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STRESS FUNCTIONS IN A HOLLOW CYLINDER UNDER HEATING AND COOLING PROCESS

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Abstract- In this paper, an attempt is made to determine the temperature displacement function and stress functions due to partially distributed heat supply $-Q_0 f(r,t)/\lambda$ at $r=\xi$ in a hollow cylinder occupying the space $a \le r \le b,\ 0 \le z \le h$ by applying transformation techniques.
Keywords- Hollow cylinder, Fourier sine transform, Marchi-Zgrablich transform and Laplace transform.

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Introduction

During the second half of the Twentieth century, non isothermal problem of the theory of elasticity became increasingly important this is due to their wide application in diverse fields. The high relocation of modern aircraft gives rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Noda (1983) studied transient thermal stress problem in a finite circular transversely isotropic solid cylinder subjected to an asymmetrical temperature distribution on a cylindrical surface.

Chu and Lee (1995) derived exact analytic solution for thermal stresses in a long coaxial cylinder of two materials. Two cases were studied: one was of plane strain and the other was of zero axial force. The boundary was coated with a thin layer of poor conductor. The temperature was derived by using Laplace transformation. Graphs of temperature distributions and thermal stress fields were provided for several cases of practical interest. Dai and Wang (2006) presented an analytical method to solve magneto-elastic wave propagation and perturbation of the magnetic field vector in an orthotropic laminated hollow cylinder with arbitrary thickness. The magnetoelastodynamic equation for each separate orthotropic hollow cylinder was solved by making use of finite Hankel transforms and Laplace transforms.

Shao and Ma (2008) studied thermo-mechanical analysis of functionally graded hollow circular cylinders subjected to mechanical loads and linearly increasing boundary temperature was carried out. Thermo-mechanical properties of functionally graded material (FGM) were assumed to be temperature independent and vary continuously in the radial direction of cylinder. Employing Laplace transform techniques and series solving method for ordinary differential equation, solutions for the time-dependent temperature and thermo-mechanical stresses were obtained. Jinwu Kang (2008) applied the method to analyze stress in a stress frame specimen casting and a cylinder block. The results were more accurate than without consideration of the contact effects on the heat transfer. Eslami et al. (2009) discussed exact solution of steady-state two-dimensional ax symmetric mechanical and thermal stresses for a short hollow cylinder made of functionally graded material was developed. Temperature, as functions of radial and longitudinal directions, was solved analytically, using the generalized Bessel function. Nabavi and Ghajar (2010) derived a general weight function to evaluate the thermal stress intensity factors of a circumferential crack in cylinders. The weight function derived was valid for a wide range of thin- to thick-walled cylinders and relative crack depth. Closed-form stress intensity factor based on the weight function method was derived as a function of the Biot number and relative depth and various inner-to-outer radius ratios of cylinders. Tokovyy and Ma (2011) presented an analytical method for solving the ax symmetric stress problem for a long hollow cylinder subjected to locally-distributed residual (incompatible) strains. This method was based on direct integration of the equilibrium and compatibility equations, which thereby have been reduced to the set of two governing equations for two key functions with corresponding boundary and integral conditions. The governing equations were solved by making use of the Fourier integral transformation.

Ozturk and Gulgec (2011) investigated Elastic-plastic deformation of a solid cylinder with fixed ends, made of functionally graded material (FGM) with uniform internal heat generation based on Tresca's yield criterion and its associated flow rule, considering four of the material properties to vary radially according to a parabolic form. Expressions for the distributions of stress, strain and radial displacement were found analytically in terms of unknown interface radii.

Here we have solved the problem of thermoelasticity for a hollow cylinder by providing partially distributed heat supply with the stated boundary conditions. Numerical results are also included. The result presented here may be useful in engineering problem, particularly in determination of the state of strain in hollow cylinder constructing foundation of containers for hot gases or liquid in the foundation for furnaces etc.

