Journal of Statistics and Mathematics
ISSN: 0976-8807 & E-ISSN: 0976-8815, Volume 3, Issue 3, 2012, pp.-134-137.
Available online at http://www.bioinfo.in/contents.php?id=85

STUDY OF MAGNETOTHERMODYNAMIC STRESS AND PERTURBATION OF MAGNETIC FIELD VECTOR IN AN ORTHOTROPIC SOLID CYLINDER

BAGADE S.H., VARGHESE V. AND KHOBRAGADE N.W.*

Department of Maths, MJP Educational Campus, RTM Nagpur University, Nagpur- 440033, MS, India.
*Corresponding Author: Email- khobragadenw@gmail.com

Received: December 10, 2012; Accepted: December 18, 2012

Abstract- In this paper, we investigate the problem of magnetothermoelastic responses and perturbation of magnetic field vector in a conducting thermoelastic orthotropic cylinder subjected to rapid temperature change due to thermal shocks. The Laplace transform and finite Marchi-Zgrablich integral transform techniques are used to obtain the analytical expression for Magnetothermodynamic stresses and perturbation response of an axial magnetic field vector in the orthotropic cylinders. Graphical representation of magnetothermostress and perturbation of magnetic field vector shows focusing effect which is then discussed.

Keywords- Orthotropic Cylinder, Magnetothermoelastic, Perturbation of magnetic field vector Focusing effect
2001 Mathematics Subject Classification- 74J25, 74H99, 74D99

Citation: Bagade S.H., Varghese V. and Khobragade N.W. (2012) Study of Magnetothermodynamic Stress and Perturbation of Magnetic Field Vector in an Orthotropic Solid Cylinder. Journal of Statistics and Mathematics, ISSN: 0976-8807 & E-ISSN: 0976-8815, Volume 3, Issue 3, pp. -134-137.

Introduction

Whenever electromagnetic pulses or thermal shocks of any kind or radiant energy in the form of γ-rays are incident on a conducting orthotropic solid cylinder then absorption of these pulses leads to rapid change on the temperature $T(r,t)$. Also the interactions between the deformations and the applied magnetic field give rise to the theory of dynamic coupled magnetothermoelasticity. Increasing attention is being devoted to this theory due to its applications in the field of engineering, magnetic structural elements, magnetic storage elements etc. The theory is also of great importance in the branches of physics which includes plasma physics, Geophysics and the corresponding measurement techniques of magnetoelasticity.

That magnetoelastic effect can be one of the reasons for the change in electromagnetic field and temperature distribution in Seismic Zones of lithosphere in course of their activation was proposed by Novik [7]. The macroscopic theory for analyzing magnetothermoelastic response in type-II superconductor was studied by Zhou [12]. Dhaliwal et al [1] dealt with the generalized elastic theory applied to the problems of magnetothermoelastic waves produced by thermal shocks in an infinite elastic solid with cylindrical cavity. Ezzat [2] found the distribution of thermal stresses and temperature in perfectly conducting half spaces when suddenly heated in absence of magnetic field. In Sherief et al [8], the Laplace transform technique was used to find the distribution of thermal stresses and temperature in a thermoelastic, electrically conducting half spaces under sudden thermal shock and permeated by magnetic field. The interactions between deformations and magnetic field in conducting orthotropic cylinder is considered by adding electromagnetic force in Kraus [4]. Solodyak et al [9] presented the analytical method for obtaining electromagnetic and temperature field and stresses in ferromagnetic solid subjected to electromagnetic field.

Till date most of the investigations about magnetothermoelasticity are done with respect to infinite isotropic structure and investigations on magnetothermoelasticity in finite orthotropic structures have been very few. In this paper we attempt to determine the response histories of magnetic thermo stresses and perturbation of magnetic field vector in finite orthotropic cylinder (solid) under sudden temperature change to a constant temperature and permeated by uniform primary magnetic field. The Laplace transform and finite Marchi-Zgrablich transform techniques have been used to analyze the above problem.

In the numerical results presented, it is seen that the response of Magnetothermodynamic stresses and perturbation of magnetic field vector near the center of an orthotropic cylinder reaches a peak value. This is called the wave-focusing effect. This focusing effect is reduced by magnetothermoelastic waves colliding with each other at the center of orthotropic cylinder and continuously accumulating at the center.

Governing Equations

Consider a long orthotropic solid cylinder of radius a placed in an axial magnetic field $H(0,0,H_Z)$. If electromagnetic pulse or γ-rays pulse radiant energy is incident on a solid cylinder, then it undergoes rapid change in temperature $T(r,t)$.

