	0.838	0.873	0.651	0.378	1.287	2.309
	CAN2	0.044	0.920	0.034	0.927	0.846	0.898	0.125	0.078	6.770	11.461
	tCAN1	0.021	0.915	0.034	0.933	0.888	0.897	0.779	0.412	1.139	2.177
	tCAN2	0.022	0.938	0.030	0.933	0.920	0.916	0.164	0.084	5.599	10.900

Table 5- Continue...

Intensity Parameters	Estimation Approaches	n=10 $\hat{\beta}(\alpha)$	n=10 $\hat{\beta}(1-\alpha)$	n=29 $\hat{\beta}(\alpha)$	n=29 $\hat{\beta}(1-\alpha)$	Coverage Percentages n=10	Coverage Percentages n=29	Average Lengths n=10	Average Lengths n=29	Relative Coverage n=10	Relative Coverage n=29
po=0.6 p1=0.4 ρ1=0.83 and ρ2=0.17	Boott1	0.023	0.871	0.028	0.905	0.815	0.862	0.564	0.369	1.444	2.339
	Boott2	0.029	0.893	0.029	0.916	0.809	0.876	0.115	0.075	7.009	11.662
	VST1	0.070	0.925	0.075	0.944	0.826	0.853	0.590	0.358	1.401	**2.383**
	VST2	0.092	0.936	0.055	0.941	0.788	0.876	0.109	0.075	**7.211**	11.668
	SB1	0.032	0.898	0.040	0.921	0.832	0.870	0.630	0.379	1.320	2.297
	SB2	0.043	0.919	0.035	0.929	0.849	0.901	0.127	0.079	6.695	11.450
	BB1	0.040	0.882	0.045	0.913	0.794	0.851	0.557	0.359	1.426	2.372
	BB2	0.053	0.910	0.038	0.925	0.804	0.880	0.112	0.075	7.202	**11.757**
	PB1	0.068	0.920	0.066	0.942	0.822	0.857	0.567	0.367	**1.451**	2.337
	PB2	0.104	0.941	0.059	0.942	0.792	0.875	0.116	0.074	6.840	11.753
	BCaB1	0.065	0.920	0.068	0.946	0.814	0.857	0.568	0.370	1.434	2.316
	BCaB2	0.101	0.938	0.058	0.941	0.794	0.870	0.114	0.074	6.979	11.720
po=0.1 p1=0.9 ρ1=0.5 and ρ2=0.9	CAN1	0.023	0.908	0.033	0.922	0.815	0.883	0.400	0.235	2.038	3.755
	CAN2	0.042	0.908	0.039	0.919	0.843	0.873	0.676	0.408	1.246	2.140
	tCAN1	0.015	0.925	0.027	0.929	0.859	0.894	0.494	0.254	1.738	3.520
	tCAN2	0.034	0.929	0.035	0.929	0.891	0.894	0.828	0.439	1.076	2.035
	Boott1	0.021	0.889	0.023	0.906	0.808	0.858	0.350	0.227	**2.306**	3.783
	Boott2	0.034	0.887	0.032	0.905	0.806	0.872	0.617	0.391	1.307	2.233
	VST1	0.066	0.942	0.052	0.942	0.810	0.857	0.364	0.231	2.225	3.713
	VST2	0.074	0.930	0.066	0.936	0.812	0.865	0.630	0.386	1.290	2.238
	SB1	0.029	0.907	0.031	0.922	0.802	0.883	0.384	0.237	2.091	3.728
	SB2	0.041	0.911	0.040	0.919	0.851	0.872	0.694	0.408	1.227	2.140
	BB1	0.035	0.896	0.038	0.916	0.773	0.861	0.345	0.222	2.242	3.877
	BB2	0.046	0.902	0.043	0.919	0.822	0.862	0.619	0.391	**1.328**	2.202
	PB1	0.056	0.929	0.051	0.933	0.803	0.856	0.353	0.222	2.275	3.862
	PB2	0.077	0.931	0.068	0.941	0.815	0.861	0.639	0.393	1.275	2.192
	BCaB1	0.053	0.931	0.051	0.933	0.804	0.860	0.356	0.221	2.256	**3.883**