Statement of the Problem

Consider a hollow cylinder of length h occupying the space $D: a \le r \le b,\ 0 \le z \le h$. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant. From time $t=0$ to $t=t_0$, the cylinder is subjected to a partially distributed and axi-symmetric heat supply $-Q_0 f(r,t)/\lambda$ for the interior point (r,ξ,t). After that the heat supply is removed and cylinder is cooled by the surrounding medium.

The displacement function $\phi(r,z,t)$ satisfies the differential equation

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\left(\frac{1+\nu}{1-\nu}\right)a_t T \quad (1)$$

with $\phi=0$ at $r=a$ and $r=b$ (2)

ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r,z,t)$ is the heating temperature of the cylinder at time t satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial T}{\partial t} \quad (3)$$

Subject to the initial condition

$$[T(r,z,t)]_{t=0}=F(r,t) \quad (4)$$

The boundary condition

$$\left[T(r,z,t)+k_1\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a}=f_1(z,t) \quad (5)$$

$$\left[T(r,z,t)+k_2\frac{\partial T(r,z,t)}{\partial r}\right]_{r=b}=f_2(z,t) \quad (6)$$

$$T(r,z,t)\,|_{z=0}=-\frac{Q_0}{\lambda}f_1(r,t) \quad (7)$$

$$T(r,z,t)\,|_{z=h}=g(r,t) \quad (8)$$

where k and λ are the thermal diffusivity and conductivity of the material of the cylinder respectively, k_1 and k_2 are radiation constants on the curved surfaces of the cylinder respectively.

The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation are

$$\nabla^2 U-\frac{U}{r^2}+(1-2\nu)^{-1}\frac{\partial e}{\partial r}=2\left(\frac{1+\nu}{1-2\nu}\right)a_t\frac{\partial T}{\partial r} \quad (9)$$

$$\nabla^2 w+(1-2\nu)^{-1}\frac{\partial e}{\partial z}=2\left(\frac{1+\nu}{1-2\nu}\right)a_t\frac{\partial T}{\partial z} \quad (10)$$

$$U=\frac{\partial\phi}{\partial r} \quad (11)$$

$$W=\frac{\partial\phi}{\partial z} \quad (12)$$

The stress functions are given by

$$\tau_{rz}(a,z,t)=0,\quad \tau_{rz}(b,z,t)=0,\quad \tau_{rz}(r,0,t)=0 \quad (13)$$

$$\sigma_r(a,z,t)=p_1,\quad \sigma_r(b,z,t)=-p_0,\quad \sigma_z(r,0,t)=0 \quad (14)$$

where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations:

$$\sigma_r=(\lambda+2G)\frac{\partial U}{\partial r}+\lambda\left(\frac{U}{r}+\frac{\partial W}{\partial z}\right) \quad (15)$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \quad (16)$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right) \quad (17)$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \quad (18)$$

Where $\lambda = \frac{2G\nu}{1-2\nu}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components. Equations (1) to (18) constitute the mathematical formulation of the problem under consideration.

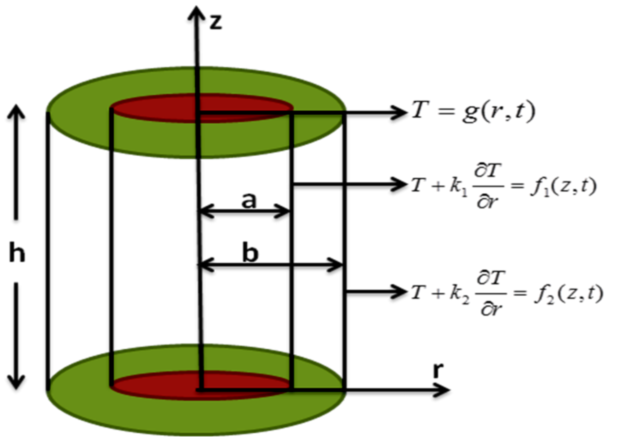

Fig. 1-

Solution of the Problem

Determination of Heating Temperature

Applying Marchi-Zgrablich transforms to equations (3), (4), (7), (8) and using equations (5) and (6), one obtains

