Journal of Statistics and Mathematics
ISSN: 0976-8807 & E-ISSN: 0976-8815, Volume 3, Issue 3, 2012, pp.-114-117.
Available online at http://www.bioinfo.in/contents.php?id=85

THERMAL DEFLECTION OF AN INVERSE THERMOELASTIC PROBLEM IN A THIN CIRCULAR PLATE

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Received: July 07, 2012; Accepted: July 12, 2012

Abstract- In this paper an attempt has been made to solve the inverse problem of thermoelasticity in a thin circular plate by determining the unknown temperature gradient, temperature distribution and the thermal deflection on the edge of the circular plate. The results are obtained in terms of series of Bessel's function and illustrated numerically.
Keywords- Circular plate, thermal deflection, inverse problem, integral transform

Citation: Roy H.S. and Khobragade N.W. (2012) Thermal Deflection of an Inverse Thermoelastic Problem in a Thin Circular Plate. Journal of Statistics and Mathematics, ISSN: 0976-8807 & E-ISSN: 0976-8815, Volume 3, Issue 3, pp.-114-117.

Introduction

Problems of normal deflection of an axisymmetrically heated circular plate in the case of fixed and simply supported edges have been considered by Boley and Weiner [1]. Further Roy Choudhuri [3] has succeeded in determining normal deflection of a thin clamped circular plate due to ramp type heating of concentric circular region of the upper face, the lower face of the plate is kept at zero temperature, while the circular edge is thermally insulated. Recently Deshmukh and Ingle [2] analyzed thermal deflection of a thin clamped circular plate due to partially distributed heat supply. The inverse thermoelastic problem consists of determination of the temperature of the heating medium, the heat flux on the boundary of the surface of the solid when the conditions of displacements and stresses are known at some points of the solid under consideration. The inverse problem is very important in view of its relevance to various industrial machines subjected to heating, such as main shaft of lathe, turbine, roll of rolling mills. Here we modify the work of Roy Choudhuri [3] and studied the inverse thermoelastic problems. Khobragade et al. studied the thermoelastic problem of a cylinder with internal heat sources [4,5].

Statement of the problem

Consider a thin circular plate of radius r and thickness h, occupying the space $D: 0 \le r \le a, -h \le z \le h$. Let the plate be initially at zero temperature. Under these more realistic prescribed conditions the thermal deflection of a plate is required to be determined.

The differential equation satisfied by the deflection $\omega(r,t)$ is

$$D\nabla_1^4 \omega = -\frac{\nabla_1^2 M_T}{(1-\nu)} \tag{1}$$

where ν is the Poisson's ratio of the plate material and

$$\nabla_1^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr},$$

and the resultant thermal moment M_T is defined as

$$M_T(r,t) = \alpha E \int_{-h}^{h} zT(r,z,t)dz, \quad (2)$$

and α, E are the coefficient of linear thermal expansion, Young's modulus respectively. Since the edge of the circular plate is fixed and clamped.

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a \quad (3)$$

The temperature of the plate at time t satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{K}\frac{\partial T}{\partial t}, \quad 0 \le r \le a, \quad -h \le z \le h, \ t>0. \quad (4)$$

subject to the initial and boundary conditions

$$\left[T(r,z,t)\right]_{t=0} = 0 \quad (5)$$

$$\left[T(r,z,t) + K_1\left(\frac{\partial T}{\partial z}\right)\right]_{z=-h} = F_1, \quad (6)$$

$$\left[T(r,z,t) + K_2\left(\frac{\partial T}{\partial z}\right)\right]_{z=h} = F_2 \quad (7)$$

$$T(a,z,t) = g(z,t) \quad \text{(Unknown)} \quad (8)$$

and the interior condition

$$\left[T(r,z,t) + \left(\frac{\partial T}{\partial r}\right)\right]_{r=\xi} = f(z,t), \quad \text{(known)}, \quad 0 < \xi < a \quad (9)$$

where, K_1 and K_2 are the radiation constants on the two plane surfaces, K is the thermal diffusivity of the material of the plate, where F_1 and F_2 are set to be zero.