This then leads to interactions between the deformations of the cylinder and the perturbation of the magnetic field vector in orthotropic cylinder.
Assuming that, the magnetic permeability, μ, of the orthotropic solid cylinder equals the magnetic permeability of the medium around it and omitting the displacement electric current, the governing electrodynamic Maxwell's equation [6] for perfectly conducting body are

$$\left.\begin{aligned} &\bar{J} = curl\ \bar{h} \\ &-\mu\frac{\partial \bar{h}}{\partial t} = curl\ \bar{e} \\ &div\ \bar{h} = 0 \\ &\bar{e} = -\mu\left(\frac{\partial \bar{U}}{\partial t}\times\bar{H}\right) \end{aligned}\right\} \quad (1)$$

where
J is current density, h is perturbation of magnetic field vector $(0,0,h_Z)$
e is perturbation of electric field vector
H is magnetic intensity vector $(0,0,H_Z)$
U is the displacement vector.
Applying the initial magnetic field $H(0,0,H_Z)$ to [Eq-1] in (r,Θ,z) i.e. cylindrical polar coordinate system , we get

$$\left.\begin{aligned} &\bar{U} = (\bar{U}_r(r,t),0,0) \\ &\bar{H} = (0,0,\bar{H}_z) \\ &\bar{e} = \left(0,-\bar{H}_z\frac{\partial \bar{U}_r}{\partial t},0\right) \end{aligned}\right\} \quad (2a)$$

$$\bar{h}_z = curl(\bar{U}\times\bar{H}) = \left[0,0,-\bar{H}_z\left(\frac{1}{r}\frac{\partial}{\partial r}(r,U_r)\right)\right] = \left[0,0,-\bar{H}_z\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right)\right] \quad (2b)$$

$$h = (0,0,\bar{h}_z), \quad J = \left(0,\frac{\partial \bar{h}_z}{\partial r},0\right) \quad (2c)$$

$$\bar{h}_z = -\bar{H}_z\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right) \quad (2d)$$

U_r is the radial displacement component.
From [Eq-1] and [Eq-2] and Fung [3] the magneto elastic dynamic equation of orthotropic cylinder becomes

$$\frac{\partial \sigma_r}{\partial r}+\frac{1}{r}(\sigma_r-\sigma_\theta)+f_r = \rho\frac{\partial^2 U_r}{\partial t^2} \quad (3)$$

where f_r is the Lorentz force [4] which is given by

$$f_r = \mu(\bar{J}\times\bar{H}) = \mu H_z^2\frac{\partial}{\partial r}\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right) \quad (4)$$

From Lekhnitskii [9] the generalized Hookes law for an orthotropic thermoelastic cylinder can be written as

$$\left.\begin{aligned} &\varepsilon_r = \frac{1}{E_r}\sigma_r - \frac{\nu_{\theta r}}{E_\theta}\sigma_\theta - \frac{\nu_{zr}}{E_z}\sigma_z + \alpha_r T(r,t) \\ &\varepsilon_\theta = -\frac{\nu_\theta r}{E_\theta}\sigma_r + \frac{1}{E_\theta}\sigma_\theta - \frac{\nu_{z\theta}}{E_z}\sigma_z + \alpha_\theta T(r,t) \\ &\varepsilon_z = -\frac{\nu_z r}{E_z}\sigma_r - \frac{\nu_{z\theta}}{E_z}\sigma_\theta + \frac{1}{E_z}\sigma_\theta + \alpha_z T(r,t) \end{aligned}\right\} \quad (5)$$

where σ_r, σ_Θ are radial and circumferential stress
E_i, $V_{i,j}$ α_i (I, j=r, Θ, z) are the Young's modulus, Poison's ratio and coefficient of thermal expansion in radial, circumferential and axial directions respectively.
$\varepsilon_i(i=r,\theta,z)$ are the strains.