$$\frac{d^2\bar{T}}{dz^2} - \mu_m^2\bar{T} + \frac{b}{k_2}S_0(k_1,k_2,\mu_m b)f_2 - \frac{a}{k_1}S_0(k_1,k_2,\mu_m a)f_1 = \frac{1}{k}\frac{d\bar{T}}{dt} \quad (19)$$

$$[\bar{T}(n,z,t)]_{t=0} = \bar{F}(n,t) \quad (20)$$

$$[\bar{T}(n,z,t)]_{z=0} = \frac{-Q_0}{\lambda}\bar{f}(n,t) \quad (21)$$

$$[\bar{T}(n,z,t)]_{z=h} = \bar{g}(n,t) \quad (22)$$

Now, applying Fourier sine transform to equation (19), one obtains

$$\frac{d\bar{T}^*}{dt} + p^2\bar{T}^* = A \quad (23)$$

Where

$$p^2 = k\left[\mu_m^2 + \left(\frac{m\pi}{h}\right)^2\right] \quad (24)$$

And

$$A = k\left[\frac{b}{k_2}S_0(k_1,k_2,\mu_m b)f_2^* - \frac{a}{k_1}S_0(k_1,k_2,\mu_m a)f_1^* - \frac{m\pi}{h}\left(\frac{Q_0}{\lambda}\bar{f} + (-1)^m\bar{g}\right)\right] \quad (25)$$

Equation (23) is the first order differential equation, whose solution is

$$\bar{T}^* = e^{p^2 t}\left[\int_0^t A e^{-p^2 t'}dt' + \bar{F}^*\right] \quad (26)$$

Applying inversion of Fourier sine transform and inversion of Marchi-Zgrablich transform on equation (26), one obtains

$$T(r,z,t) = \frac{2}{h^2}\sum_{m,n=1}^{\infty}\sin(m\pi z)\frac{S_0(k_1,k_2,\mu_n r)}{C_n} \times e^{p^2 t}\left[\int_0^t A e^{-p^2 t'}dt' + \bar{F}^*\right] \quad (27)$$

Cooling Process

The temperature change $T'(r,z,t)$ for the cooling process satisfies the equation

$$\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + \frac{\partial^2 T'}{\partial z^2} = \frac{1}{k}\frac{\partial T'}{\partial t} \quad (28)$$

$$[T'(r,z,t)]_{t=t_0} = [T(r,z,t)]_{t=t_0} = T(r,z,t_0) \quad (29)$$

$$\left[T'(r,z,t) + k_1\frac{\partial T'(r,z,t)}{\partial r}\right]_{r=a} = f_1(z,t) \quad (30)$$

$$\left[T'(r,z,t) + k_2\frac{\partial T'(r,z,t)}{\partial r}\right]_{r=b} = f_2(z,t) \quad (31)$$

$$[T'(r,z,t)]_{z=0} = f(r,t) \quad (32)$$

$$[T'(r,z,t)]_{z=h} = g(r,t) \quad (33)$$

where $T'(r,z,t)$ is the heating temperature of the cylinder at time t satisfying the differential equation (1).

Determination of Temperature for Cooling Process

Applying finite Marchi-Zgrablich transform to the equations (28), (29), (32), (33) and using equations (30), (31) one obtains

$$\frac{d^2\bar{T}'}{dz^2} - \mu_m^2\bar{T}' + \frac{b}{k_2}S_0(k_1,k_2,\mu_m b)f_2 - \frac{a}{k_1}S_0(k_1,k_2,\mu_m a)f_1 = \frac{1}{k}\frac{d\bar{T}'}{dt} \quad (34)$$