Solution of the problem

By applying Marchi- Fasulo to the equations (6) and (7) one obtains

$$\frac{d^2\overline{T}}{dr^2} + \frac{1}{r}\frac{d\overline{T}}{dr} - a_n^2\overline{T} = \theta + \frac{1}{K}\frac{d\overline{T}}{dt} \quad (10)$$

Where, $\theta = \frac{P_n(-h)}{K_2}F_1 - \frac{P_n(h)}{K_1}F_2$ and the eigen values a_n are the solutions of the equation

$$\left[\alpha_1 a\cos(ah) + \beta_1\sin(ah)\right]\times\left[\beta_2\cos(ah) + \alpha_2 a\sin(ah)\right]$$
$$= \left[\alpha_2 a\cos(ah) - \beta_2\sin(ah)\right]\times\left[\beta_1\cos(ah) - \alpha_1 a\sin(ah)\right]$$

α_1, α_2, β_1 and β_2 are constants.

where,

$$P_n(z) = Q_n\cos(a_n z) - W_n\sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_1 - \alpha_2)a_n\sin(a_n h)$$

Applying Laplace transform to the equation (10), we obtain

$$\frac{d^2\overline{T}^*}{dr^2} + \frac{1}{r}\frac{d\overline{T}^*}{dr} - q^2\overline{T}^* = 0 \quad (11)$$

where,

$$q^2 = a_n{}^2 + \frac{s}{k}$$

$$\overline{T}^*(a,n,s) = \overline{g}^*(n,s) \quad (12)$$

$$\left[\overline{T}^*(r,n,s) + \frac{d\overline{T}^*}{dr}\right]_{r=\xi} = \overline{f}^*(n,s) \quad (13)$$

where $\overline{T}^*$ denotes Laplace transform of $\overline{T}$ and s is a Laplace transform parameter.

The equation (11) is a Bessel's equation whose solution is given by

$$\overline{T}^*(r,n,s) = AI_0(qr) + BK_0(qr) \quad (14)$$

where, A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.

As $r \to 0, K_0(qr) \to \infty$, but by physical consideration, $\overline{T}^*(r,n,s)$ remains finite. Therefore B must be zero.

Using equation (13) in (14), one obtains

$$\overline{T}^*(r,n,s) = \frac{\overline{f}^*(n,s)}{\left[I_0(q\xi) + qI_0'(q\xi)\right]}I_0(qr) \quad (15)$$

$$\overline{g}^*(r,n,s) = \frac{\overline{f}^*(n,s)}{\left[I_0(q\xi) + qI_0'(q\xi)\right]}I_0(qa) \quad (16)$$

By applying Inverse Laplace Transform to the equation (15), one obtains

$$\overline{T}(r,n,t) = \frac{2K}{\xi}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(r\lambda_m)}{\left[J_1(\xi\lambda_m) + \lambda_m J_2(\xi\lambda_m)\right]}\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2 + a_n^2)(t-t')}dt' \quad (17)$$

$$\overline{g}(n,t) = \frac{2K}{\xi}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(a\lambda_m)}{\left[J_1(\xi\lambda_m) + \lambda_m J_2(\xi\lambda_m)\right]}\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2 + a_n^2)(t-t')}dt' \quad (18)$$

By applying inverse Marchi-Fasulo integral transform to the equations (17) and (18), one obtains the expressions for the temperature distribution(r,z,t) and unknown temperature gradient g(z,t) as

$$T(r,z,t) = \frac{4K}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(r\lambda_m)}{\left[J_1(\xi\lambda_m) + \lambda_m J_2(\xi\lambda_m) - \lambda_m J_0(\xi\lambda_m)\right]}$$
$$\times\int_0^t \overline{f}(n,t')e^{-K(\lambda_m^2 + a_n^2)(t-t')}dt' \quad (19)$$

$$g(z,t)=\frac{4K}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(a\lambda_m)}{[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}$$

$$\times\int_0^t \overline{f}(n,t')e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{20}$$

where, m, n are positive integers and λ_m are positive roots of the transcendental equation $J_0(\lambda_m\xi)=0$.