Considering a generalized plane strain problem, $\varepsilon_z = 0$ and solving [Eq-5] we get

$$\left.\begin{aligned} &\sigma_r = s_1\left[\beta_{22}\frac{\partial U_r}{\partial_r}+\beta_{12}\frac{U_r}{r}-s_2T(r,t)\right] \\ &\sigma_\theta = s_1\left[\beta_{12}\frac{\partial U_r}{\partial_r}+\beta_{11}\frac{U_r}{r}-s_3T(r,t)\right] \end{aligned}\right\} \quad (6)$$

where engineering constants β_{ij}, S_1,S_2,S_3, are as shown

$$\beta_{11} = \frac{1}{E_r}-\frac{\nu_{zr}^2}{E_z}, \quad \beta_{22} = \frac{1}{E_\theta}-\frac{\nu_{z\theta}^2}{E_z}, \quad \beta_{12} = \frac{\nu_{\theta r}}{E_\theta}-\frac{\nu_{z\theta}\nu_{zr}}{E_\theta}$$

$$s_1 = \frac{1}{(\beta_{11}\beta_{22}-\beta_{12}^2)}, \quad s_2 = \beta_{22}b_1+\beta_{12}b_2, \quad s_3 = \beta_{12}b_1+\beta_{11}b_2 \quad (7)$$

$$b_1 = \alpha_r+\nu_{zr}\alpha_z, \quad b_2 = \alpha_\theta+\nu_{z\theta}\alpha_z$$

Substituting [Eq-6] into [Eq-3], the basic displacement equation of magnetothermoelastic motion in an orthotropic cylinder may be expressed as in Wang, et al. [11].

$$\frac{\partial^2 U_r(r,t)}{\partial_r^2}+\frac{1}{r}\frac{\partial U_r(r,t)}{\partial_r}-N^2\frac{1}{r^2}U_r(r,t) = \frac{1}{V^2}\frac{\partial^2 U_r(r,t)}{\partial t^2}+g(r,t) \qquad 0\le r\le a\ ,t\ge 0 \quad (8a)$$

Where

$$g(r,t) = k_1\frac{\partial T(r,t)}{\partial_r}+k_2\frac{T(r,t)}{r}$$

$$N^2 = \frac{S_1\beta_{11}+\mu H_z^2}{S_1\beta_{22}+\mu H_z^2}, \quad v^2 = \frac{S_1\beta_{22}+\mu H_z^2}{\rho}, \quad k_1 = \frac{S_1S_2}{S_1\beta_{22}+\mu H_z^2}, \quad k_2 = \frac{S_1(S_2-S_3)}{S_1\beta_{22}-\mu H_z^2}$$

Omitting the Maxwell tensor on the surface of Orthotropic cylinder, the corresponding boundary conditions are

$$U_r(0,t) = 0$$

$$\sigma_r(0,t)|_{r=a} = \left[S_1\left(\beta_{22}\frac{\partial U_r}{\partial_r}+\beta_{12}\frac{U_r}{r}-S_2T(r,t)\right)\right]_{r=a} = 0 \quad (8b)$$

The initial conditions are

$$U_r(r,0) = 0 \quad (8c)$$

$$\frac{\partial U_r}{\partial t}(r,0) = 0 \quad (8d)$$

Solution of the Problem

Let us assume that the general solution to [Eq-8a] may be expressed in the form

$$U(r,t) = U_s(r,t)+U_d(r,t) \quad (9)$$

where $U_s(r,t)$ is the static part of solution to [Eq-8] and U_d is the dynamic part of solution to [Eq-8a]. The static part U_s satisfies the solution with inhomogeneous boundary conditions while the dynamic part U_d satisfies the solution with homogenous boundary conditions.

For the static part $U_s(r,t)$, the governing equations and boundary conditions become

$$\frac{\partial^2 U_s(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}U_s(r,t)-N^2\frac{1}{r^2}U_s(r,t) = g(r,t) \quad (10a)$$

$$U_s(0,t) = 0 \quad (10b)$$

$$\left.\beta_{22}\frac{\partial}{\partial r}U_s(r,t)+\beta_{12}\frac{U_s(r,t)}{r}\right]_{r=a} = S_2T(a,t) \quad (10c)$$

[Eq-10a] simplifies to

$$\frac{\partial}{\partial r}\left[r^{-(2N-1)}\frac{\partial}{\partial r}(r^N U_s(r,t))\right] = r^{-N+1}g(r,t) \quad (10d)$$

From [Eq-10d] the solution of equation 10(a) which satisfies the boundary conditions 10(b) is written as

$$U_s(r,t) = C_1 r^N + r^{-N}\int_0^r r^{2N-1}[\int_0^r r^{-N+1} g(r,t)\, dr]\, dr \quad (11a)$$

where

$$N = \sqrt{\frac{S_1\beta_{11} + \mu H_z^2}{S_1\beta_{22} + \mu H_z^2}} > 0 \quad (11b)$$