$$[T'(r,z,t)]_{t=t_0} = \bar{T}(r,z,t_0) \quad (35)$$

$$[T'(r,z,t)]_{z=0} = \bar{f}(n,t) \quad (36)$$

$$\left[\overline{T'}(n,z,t)\right]_{z=h} = \overline{g}(r,t) \tag{37}$$

Now, on applying Laplace transform to equation (34) one obtains

$$\frac{d\overline{T}'^*}{dz^2} + p^2\overline{T}'^* = \chi \tag{38}$$

Where $p^2 = \mu_m^2 + \frac{s}{k}$

$$\chi = \frac{a}{k_1}S_0(k_1,k_2,\mu_m a)f_1^* - \frac{b}{k_2}S_0(k_1,k_2,\mu_m b)f_2^* - \frac{\overline{F}}{k}$$

and

Equation (38) is the second order differential equation, whose solution given by

$$\overline{T'}^* = Ae^{pz} + Be^{-pz} + \psi \tag{39}$$

where $\psi = P.I. = \frac{1}{D^2 - p^2}\chi$

A and B are arbitrary constants, which are found to be

$$A = -\left[\frac{e^{-ph}(\bar{f}^* - \psi(0) + \psi(h) - \overline{g}^*}{2\sinh(ph)}\right] \tag{40}$$

$$B = \left[\frac{e^{ph}(\bar{f}^* - \psi(0)) + \psi(h) - \overline{g}^*}{2\sinh(ph)}\right] \tag{41}$$

Substituting the values of A and B in equation (39) one obtains

$$\overline{T'}^* = -(\bar{f}^* - \psi(0)\frac{\sinh(p(z-h))}{\sinh(ph)} - \psi(h)\frac{\sinh(pz)}{\sinh(ph)} + \overline{g}^*\frac{\sinh(pz)}{\sinh(ph)} + \psi \tag{42}$$

On taking inversion of Laplace transform and then inversion of Marchi-Zgrablich transform to equation (42), one obtains

$$T'(r,z,t) = \frac{2\alpha\pi}{h^2}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{C_m} \times \int_0^t \sum_{n=1}^{\infty}(-1)^{n+1} n e^{-\alpha u\left(\mu_n^2 + \frac{n^2\pi^2}{h^2}\right)} \left\{[\Psi(t-u) - \bar{f}(t-u)]\sin\left(\frac{n\pi}{h}(z-h)\right) - [\Phi(t-u) - \overline{g}(t-u)]\sin\left(\frac{n\pi}{h}\right)z\right\}du + L^{-1}(\psi) \tag{43}$$

Where $\psi = L^{-1}(\psi(0))$ and $\Phi = L^{-1}(\psi(h))$

Determination of Displacement Function

Substituting the value of $T(r,z,t)$ in equation (1), one obtains the thermoelastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t) = \frac{r^2}{2h^2}\left(\frac{1+w}{1-w}\right)\sum_{m,n=1}^{\infty}\sin(m\pi z)\frac{S_0(k_1,k_2,\mu_n r)}{C_n} \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \tag{44}$$

Using equation (44) in the equation (11) and (12) one obtains

$$U = \frac{r}{2h^2}\left(\frac{1+w}{1-w}\right) \times \sum_{m,n=1}^{\infty}\sin(m\pi z)\frac{rS_0'(k_1,k_2,\mu_n r) + 2S_0(k_1,k_2,\mu_n r)}{C_n} \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \tag{45}$$

$$W = \frac{\pi r^2}{2h^2}\left(\frac{1+w}{1-w}\right) \times \sum_{m,n=1}^{\infty} m\cos(m\pi z)\frac{S_0(k_1,k_2,\mu_n r)}{C_n} \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \tag{46}$$