	BCaB2	0.075	0.931	0.071	0.939	0.811	0.865	0.638	0.386	1.271	**2.238**
po=0.4 p1=0.6 ρ1=0.63 and ρ2=0.75	CAN1	0.034	0.911	0.036	0.907	0.835	0.861	0.480	0.283	1.740	3.047
	CAN2	0.041	0.916	0.042	0.925	0.836	0.869	0.574	0.342	1.456	2.542
	tCAN1	0.017	0.929	0.032	0.914	0.892	0.880	0.618	0.301	1.442	2.925
	tCAN2	0.025	0.944	0.034	0.927	0.912	0.883	0.752	0.367	1.213	2.405
	Boott1	0.022	0.882	0.031	0.891	0.815	0.833	0.433	0.267	1.884	3.125
	Boott2	0.029	0.886	0.031	0.908	0.796	0.854	0.520	0.330	1.531	2.588
	VST1	0.069	0.932	0.060	0.925	0.817	0.842	0.446	0.272	1.831	3.092
	VST2	0.086	0.936	0.057	0.935	0.792	0.850	0.509	0.332	1.557	2.560
	SB1	0.036	0.910	0.036	0.907	0.832	0.859	0.473	0.282	1.759	3.046
	SB2	0.042	0.918	0.042	0.923	0.836	0.869	0.583	0.341	1.434	2.547
	BB1	0.048	0.896	0.039	0.903	0.780	0.842	0.414	0.270	**1.886**	3.115
	BB2	0.055	0.902	0.044	0.918	0.794	0.850	0.500	0.326	**1.587**	**2.607**
	PB1	0.062	0.930	0.057	0.922	0.824	0.849	0.441	0.265	1.867	**3.199**
	PB2	0.096	0.945	0.065	0.940	0.791	0.854	0.547	0.329	1.447	2.594
	BCaB1	0.061	0.926	0.057	0.920	0.819	0.838	0.435	0.264	1.883	3.180
	BCaB2	0.098	0.941	0.064	0.941	0.780	0.857	0.531	0.330	1.468	2.597
po=0.9 p1=0.1 ρ1=0.5 and ρ2=0.1	CAN1	0.022	0.899	0.035	0.906	0.836	0.868	0.401	0.226	2.083	3.848
	CAN2	0.038	0.911	0.037	0.931	0.831	0.871	0.076	0.047	10.927	18.612
	tCAN1	0.010	0.917	0.032	0.912	0.877	0.884	0.526	0.239	1.666	3.705
	tCAN2	0.029	0.930	0.030	0.940	0.889	0.898	0.094	0.051	9.468	17.548
	Boott1	0.014	0.876	0.028	0.896	0.820	0.853	0.357	0.217	2.297	3.936
	Boott2	0.031	0.887	0.027	0.918	0.796	0.858	0.069	0.046	11.600	18.843
	VST1	0.067	0.929	0.062	0.928	0.817	0.863	0.360	0.216	2.272	3.988
	VST2	0.081	0.930	0.061	0.948	0.798	0.857	0.068	0.045	**11.768**	19.151
	SB1	0.023	0.896	0.040	0.904	0.835	0.859	0.396	0.220	2.110	3.908
	SB2	0.039	0.913	0.036	0.930	0.834	0.865	0.077	0.047	10.782	18.394
	BB1	0.031	0.885	0.042	0.904	0.804	0.854	0.349	0.213	2.302	4.002
	BB2	0.048	0.905	0.044	0.925	0.796	0.848	0.068	0.044	11.685	**19.330**
	PB1	0.049	0.919	0.061	0.923	0.824	0.864	0.353	0.210	2.337	**4.118**
	PB2	0.087	0.935	0.061	0.953	0.803	0.859	0.070	0.046	11.395	18.596
	BCaB1	0.051	0.915	0.060	0.924	0.812	0.862	0.346	0.211	**2.347**	4.094
	BCaB2	0.085	0.932	0.060	0.951	0.792	0.858	0.069	0.046	11.403	18.736

Table 6- Queueing network model: $E4/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$.