The unknown constant C_1 in [Eq-11a] can be easily determined from boundary condition (10c). The dynamic part of the solution $U_d(r,t)$ should satisfy the inhomogeneous [Eq-12a] and corresponding homogenous boundary conditions (12b) and (12c) and initial condition (12d).

$$\frac{\partial^2}{\partial r^2}U_d(r,t) + \frac{1}{r}\frac{\partial}{\partial r}U_d(r,t) - \frac{N^2}{r^2}U_d(r,t) = \frac{1}{V^2}\left[\frac{\partial^2}{\partial t^2}U_d(r,t) + \frac{\partial^2}{\partial t^2}U_s(r,t)\right] \quad (12a)$$

$$U_d(0,t) = 0 \quad (12b)$$

$$\left[\beta_{22}\frac{\partial U_d(r,t)}{\partial r} + \beta_{12}\frac{U_d(r,t)}{r}\right]_{r=a} = 0 \quad (12c)$$

$$U_d(r,0) = -U_d(r,0) = U_0, \quad \frac{\partial U_d(r,0)}{\partial t} = -\frac{\partial U_s(r,0)}{\partial t} = V_0 \quad (12d)$$

where $U_s(r,t)$ is the static solution as on [Eq-11].

$V \rightarrow$ magneto thermoelastic wave speed.

In order to remove the left hand side group of the [Eq-12a] for the homogenous boundary conditions (12b) of the third kind we apply the finite Marchi-Zgrablich integral transform with respect to the radial coordinate r to [Eq-12a].

Applying finite Marchi-Zgrablich transform we get

$$-\mu_n^2 U_d(n,t) = \frac{1}{V^2}\left[\frac{d^2}{dt^2}\overline{U}_s(n,t) + \frac{d^2}{dt^2}\overline{U}_d(n,t)\right] \quad (13)$$

where $\overline{U}_s$ and $\overline{U}_d$ are Marchi-Zgrablich transform of U_s and U_d respectively where the eigen values μ_n are the positive root of the frequency of characteristic equation.

$$J_p(\alpha, \mu a)\, Y_p(\beta, \mu b) - J_p(\alpha, \mu b)\, Y_p(\beta, \mu a) = 0 \quad (14)$$

Applying the Laplace transform to [Eq-13] one obtains

$$\overline{U}_d^*(n,s) = -\overline{U}_s^*(n,s) + \frac{\omega_i^2}{(\omega_i^2 + s^2)}\overline{U}_s^*(n,s) + \frac{s^2}{(\omega_i^2 + s^2)}\overline{U}_0 + \frac{1}{\omega_i^2 + s^2}\overline{V}_0 \quad (15)$$

Where S is the Laplace transform parameter & $\omega_i = \mu_n V$

Using the inverse Laplace transform and its convolution property in [Eq-15] gives

$$\overline{U}_d(n,t) = -\overline{U}_s(n,t) + \omega_i\int_0^t \overline{U}_s \sin\left[\omega_i(t - t^1)dt^1\right] + \overline{U}_0\cos(\omega_i t) + \frac{V_0}{\omega_i}\sin(\omega_i t) \quad (16)$$

Applying the finite inverse Marchi-Zgrablich integral transform to [Eq-16] the solution $U_d(r,t)$ is expressed as

$$U_d(r,t) = \sum_{n=1}^{\infty}\left[\frac{S_p(\alpha_1\, \beta_1\, \mu_n r)}{\int_0^a r[S_p(\alpha, \beta, \mu_n r)]^2\, dr}\right]\overline{U}_d(n,t) \quad (17)$$

where $\overline{U}_d(n,t)$ is finite Marchi-Zgrablich integral transform of $U_d(r,t)$ with respect to the Kernel function $S_p(\alpha, \beta, \mu_n r)$ and weight function r. Here the Kernel $S_p(\alpha, \beta, \mu_n r)$ can be defined as

$$S_p(\alpha, \beta, \mu_n r) = J_p(\mu_n r)$$

$$[Y_p(\alpha, \mu_n 0) + Y_p(\beta, \mu_n a)] - Y_p(\mu_n r)\,[J(\alpha, \mu_n 0) + J_p(\beta, \mu_n a)] \quad (18)$$

Where $J_p(k_i, \mu\xi) = J_p(\mu\xi) + k_i \mu J_p^1(\mu\xi)$

$Y_p(k_i\ \mu\xi) = Y_p(\mu\xi) + k_i\mu\, Y_p^1(\mu\xi)$ for i=1,2,3…. (19)

$J_p(\mu r)$ and $Y_p(\mu r)$ are Bessel's function of first and second kind respectively.

