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THERMOELASTIC SOLUTION OF A SEMI INFINITE RECTANGULAR BEAM DUE TO HEAT GENERATION

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Abstract- This paper deals with the study of determination of the temperature distribution, displacement and stress functions at any point of a rectangular beam occupying the space *D*: $-a\le x\le a$; $-b\le y\le b$; $0\le z<\infty$ by applying Fourier sine transform and Marchi-Fasulo transform techniques.
Keywords- Semi-infinite Rectangular Beam, Fourier sine- transforms and Marchi-Fasulo transform.
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Introduction

Adams and Bert [1], Tanigawa and Komatsubara [4] and Vihak et al. [5] have studied the direct problem of thermo elasticity in a rectangular plate under thermal shock. Khobragade et al. [2] has studied the inverse steady-state thermoelastic problem to determine the temperature, displacement function and thermal stresses at the boundary of a thin rectangular plate. They have used the finite Fourier sine transform technique. Khobragade and Lamba [3] have studied study three dimensional coupled thermoelastic response of infinitely long hollow circular cylinder due to axi symmetrical heating, considered under the thermo-mechanical coupling effect. This approach is based upon integral transform techniques, to find the thermoelastic solution. The expression for both the temperature and the stress distribution are determined from field equation of motion. Numerical calculations are carried out and results are depicted graphically.

In the present paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a thin rectangular beam occupying the region *D*: $-a\le x\le a$; $-b\le y\le b$; $0\le z<\infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier sine transform techniques have been used to find the solution of the problem.

Statement of the Problem

Consider a rectangular beam occupying the space *D*: $-a\le x\le a$; $-b\le y\le b$; $0\le z<\infty$. The displacement components u_x, u_y, u_z in the x, y and z directions respectively as [1] are

$$u_x=\int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\nu\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{1}$$

$$u_y=\int_{-b}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-\nu\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{2}$$

$$u_z=\int_{0}^{\infty}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-\nu\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{3}$$

where E, ν and λ are the Young's modulus, Poisson's ratio and the linear coefficient of thermal expansion of the material of the beam respectively and $U(x,y,z,t)$ is the Airy's stress functions which satisfy the differential equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t)=-\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)T(x,y,z,t) \tag{4}$$

Where $T(x,y,z,t)$ denotes the temperature of a rectangular beam satisfying the following differential equation:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5}$$

where k is the thermal conductivity and is α the thermal diffusivity of the material,

subject to the initial condition

$$T(x,y,z,0)=0 \tag{6}$$

The boundary conditions are

$$\left[T(x,y,z)+k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a}=f_1(y,z,t) \tag{7}$$

$$\left[T(x,y,z)+k_2\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a}=f_2(y,z,t) \quad (8)$$

$$\left[T(x,y,z,t)+k_3\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b}=f_3(x,z,t) \quad (9)$$

$$\left[T(x,y,z,t)+k_4\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b}=f_4(x,z,t) \quad (10)$$

$$\left[T(x,y,z,t)+k_5\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=0}=f_5(x,y,t) \quad (11)$$

$$\left[T(x,y,z,t)+k_6\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=\infty}=f_6(x,y,t) \quad (12)$$

The stress components in terms of $U(x,y,z,t)$ are given by

$$\sigma_{xx}=\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right) \quad (13)$$

$$\sigma_{yy}=\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right) \quad (14)$$

$$\sigma_{zz}=\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \quad (15)$$

The [Eq-1] to [Eq-15] constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

By applying finite Marchi- Fasulo transform (twice) and Fourier sine transform to the [Eq-5] to [Eq-12], one obtains

$$\overline{\overline{T}}^{*}(l,m,\eta,t)=\int_0^t\left\{\frac{\alpha\overline{\overline{g}}^{*}}{k}+\Omega_1\right\}e^{-\alpha q^2(t-t')}dt' \quad (16)$$

where

$$q^2=a_l{}^2+c_m{}^2+\eta^2$$

$$\Omega_1=\alpha\Omega,\ \ \Omega=\bar{\phi}+\eta f_5+\Phi$$

$$\phi=\frac{R_l(a)}{k_1}f_1-\frac{R_l(-a)}{k_2}f_2$$

$$\Phi=\frac{P_m(b)}{k_3}f_3-\frac{P_m(-b)}{k_4}f_4$$

Applying inversion of semi infinite Fourier sine transform and finite Marchi-Fasulo transform to the [Eq-16], one obtain the expression for temperature distribution as,

$$T(x,y,z,t)=\frac{2\eta}{\pi}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)$$

$$\int_0^{\infty}\left\{\int_0^t\left(\frac{\alpha\overline{\overline{g}}^{*}}{k}+\Omega_1\right)e^{-\alpha q^2(t-t')}dt'\right\}\sin\eta z dz \quad (17)$$

which is the required solution.