Intensity Parameters	Estimation Approaches	*n*=10		*n*=29		Coverage Percentages		Average Lengths		Relative Coverage	
		$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	$\hat{\beta}(\alpha)$	$\hat{\beta}(1-\alpha)$	*n*=10	*n*=29	*n*=10	*n*=29	*n*=10	*n*=29
po=0.2 p1=0.8 ρ1=0.5 and ρ2=0.2	CAN1	0.027	0.891	0.035	0.919	0.833	0.843	0.387	0.228	2.150	3.691
	CAN2	0.037	0.888	0.033	0.927	0.820	0.878	0.146	0.095	5.601	9.240
	tCAN1	0.015	0.904	0.027	0.921	0.870	0.862	0.488	0.246	1.782	3.505
	tCAN2	0.025	0.909	0.032	0.934	0.884	0.902	0.183	0.101	4.821	8.942
	Boott1	0.020	0.859	0.026	0.902	0.784	0.848	0.338	0.219	2.321	3.873
	Boott2	0.025	0.874	0.029	0.907	0.778	0.860	0.136	0.089	5.729	**9.694**
	VST1	0.052	0.906	0.061	0.930	0.812	0.837	0.373	0.215	2.179	3.886
	VST2	0.078	0.910	0.052	0.946	0.777	0.877	0.132	0.093	5.887	9.459
	SB1	0.029	0.884	0.036	0.917	0.827	0.838	0.379	0.226	2.184	3.705
	SB2	0.039	0.892	0.035	0.924	0.827	0.871	0.148	0.094	5.576	9.261
	BB1	0.033	0.874	0.042	0.913	0.793	0.823	0.342	0.214	2.316	3.842
	BB2	0.047	0.884	0.041	0.920	0.772	0.854	0.132	0.088	5.858	9.656
	PB1	0.047	0.911	0.058	0.929	0.824	0.840	0.349	0.213	2.362	3.953
	PB2	0.086	0.917	0.056	0.946	0.774	0.870	0.132	0.092	5.858	9.478
	BCaB1	0.043	0.904	0.055	0.926	0.819	0.837	0.344	0.212	**2.380**	**3.953**
	BCaB2	0.078	0.912	0.062	0.946	0.781	0.863	0.132	0.090	**5.934**	9.545
po=0.6 p1=0.4 ρ1=0.83 and ρ2=0.17	CAN1	0.026	0.913	0.033	0.922	0.857	0.861	0.674	0.385	1.272	2.237
	CAN2	0.038	0.905	0.033	0.921	0.840	0.868	0.126	0.077	6.657	11.226
	tCAN1	0.018	0.926	0.029	0.930	0.891	0.887	0.821	0.413	1.086	2.150
	tCAN2	0.031	0.923	0.028	0.926	0.889	0.889	0.153	0.083	5.812	10.694
	Boott1	0.021	0.886	0.023	0.896	0.825	0.843	0.590	0.363	1.398	2.321
	Boott2	0.033	0.867	0.026	0.901	0.776	0.860	0.109	0.073	7.116	11.781
	VST1	0.063	0.937	0.047	0.946	0.826	0.874	0.620	0.389	1.333	2.249
	VST2	0.082	0.922	0.053	0.934	0.791	0.864	0.112	0.074	7.079	11.653
	SB1	0.029	0.913	0.032	0.919	0.851	0.859	0.660	0.384	1.289	2.239
	SB2	0.041	0.909	0.032	0.921	0.849	0.865	0.128	0.078	6.657	11.099
	BB1	0.038	0.897	0.036	0.912	0.809	0.835	0.579	0.363	1.398	2.301
	BB2	0.048	0.888	0.037	0.910	0.803	0.839	0.111	0.072	**7.255**	11.576
	PB1	0.059	0.928	0.047	0.935	0.830	0.860	0.594	0.369	**1.398**	2.332
	PB2	0.093	0.931	0.056	0.934	0.789	0.861	0.115	0.073	6.845	**11.849**