By substituting [Eq-17] and [Eq-11a] in [Eq-9] the general solution for the basic governing [Eq-8] becomes

$$U(r,t) = U_s(r,t) + U_d(r,t)$$

$$U(r,t) = C_1 r^N + r^{-N}\int_0^r r^{2N-1}\left[\int_0^r r^{-N+1} g(r,t) dr\right] dr + \sum_{n=1}^{\infty}\left[\frac{S_p(\alpha_1\beta_1\mu_n r)}{\int_0^a r[s_p(\alpha, \beta, \mu_n r)]^2\, dr}\right]\overline{U}_d(n,t) \quad (20)$$

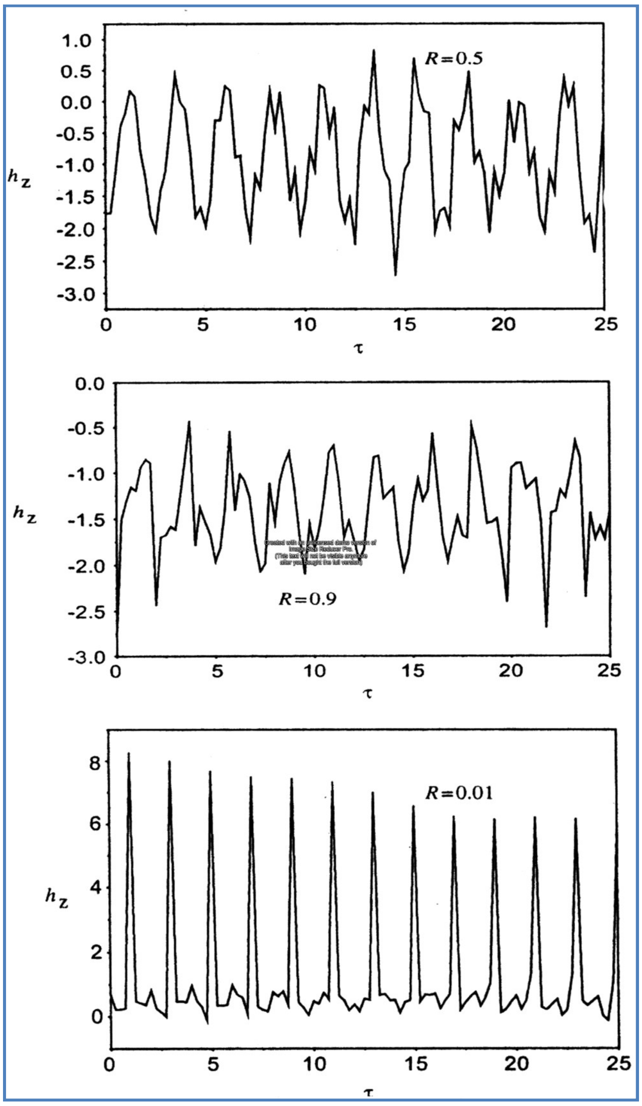

Fig. 1- Perturbation response histories of magnetic field vector in an orthopedic cylinder subjected to thermal shock and permeated by a uniform axial magnetic field, where α=0.01m, $R=r/\alpha$, $\tau=tV/\alpha$