Airy's Stress Function

Substituting the value of $T(x,y,z,t)$ from [Eq-17] in [Eq-4] one obtains

$$U(x,y,z,t)=-\frac{2\eta\lambda E}{\pi}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz \quad (18)$$

Where

$$B(t)=\int_0^t\left(\frac{\alpha\overline{\overline{g}}^{*}}{k}+\Omega_1\right)e^{-\alpha q^2(t-t')}dt'$$

Displacement Components

Substituting the value of [Eq-18] in the [Eq-1] to [Eq-3] one obtains

$$u_x=-\frac{2\eta\lambda}{\pi}\int_{-a}^{a}\left[\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m''(y)}{\mu^2{}_m}\right)\int_0^{\infty}B(t)\sin\eta z dz+\right.$$

$$\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\eta^2\sin\eta z dz$$

$$-\nu\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l''(x)}{\xi_l^2}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz-$$

$$\left.\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz\right]dx \quad (19)$$

$$u_y=-\frac{2\eta\lambda}{\pi}\int_{-b}^{b}\left[\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\eta^2\sin\eta z dz\right.$$

$$+\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l''(x)}{\xi^2{}_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz$$

$$-\nu\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m''(y)}{\mu^2{}_m}\right)\int_0^{\infty}B(t)\sin\eta z dz-$$

$$\left.\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz\right]dy \quad (20)$$

$$u_z=-\frac{2\eta\lambda}{\pi}\int_0^{\infty}\left[\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l''(x)}{\xi^2{}_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz+\right.$$

$$\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m''(y)}{\mu^2{}_m}\right)\int_0^{\infty}B(t)\sin\eta z dz$$

$$+\nu\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\eta^2\sin\eta dz-$$

$$\left.\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz\right]dz \quad (21)$$

Determination of Stress Functions

Substituting the value of $U(x,y,z,t)$ from [Eq-18] in the [Eq-13] [Eq-15], one obtain

$$\sigma_{xx}=-\frac{2\eta\lambda E}{\pi}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left[\left(\frac{P_m''(y)}{\mu^2{}_m}\right)\int_0^{\infty}B(t)\sin\eta z dz\right.$$

$$\left.\sum_{m=1}^{\infty}\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}B(t)\sin\eta z dz\right] \quad (22)$$

$$\sigma_{yy}=-\frac{2\eta\lambda E}{\pi}\sum_{m=1}^{\infty}\left(\frac{P_m(y)}{\mu_m}\right)\left[\frac{R_l''(x)}{\xi^2{}_l}\int_0^{\infty}B(t)\sin\eta z dz-\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\int_0^{\infty}B(t)\sin\eta z dz\right] \quad (23)$$

$$\sigma_{zz} = -\frac{2\eta\lambda E}{\pi}\left[\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l''(x)}{\xi^2{}_l}\right)\left(\frac{P_m(y)}{\mu_m}\right) + \sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m''(y)}{\mu^2{}_m}\right)\right]\int_0^{\infty} B(t)\sin\eta z dz \quad (24)$$

Special Case and Numerical Results

Set $f(x,y,z,t) = (x+a)^2(x-a)^2(y+b)^2(y-b)^2 z(1-e^{-t})$

and $g(x,y,t) = \delta(x-x_1)\delta(y-y_1)\delta(z-z_1)\delta(t-t_1)$

Applying twice finite Marchi- Fasulo transform and then Fourier sine integral transform, we get

$$\bar{\bar{g}}^{*} = A \quad (25)$$

$$\Omega = \bar{\bar{f}} = 16(k_1+k_2)(k_3+k_4)z(1-e^{-t}) \times \left[\frac{(a_l a)\cos^2(a_l a) - \cos(a_l a)\sin(a_l a)}{a_l{}^2}\right]$$

$$\times\left[\frac{(a_m b)\cos^2(a_m b) - \cos(a_m b)\sin(a_m b)}{a_m{}^2}\right] \quad (26)$$

Substituting these values in [Eq-17], we get

$$T(x,y,z,t) = \frac{2\eta}{\pi}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}\left\{\int_0^t\left(\frac{\alpha A}{k}+\Omega_1\right)e^{-\alpha q^2(t-t')}dt'\right\}\sin\eta z dz \quad (27)$$

Numerical Results

Set A=1, $\delta = \frac{2\eta}{\pi},\ a=3,\ k=117,\ \alpha=3.33,\ b=2,\ h=10^3,\ t=1$

in the [Eq-27] to obtain

$$\frac{T(x,y,z,t)}{\delta} = \sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{R_l(x)}{\xi_l}\right)\left(\frac{P_m(y)}{\mu_m}\right)\int_0^{\infty}\left\{\int_0^t\left(\frac{\alpha A}{k}+\Omega_1\right)e^{-0.86q^2(t-t')}dt'\right\}\sin\eta z dz \quad (28)$$

Material Properties

The numerical calculation has been carried out for an Aluminum (Pure) rectangular beam with the material properties as,
Density *p*= 169 lb/ft3
Specific heat = 0.208 Btu/ lb0F
Thermal conductivity *k* = 117 Btu/(hr.ft0F)
Thermal diffusivity α = 3.33 ft2/hr
Poisson ratio *v* = 0.35
Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6} 1/F
Lame constant μ = 26.67
Young's Modulus of elasticity *E* =70GPa

Dimensions

The constants associated with the numerical calculation are taken as
Length of rectangular plate *a* = 3ft
Breadth of rectangular plate b = 2ft
Height of rectangular plate h = 10^3 ft

Conclusion

The temperature distribution, displacements and thermal stresses at any point of a rectangular beam have been obtained; when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and Fourier sine transform techniques.

The results are obtained in the form of infinite series. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

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Reference

[1] Adams R.J. and Best C.W. (1999) *Journal of Thermal Stresses*, 22, 875-895.
[2] Durge M.H. and Khobragade N.W. (2003) *Bull. of the Cal. Math. Soc.*, 95(6), 497-500.
[3] Khobragade N.W. and Lamba N.K. (2011) *International journal of Latest trend in Mathematics*, 1(2), 29-32.
[4] Tanigawa Y. and Komatsubara Y. (1997) *Journal of Thermal Stresses*, 20, 517.
[5] Vihak V.M., Yuzvyak M.Y. and Yasinskij A.V. (1998) *Journal of Thermal Stresses*, 21, 545-561.