	BCaB1	0.060	0.928	0.049	0.933	0.823	0.850	0.591	0.364	1.393	**2.334**
	BCaB2	0.089	0.931	0.054	0.931	0.787	0.856	0.115	0.072	6.823	11.827
po=0.1 p1=0.9 ρ1=0.5 and ρ2=0.9	CAN1	0.017	0.902	0.025	0.920	0.847	0.867	0.418	0.241	2.027	3.603
	CAN2	0.040	0.911	0.047	0.928	0.857	0.857	0.681	0.405	1.259	2.118
	tCAN1	0.008	0.926	0.022	0.924	0.889	0.882	0.553	0.255	1.606	3.454
	tCAN2	0.029	0.929	0.044	0.935	0.907	0.878	0.843	0.430	1.075	2.040
	Boott1	0.008	0.873	0.021	0.908	0.815	0.876	0.372	0.227	2.191	3.853
	Boott2	0.033	0.894	0.041	0.914	0.821	0.837	0.624	0.388	1.315	**2.160**
	VST1	0.064	0.930	0.056	0.940	0.809	0.871	0.367	0.225	2.207	3.879
	VST2	0.083	0.927	0.067	0.950	0.802	0.852	0.605	0.398	1.326	2.143
	SB1	0.017	0.901	0.026	0.918	0.842	0.864	0.415	0.238	2.028	3.635
	SB2	0.040	0.914	0.046	0.931	0.858	0.861	0.695	0.410	1.234	2.101
	BB1	0.027	0.893	0.031	0.916	0.819	0.847	0.363	0.226	2.255	3.743
	BB2	0.055	0.905	0.050	0.926	0.811	0.838	0.603	0.389	**1.346**	2.154
	PB1	0.047	0.928	0.056	0.934	0.830	0.854	0.366	0.218	2.269	**3.925**
	PB2	0.079	0.941	0.069	0.954	0.808	0.858	0.658	0.412	1.227	2.085
	BCaB1	0.048	0.927	0.054	0.937	0.825	0.859	0.363	0.220	**2.274**	3.899
	BCaB2	0.072	0.935	0.069	0.952	0.808	0.855	0.646	0.406	1.251	2.105
po=0.4 p1=0.6 ρ1=0.63 and ρ2=0.75	CAN1	0.028	0.892	0.033	0.931	0.837	0.883	0.481	0.299	1.739	2.953
	CAN2	0.034	0.904	0.042	0.932	0.838	0.846	0.572	0.344	1.465	2.461
	tCAN1	0.017	0.912	0.030	0.934	0.876	0.889	0.603	0.315	1.452	2.820
	tCAN2	0.025	0.917	0.039	0.936	0.886	0.874	0.699	0.363	1.268	2.405
	Boott1	0.023	0.865	0.027	0.919	0.773	0.872	0.420	0.286	1.838	3.046
	Boott2	0.025	0.870	0.036	0.911	0.795	0.828	0.505	0.323	1.574	**2.561**
	VST1	0.065	0.920	0.066	0.945	0.804	0.851	0.449	0.276	1.792	3.082
	VST2	0.075	0.917	0.064	0.950	0.796	0.837	0.507	0.332	1.571	2.521
	SB1	0.028	0.891	0.033	0.931	0.834	0.878	0.478	0.298	1.745	2.945
	SB2	0.037	0.902	0.044	0.930	0.835	0.843	0.571	0.340	1.463	2.478
	BB1	0.033	0.876	0.041	0.928	0.799	0.863	0.427	0.281	1.871	3.068
	BB2	0.049	0.891	0.049	0.924	0.794	0.819	0.496	0.322	**1.602**	2.546
	PB1	0.065	0.916	0.062	0.942	0.801	0.850	0.424	0.277	1.887	3.067
	PB2	0.085	0.929	0.071	0.953	0.798	0.833	0.517	0.338	1.544	2.466
	BCaB1	0.060	0.913	0.068	0.941	0.806	0.841	0.423	0.273	**1.905**	**3.084**
	BCaB2	0.076	0.922	0.071	0.945	0.793	0.831	0.510	0.326	1.553	2.545