Determination of Thermal Deflection

Substituting Eq. (19) in Eq. (2), one obtains

$$M_T(r,t)=\frac{4K\alpha E}{\xi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(r\lambda_m)}{\lambda_n[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}$$

$$\times\int_{-h}^{h} zP_n(z)dz\int_0^t \overline{f}(n,t')e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{21}$$

As a solution of equation (1) satisfying (3) we assume

$$\omega(r,t)=\sum_{n=1}^{\infty}C_n(t)\left[2aJ_0(\lambda_n r)-2aJ_0(\lambda_n a)+\lambda_n(r^2-a^2)J_1(\lambda_n a)\right] \tag{22}$$

Substituting (21) and (22) in (1), one obtains

$$C_n(t)=\frac{2K\alpha E}{D(1-\nu)\xi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\lambda_n[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}$$

$$\times\int_{-h}^{h} zP_n(z)dz\int_0^t \overline{f}(n,t')e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{23}$$

Substituting Eq. (23) in Eq. (22), one obtains the expressions for thermal deflection as

$$\omega(r,t)=\frac{2K\alpha E}{D(1-\nu)\xi}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}$$

$$\times\left[2aJ_0(\lambda_n r)-2aJ_0(\lambda_n a)+\lambda_n(r^2-a^2)J_1(\lambda_n a)\right]$$

$$\times\int_{-h}^{h} P_n(z)dz\int_0^t \overline{f}(n,t')e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{24}$$

Special case

Set $f(z,t)=r(z-h)^2(z+h)^2(1-e^{-t})$ (25)

By applying finite Marchi-Fasulo integral transform to the equation (25) one obtains

$$\overline{f}(n,t)=\int_{-h}^{h} r(z-h)^2(z+h)^2(1-e^{-t})P_n(z)dz$$

$$=4(K_1+K_2)r(1-e^{-t})\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right) \tag{26}$$

Substituting the value of $\overline{f}(n,t)$ from (26) in equations (19), (20) and (24) one obtains the expression for unknown temperature gradient, temperature distribution and thermal deflection respectively as

$$T(r,z,t)=\frac{16K(K_1+K_2)r}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}$$

$$\times\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right)$$

$$\times\sum_{m=1}^{\infty}\frac{\lambda_m J_0(r\lambda_m)}{[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}\times\int_0^t(1-e^{-t'})e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{27}$$

$$g(z,t)=\frac{16K(K_1+K_2)a}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}$$

$$\times\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right)$$

$$\times\sum_{m=1}^{\infty}\frac{\lambda_m J_0(a\lambda_m)}{[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}\times\int_0^t(1-e^{-t'})e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{28}$$

$$M_T(r,t)=\frac{16K(K_1+K_2)\alpha Er}{\xi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right)$$

$$\times\frac{\lambda_m J_0(r\lambda_m)}{\lambda_n[J_1(\xi\lambda_m)+\lambda_m J_2(\xi\lambda_m)-\lambda_m J_0(\xi\lambda_m)]}\times\int_{-h}^{h} zP_n(z)dz$$