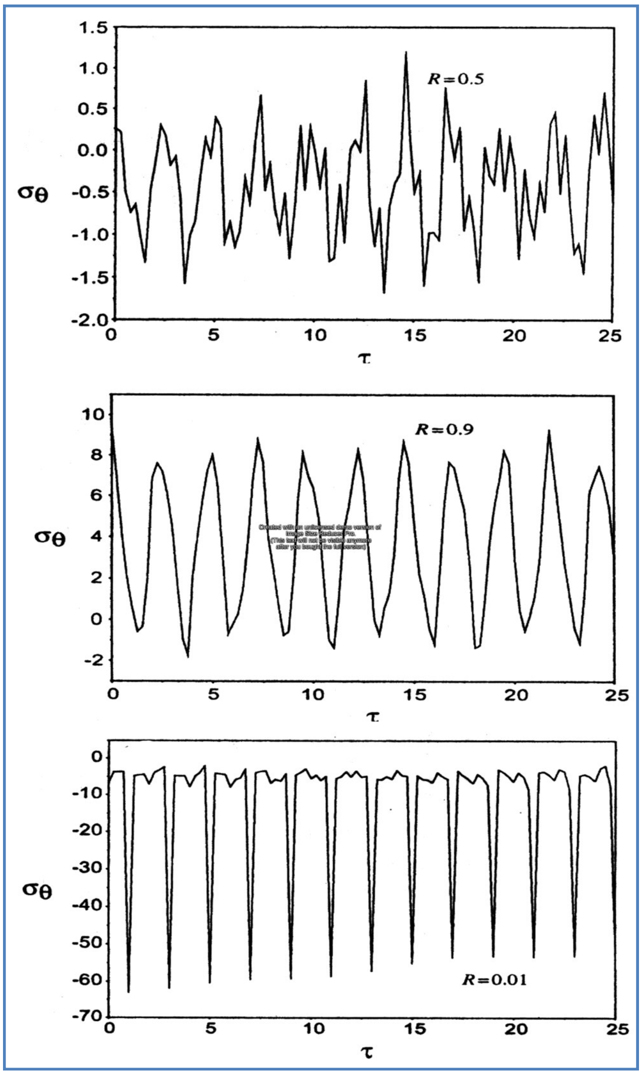

Fig. 2- Response histories circumferential stress in an orthopedic cylinder subjected to thermal shock and permeated by a uniform axial magnetic field, where α=0.01m, $R=r/\alpha$, $\tau=tV/\alpha$

Numerical Analysis and Discussion

The symmetric magnetothermoelastic problem of an orthotropic solid cylinder placed in uniform axial magnetic field and subjected to thermal shocks due to any source is considered.

Here for the orthotropic solid cylinder the various material constants taken are :

$$E_z = 150\, GP_a \,,\quad E_r = 7.5\, GP_a \,,\quad E_\theta = 20 GP_a$$

$$\nu_{zr} = 0.25 \,,\quad \nu_{z\theta} = 0.3 \,,\quad \nu_{\theta r} = 0.2$$

$$\alpha_r = 1\times10^{-4}\,\frac{1}{{}^0C} \,,\quad \alpha_\theta = 1\times10^{-5}\,\frac{1}{C^0} \,,\quad \alpha_z = 1\times10^{-6}\,\frac{1}{{}^0C}$$

Magnetic interference wave speed V=1000 m/s .

Further we define some non-dimensional parameters

$$R = \frac{r}{a} \quad \text{and} \quad \tau = \frac{tV}{a}$$

For the sake of study, three values of R i.e. R=0.01, R=0.5, R=0.9 are considered [Fig-1], [Fig-2].

Conclusion

By using the Laplace transform and finite Marchi-Zgrablich transforms, the solutions for displacement, stress components and magnetic field vector perturbations were found out for conducting orthotropic solid cylinder placed in axial magnetic field. Large amplitude of stresses and magnetic field vector perturbations near the center indicate focusing effect. By using this knowledge we can design various magnetoelastic elements for specific engineering requirements. The knowledge can also be used for proper selection of materials for in designing and manufacturing field; and to control the magnetothermoelastic stresses and magnetic field vector perturbations.

Acknowledgement

The authors are thankful to University Grant Commission, New Delhi for providing the partial financial assistance under major research project scheme.

References

[1] Dhaliwal R.S. and Saxena H.S. (1991) *Journal of Thermal Stresses,* 14, 353-369.
[2] Ezzat M.A. (1965) *Journal of Thermal Stresses,* 20, 617-633.
[3] Fung Y.C. (1965) *Foundation of Solid Mechanics*, Prentice Hall, Inc Englewood Clifts, USA.
[4] Kraus J.D. (1984) *Electro Magnetics*, McGraw Hill, Inc, USA.
[5] Lekhnitskii S.G. (1981) *Theory of Elasticity on Anisotropic Body*, Mir Publishers Moscow.
[6] Marchi E. and Zgrablich G. (1966) *Bulletin of the Calcutta Mathematical Society*, 58, 73-76.
[7] Novik O.B. (1996) *Transaction of the Russian Academy of Sciences*, Earth Science Section 341A , 5-11.
[8] Sherief H.H., Ezzat M.A. (1996) *International Journal of Solids and Structures*, 33, 4449-4459.
[9] Solodyak M., Gachkevich A. (1996) *Mathematical Methods in Electromagnetic Theory*, *Conference Proceedings*, 219-222.
[10]Sneddon I.N. (1974) *The use of Integral Transforms*, McGraw Hill Book, Co.
[11]Wang X., Dai H.L. (2004) *International Journal of Engineering Sciences.,* 42, 539-556.
[12]Zhou S.A. (1994) *International Journal of Applied Electromagnetic in Material*, 53, 211-222.