Determination of Stress Functions

Substituting the values of equations (45) and (46) in equation (15) to (18) one obtains the stress functions as

$$\sigma_r = \frac{1}{2h^2}\left(\frac{1+w}{1-w}\right)\sum_{m,n=1}^{\infty}\frac{\sin(m\pi z)}{C_n} \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \times [(\lambda + 2G)r^2 S_0''(k_1,k_2,\mu_n r) + (5\lambda r + 8rG)S_0'(k_1,k_2,\mu_n r) + (4\lambda + 4G - \lambda_m^2\pi^2 r^2)S_0(k_1,k_2,\mu_n r)] \tag{47}$$

$$\sigma_z = \frac{1}{2h^2}\left(\frac{1+w}{1-w}\right)\sum_{m,n=1}^{\infty}\frac{\sin(m\pi z)}{C_n} \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \times [\lambda r^2 S_0''(k_1,k_2,\mu_n r) + 5\lambda r S_0'(k_1,k_2,\mu_n r) + (4\lambda + 4m^2\pi^2 r^2\lambda - 2m^2\pi^2 r^2 G)S_0(k_1,k_2,\mu_n r)] \tag{48}$$

$$\sigma_\theta = \frac{1}{2h^2}\left(\frac{1+w}{1-w}\right)\sum_{m,n=1}^{\infty}\frac{\sin(m\pi z)}{C_n} \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \times [\lambda r^2 S_0''(k_1,k_2,\mu_n r) + (5\lambda r + 2rG)S_0'(k_1,k_2,\mu_n r) + (4\lambda + 4G - \lambda m^2\pi^2 r^2)S_0(k_1,k_2,\mu_n r)] \tag{49}$$

$$\tau_{rz} = \frac{G\pi}{h^2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{m,n=1}^{\infty} m\cos(m\pi z) \times r^2 S_0'(k_1,k_2,\mu_n r) + 2rS_0(k_1,k_2,\mu_n r) \times e^{p^2 t}\left[\int_0^t Ae^{-p^2 t'}dt' + \overline{F}^*\right] \tag{50}$$

Special Case and Numerical Results

To study the mathematical thermoelastic behavior of a hollow cylinder, we consider the following functions and parameters:

$$F(r,z)=\delta(r)\left[z^2(z-h)^2\right]$$

Applying finite Marchi-Zgrablich transform to the equation (6.1) one obtains

$$\overline{F}(n,z)=(0.75)S_0(k_1,k_2,0.75\mu_n) \qquad \beta=(1.5)S_0(0.25,0.25,0.75\mu_n),$$

a = 1 mm, b= 2 mm, h =10 mm, t =1 sec, k = 0.86 and μ_n are the roots of the transcendental equation

$$S_0(k_1,k_2,\mu_m b)-S_0(k_1,k_2,\mu_m a)_1=0$$

to get

$$\frac{T(r,z,t)}{\beta}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[\sin(m\pi z)]\int_0^t\frac{e^{-(0.86)(\mu_n^2+p^2)t'}}{\mu_n^2}dt'$$

$$\times[S_0(0.25,0.25,0.75\mu_n)S_0(0.25,0.25,0.75\mu_n r)]$$

Material Properties

The numerical calculation has been carried out for hollow cylinder with the material properties as,

Table 1-Material Properties

Materials	K Btu/hr ft⁰ F	C p Btu/ lb ⁰ F	P lb/ ft³	α ft²/ hr	λ 1/F	E GPa	V
Aluminum (Al)	117	0.208	169	3.33	12.84x10^{-6}	70	0.35
Copper (Cu)	224	0.091	558	4.42	9.3x10^{-6}	117	0.36
Iron(Fe)	36	0.104	491	0.7	6.7x10^{-6}	193	0.21
Silver(Ag)	242	0.056	655	6.6	10.7x10^{-6}	83	0.37

Dimensions

The constants associated with the numerical calculation are taken as

Inner radius of hollow cylinder *a* =1 mm

Outer radius of hollow cylinder *b* =2 mm

height of the hollow cylinder h = 10 mm

time t = 1 sec.

Graphical Analysis

The numerical calculations has been carried out with the help of mathematical software MATCAD and graphs are plotted with the help of excel.