$$\times\int_0^t(1-e^{-t'})e^{-K(\lambda_m^2+a_n^2)(t-t')}dt' \tag{29}$$

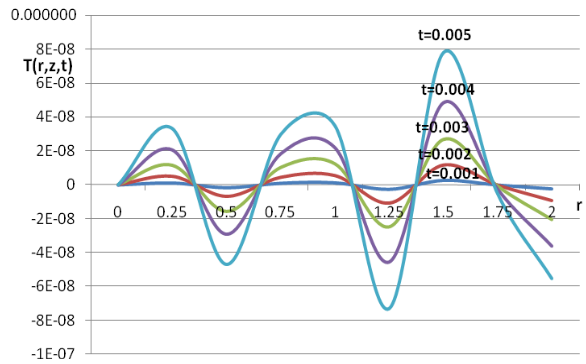

Fig. 1- Temperature gradient T(r,z,t) versus r for different values of t

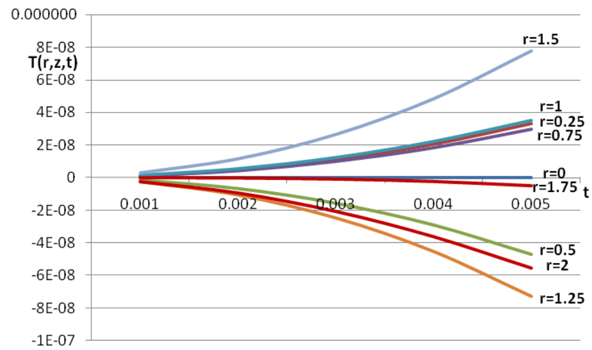

Fig. 2- Temperature gradient T(r,z,t) versus t for different values of r

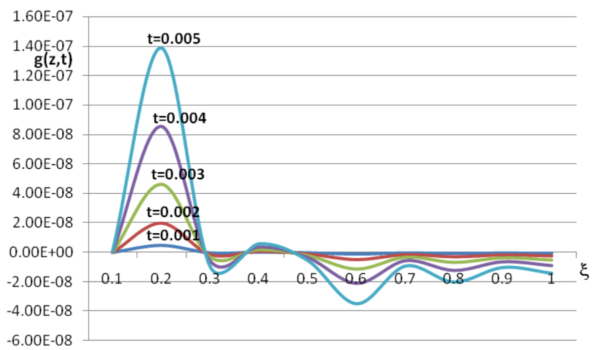

Fig. 3- Temperature distribution g(z,t) versus for different values of t

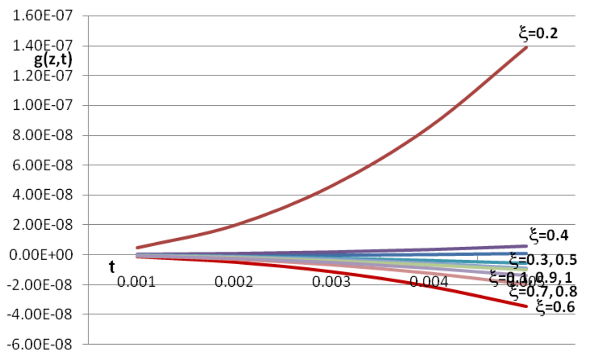

Fig. 4- Temperature distribution g(z,t) versus t for different values of ξ .

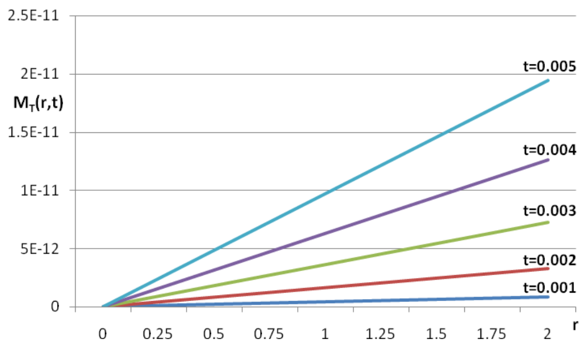

Fig. 5- Thermal deflection $M_T(r,t)$ versus r for different values of t

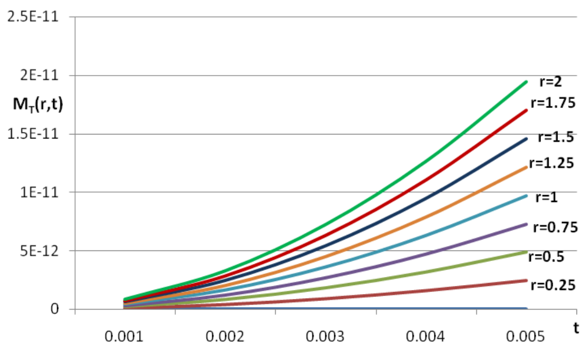

Fig. 6- Thermal deflection $M_T(r,t)$ versus t for different values of r

Acknowledgements

The authors are thankful to University Grant Commission, New Delhi for providing the partial financial assistance under major research project scheme.

Conclusion

In this paper, the temperature distribution, unknown temperature gradient and thermal deflection of a thin circular plate have been determined with the help of finite Marchi-Fasulo transform and Laplace transform techniques. The expressions are obtained in terms of Bessel's function in the form of infinite series and depicted graphically. The results that are obtained can be applied to the design of useful structures or machines in engineering applications

References

[1] Boley B.A. and Weiner J.H. (1960) *Theory of Thermal Stresses, Wiley, NewYork*.

[2] Deshmukh K.C. and Ingle S.G. (2001) *Bull. Pure Appl. Sci.*, 20E(1),159-167

[3] Roy Choudhari S.K. (1973) *J. Franklin Inst.,* 296(3), 231-249

[4] Hiranwar P.C. and Khobragade N.W. (2012) *Journal of Statistics and Mathematics*, 3(2), 109-113.

[5] Gahane T.T. and Khobragade N.W. (2012) *Journal of Statistics and Mathematics*, 3(2), 87-93.