Table 6- Continue...

Intensity Parameters	Estimation Approaches	*n*=10 $\hat{\beta}(\alpha)$	*n*=10 $\hat{\beta}(1-\alpha)$	*n*=29 $\hat{\beta}(\alpha)$	*n*=29 $\hat{\beta}(1-\alpha)$	Coverage Percentages *n*=10	Coverage Percentages *n*=29	Average Lengths *n*=10	Average Lengths *n*=29	Relative Coverage *n*=10	Relative Coverage *n*=29
ρo=0.9 ρ1=0.1 ρ1=0.5 and ρ2=0.1	CAN1	0.027	0.892	0.044	0.926	0.830	0.875	0.382	0.227	2.174	3.861
	CAN2	0.038	0.913	0.039	0.942	0.855	0.891	0.076	0.048	11.284	18.599
	tCAN1	0.013	0.906	0.036	0.929	0.862	0.893	0.491	0.243	1.756	3.682
	tCAN2	0.023	0.935	0.031	0.948	0.908	0.915	0.098	0.052	9.264	17.493
	Boott1	0.015	0.860	0.035	0.904	0.796	0.855	0.341	0.215	2.331	**3.981**
	Boott2	0.030	0.891	0.029	0.929	0.821	0.887	0.069	0.047	11.898	18.962
	VST1	0.060	0.916	0.068	0.940	0.810	0.855	0.359	0.217	2.257	3.940
	VST2	0.089	0.933	0.062	0.959	0.808	0.882	0.066	0.046	**12.225**	19.116
	SB1	0.029	0.889	0.042	0.922	0.818	0.873	0.374	0.226	2.184	3.867
	SB2	0.043	0.913	0.036	0.940	0.854	0.891	0.076	0.048	11.271	18.408
	BB1	0.037	0.877	0.045	0.919	0.774	0.856	0.332	0.217	2.334	3.947
	BB2	0.053	0.901	0.046	0.937	0.808	0.874	0.066	0.045	12.213	**19.385**
	PB1	0.054	0.910	0.061	0.936	0.797	0.857	0.336	0.218	2.369	3.936
	PB2	0.096	0.938	0.065	0.956	0.807	0.878	0.070	0.047	11.538	18.799
	BCaB1	0.057	0.909	0.061	0.934	0.788	0.855	0.332	0.216	**2.373**	3.954
	BCaB2	0.090	0.937	0.064	0.952	0.803	0.877	0.070	0.046	11.514	19.094

Note [for Table-3 to Table-6]

1. *Boldface denotes the greatest relative coverage among estimation approaches.*
2. *Calibrated confidence intervals of ρ_1 under different estimation approaches are denoted by CAN1, Exact-t1, Boot-t1, VST1, SB1, BB1, PB1, BCaB1 and that of ρ_2 are denoted by CAN2 Exact-t2, Boot-t2, VST2, SB2, BB2, PB2 and BCaB2.*