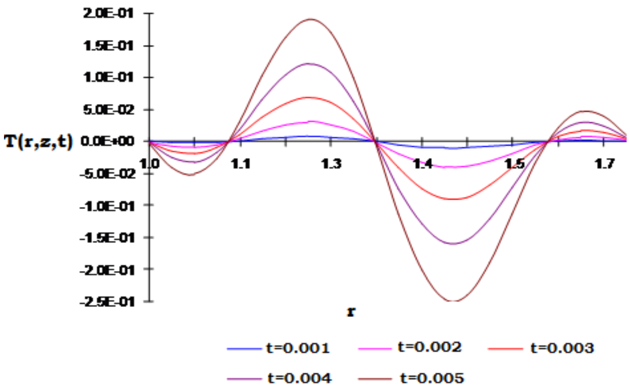

Fig. 2- Distribution of the temperature versus radius for z = 0.02 and different values of time in heating processes

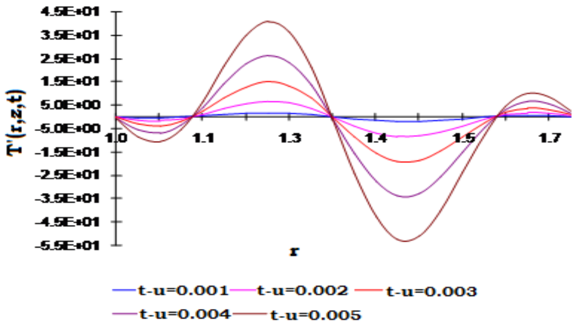

Fig. 3- Distribution of the temperature versus radius for z=0.02 and different values of time in cooling processes

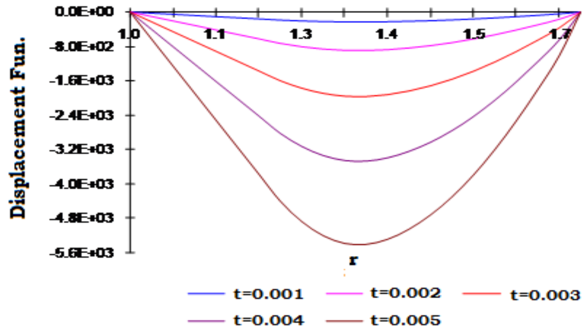

Fig. 4-Distribution of the displacement function ϕ versus radius For z=0.02 and different values of time

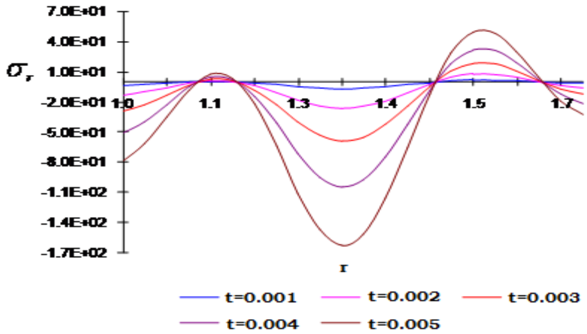

Fig. 5- Distribution of the radial stress σ_r versus radius for z=0.02 and different values of time

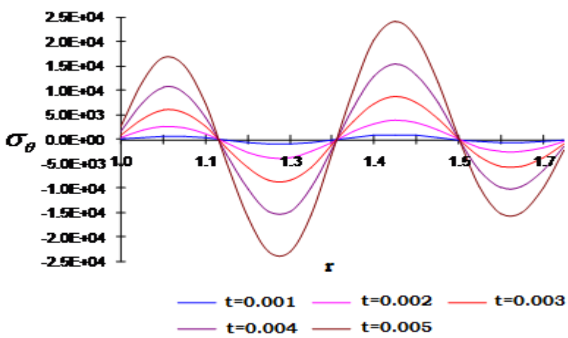

Fig. 6- Distribution of the axial stress σ_θ versus radius for z=0.02 and different values of time

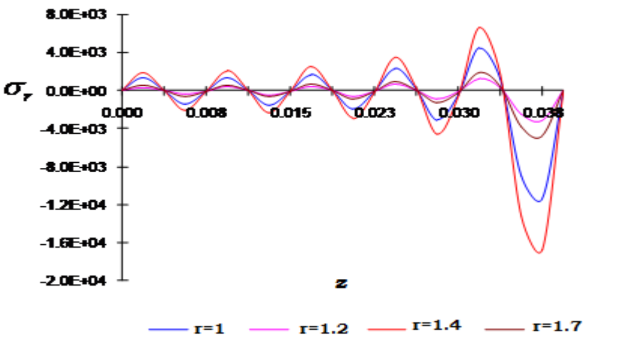

Fig. 7- Distribution of the radial stress σ_r versus thickness *z* for t = 0.02 and different values of radius *r*

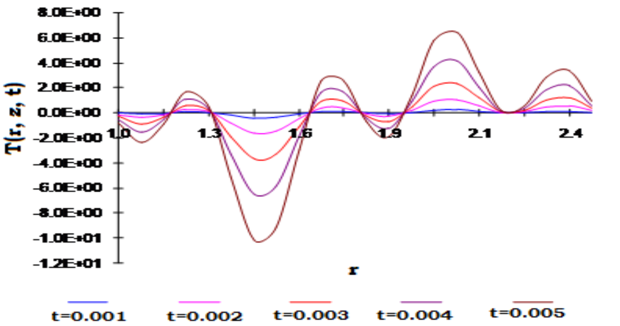

Fig. 8- Distribution of the temperature versus radius For *z*=0.02 and different values of time *t* in heating processes with *n*=0.02

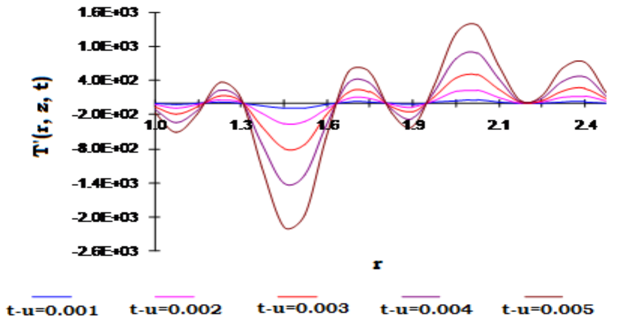

Fig. 9- Distribution of temperature versus radius for *z*=0.02 and different values of time in cooling processes with *n*=0.02

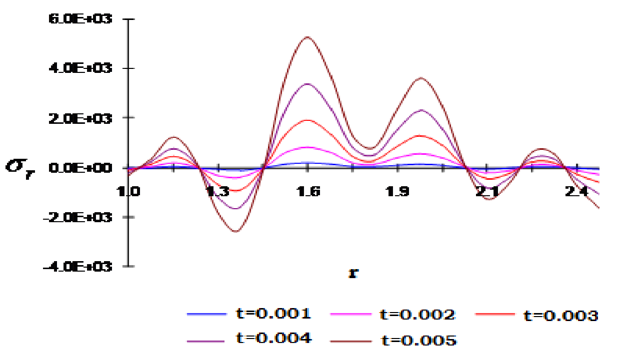

Fig. 10- Distribution of the radial stress versus radius for *z*=0.02 and different values of time with *n*=0.02