Table 7- Performances of the estimation approaches of intensities

Queueing Network Type	Queuing Network simulated	Queueing Network with greater relative coverage	Intensity Parameters	Estimation approach with greatest relative coverage n=10	Estimation approach with greatest relative coverage n=29
M/G/1 to G/M/1	$M/E_4/1$ to $E_4/M/1$ and $M/H_4^{Pe}/1$ to $H_4^{Pe}/M/1$	$M/E_4/1$ to $E_4/M/1$	ρ_1=0.50, ρ_2=0.20	VST	Boot-t
				BCaB	BCaB
			ρ_1=0.83, ρ_2=0.17	VST	VST
				BCaB	BCaB
			ρ_1=0.50, ρ_2=0.90	VST	Boot-t
				BCaB	PB
			ρ_1=0.63, ρ_2=0.75	VST	VST
				BCaB	BCaB
			ρ_1=0.50, ρ_2=0.10	VST	VST
				BCaB	BCaB
G/G/1 to G/G/1	$E_4/H_4^{Pe}/1$to $H_4^{Pe}/E_4/1$ and $E_4/H_4^{Po}/1$to $H_4^{Po}/E_4/1$	$E_4/H_4^{Pe}/1$to$H_4^{Pe}/E_4/1$	ρ_1=0.50, ρ_2=0.20	PB	VST
				VST	BB
			ρ_1=0.83, ρ_2=0.17	PB	VST
				VST	BB
			ρ_1=0.50, ρ_2=0.90	Boot-t	BCaB
				BB	BCaB
			ρ_1=0.63, ρ_2=0.75	BB	PB
				BB	BB
			ρ_1=0.50, ρ_2=0.10	BCaB	PB
				VST	BB

References

[1] Beran R. (1987) *Biometrika,* 74, 457-468.

[2] Burke P.J. (1956) *Operations Research,* 4, 699-704.

[3] Disney R.L. (1975) *Trans. A.I.E.E.*, 7, 268-288.

[4] Efron B. and Gong G. (1983) *Am. Stat.,* 37, 36-48.

[5] Efron B. and Tibshirani R.J. (1986) *Stat. Sci.,* 1, 54-77.

[6] Efron B. and Tibshirani R. (1993) *An Introduction to the Bootstrap*, Chapman and Hall, New York.

[7] Efron B. (1979) *Annals of Statistics*, 7, 1-26.

[8] Efron B. (1982) *SIAM Monograph*, 38.

[9] Efron B. (1987) *J. Am. Stat. Assoc.*, 82, 171- 200.

[10] Guntur B. (1991) *Quality Progress*, 97-103.

[11] Hall P. (1986) *Ann. Statist.*, 14, 1431-1452.

[12] Hall P. and Martin M.A. (1988) *Biometrika,* 75, 661-671.

[13] Hogg R.V. and Craig A.T. (1995) *Introduction to Mathematical Statistics*, Prentice- Hall, Inc.

[14] Jackson J.R. (1957) *Operations Research*, 5, 518- 521.

[15] Ke J.C. and Chu Y.K. (2009) *Computational Statistics,* 24(4), 567-582.

[16] Loh W.Y. (1987) *J. Amer. Statist. Assoc*., 82,155-162.

[17]Loh W.Y. (1991) *Statistica sinica*, 1, 477-491.

[18]Miller R.G. (1974) *Biometrika,* 61, 1-15.

[19]Mooney C.Z. and Duval R.D. (1993) *Bootstrapping: a nonparameric approach to Statistical inference*, SAGE Newbury Park.

[20]Rousses G.G. (1997) *A Course in Mathematical Statistics*, 2nd ed., Academic Press, New York.

[21]Rubin D.B. (1981) *The Annals of Statistics*, 9, 130-134.

[22]Thiruvaiyaru D. and Basava I.V. (1996) *Stochastic Processes and Statistical Inference*, New Age International Publications, New Delhi, 132-149.

[23]Thiruvaiyaru D., Basava I.V. and Bhat U.N. (1991) *Queueing Systems*, 9, 301-312.

[24]Young G.A. (1994) *Stat Sci.,* 9, 382-415.