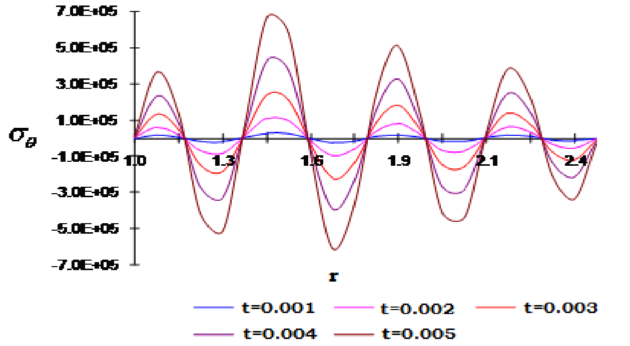

Fig. 11- Distribution of the axial stress versus radius for *z*=0.02 and different values of time with *n*=0.02

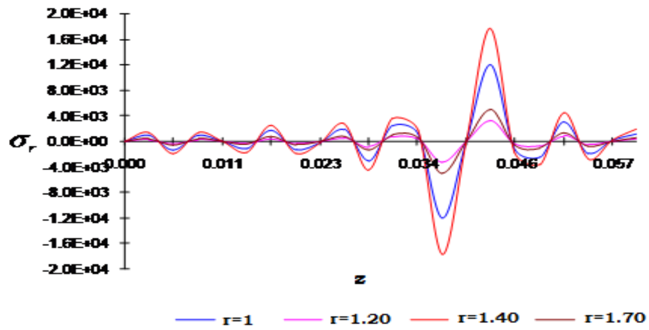

Fig. 12- Distribution of radial stress σ_r versus thickness z(=0.60) for t=0.02 and different values of radius

The foregoing analysis will be illustrated by the numerical results shown in Fig. 2 to 12. Fig. 2 and 3 depicts the distributions of the temperature increment $T(r,z,t)$ and $T'(r,z,t)$ verses radius at different values of time with z=0.02. It shows that heat gain on both boundary is zero and then initially temperature increment decreases slowly with increase of radius and the physical meaning emphasis for this phenomenon is that there is reduction in the rate of heat propagation then follows the sinusoidal nature crossing the inner core approaching towards outer edge leading to compressive radial stress at inner part and expand more on outer due to partially distributed heat supply. The difference in both the results lies with the increasing slope in Fig. 3 compared to Fig. 2 may be due to the cooling process once the partial heating is removed. Fig. 4 depicts the displacement function and it is noteworthy that it is in agreement with the boundary condition and attains minimum at the centre. Fig. 5 and 6 shows the distributions of the radial and axial thermal stresses at different value of time. The stress σ_r is smaller than stress σ_θ from Fig. 5, 6. Fig. 7 depicts the distributions of the radial stresses versus thickness showing the sinusoidal nature with increasing trend of peak with the fixed value of dimensionless time t=0.02. Further in order to investigate the outer radius-to-inner radius ratio $\eta = b/a$ and effects of thickness on the dimensionless temperature distributions and stresses are shown in Fig. 8 to 11. Selecting higher outer radius-to-inner radius ratio $\eta = 2.5$ temperature increment and stresses versus dimensionless radius with fixed value of dimensionless thickness $z = 0.02$ attains maximum peak comparative to Fig. 2,3,5 and 6. Similarly with increase in thickness $z = 0.06$ as shown in Fig. 12 temperature radial stress versus dimensionless radius with fixed value of dimensionless thickness $z = 0.02$ attains maximum peak comparative to Fig. 7.

Conclusion

In this problem, we modified the conceptual ideal proposed by Noda, et al (1983) for circular plate. The temperature distribution, thermal stresses and displacement functions are investigated of a thin hollow cylinder of length h occupying the space $D: a \le r \le b,\ 0 \le z \le h$. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant. From time $t = 0$ to $t = t_0$, the cylinder is subjected to a partially distributed and axi-symmetric heat supply $-Q_0 f(r,t)/\lambda$. After that the heat supply is removed and cylinder is cooled by the surrounding medium. The governing heat conduction equation has been solved by the applying Marchi-Zgrablich and Fourier sine transform techniques. The results are obtained in a series form in terms of Bessel's functions. Mathematical model has been constructed for the Aluminum material with the help of numerical illustration and the results obtained are illustrated graphically.

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