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THERMOELASTIC PROBLEM OF A CYLINDER WITH INTERNAL HEAT SOURCES

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Abstract- We apply integral transformation techniques to study thermoelastic response of a hollow cylinder in general, in which sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. Numerical calculations are carried out for a particular case of a hollow cylinder made of Aluminum metal and result are depicted graphically.
Keywords- Transient Response, Hollow Cylinder, Temperature Distribution Thermal Stress, Integral Transform.

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Introduction

Nowacki has determined steady state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede has determined the quasi - state thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there are not many investigation on transient state. S.K. Roy Choudhary has succeeded in determining the quasi - static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In the recent work, some problems have been solved by Noda and Deshmukh. In all aforementioned investigations an axisymmetrically heated plate has been considered. Recently, Nasser proposed the concept of heat sources in generalized thermoelasticity and applied to the thick plate problem.. Khobragade et al. studied an inverse unsteady state problem of finite length hollow cylinder. They have not considered any thermoelastic problems with boundary conditions of radiation type, in which sources are generated according to the linear functions of radiation type, in which sources are generated according to the linear function of the temperature, which satisfies the time dependent heat conduction equation. From the previous literature regarding finite length hollow cylinder as considered it was observed by the author that no analytical procedure has been established considering internal heat source generation within the body.

This paper is concerned with the transient thermoelastic problem of a hollow cylinder in which sources are generated according to the linear function of temperature, occupying the space $a \le r \le b$, $-h \le z \le h$ with radiation type boundary conditions.

Statement of the Problem

Consider a hollow cylinder of length 2h in which sources are generated according to linear function of temperature. The material is isotropic, homogeneous and all properties are assumed to be constant. Heat conduction with internal source and prescribed boundary conditions of the radiation type are considered. The equation for heat conductions is $\theta(r,z,t)$, the temperature in cylindrical coordinate is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2}\right]+\frac{1}{k}\Theta(r,z,t,\theta)=\frac{1}{\alpha}\frac{\partial\theta}{\partial t} \tag{1}$$

where $\Theta(r,z,t,\theta)$ is the source function and $\alpha = \frac{\lambda}{\rho C}$, λ being the thermal conductivity of the material, ρ is the density and C is the calorific capacity, assumed to be constant.
For convenience, we consider the under given functions as the superposition of the simpler function.

$$\Theta(r,z,t,\theta) = \Omega(r,z,t) + \chi(t)\,\Theta(r,z,t) \tag{2}$$

and

$$T(r,z,t) = \Theta(r,z,t)\exp\left[-\int_0^t \chi(\zeta)\,d\zeta\right] \tag{3}$$

$$\chi(r,z,t) = \Omega(r,z,t)\exp\left[-\int_0^t \chi(\zeta)\,d\zeta\right]$$

Consider

$$\chi(r,z,t) = \frac{\delta(r-r_0)\,\delta(z-z_0)}{2\pi r_0}\exp(-\omega t), \quad a \le r \le b; \quad -h \le z \le h, \quad \omega > 0 \tag{4}$$

Substituting equations (2) and (3) in the heat conduction equation (1), one obtain

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \chi(r,z,t) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5}$$

where k is the thermal diffusivity of the material of the cylinder (which is assumed to be constant). Subject to the initial and boundary condition

$M_t(T,1,0,0) = F(r,z)$ for all $a \le r \le b$, $0 \le r \le b$ (6)

$M_r(T,1,k_1,a) = f(z,t)$ for all $a \le r \le b$, $t > 0$ (7)

$M_r(T,1,k_2,b) = g(z,t)$ for all $0 \le r \le b$, $t > 0$ (8)

$M_z(T,1,k_3,h) = \exp(\omega t)\,\delta(r-r_0)$

$M_z(T,1,k_4,-h) = \exp(-\omega t)\,\delta(r-r_0)$ for all $a \le r \le b$, $t > 0$ (9)

The most general expression for these conditions can be given by

$$M_v(f,\bar{k},\bar{\bar{k}},\$) = (\bar{k}f + \bar{\bar{k}}\hat{f})_{v=\$}$$

where the prime $(\wedge)$ denotes differentiation with respect to $v:\delta(r-r_0)$ are the Dirac Delta functions having $a \le r \le b$; $\omega > 0$ is constants. $\exp(-\omega t)$ $\delta(r-r_0)$ is the additional sectional heat available on its surface at $z=-h,h$ and $\bar{k}$, $\bar{\bar{k}}$ are radiation constants on the upper and lower surface of cylinder respectively.

The radiation and axial displacement U and W satisfy the uncoupled thermoelastic equation as (**Sierakowski and Sun**) are

$$\nabla^2 U - \frac{U}{r^2} + (1-2v)^{-1}\frac{\partial e}{\partial r} = 2\left(\frac{1+v}{1-2v}\right)\alpha_t\frac{\partial \theta}{\partial r} \tag{10}$$

$$\nabla^2 W + (1+2v)^{-1}\frac{\partial e}{\partial z} = 2\left(\frac{1+v}{1-2v}\right)\alpha_t\frac{\partial \theta}{\partial z} \tag{11}$$

Where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilatation.

And

$$U = \frac{\partial \Omega}{\partial r}, \tag{12}$$

$$W = \frac{\partial \Omega}{\partial z} \tag{13}$$

The thermoelastic displacement function $\Omega(r,z,t)$ as (Nowacki) is governed by the Poisson's equation

$$\nabla^2\Omega = \left(\frac{1+v}{1-v}\right)\alpha_t\theta \tag{14}$$

With $\Omega = 0$ at $r = a$ and $r = b$.

Where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$,

v And α_t are poisons ratio and the linear coefficient of thermal expansion of the material of the cylinder respectively.
The stress functions are given by

$$\tau_{rz}(a,z,t) = 0, \quad \tau_{rz}(b,z,t) = 0 \quad \tau_{rz}(r,0,t) = 0$$

$$\sigma_r(a,z,t) = p_1, \quad \sigma_r(b,z,t) = -p_0 \quad \sigma_z(r,0,t) = 0$$

Where p_1 and p_0 are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of displacement components by the relations

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left(\frac{U}{r} + \frac{\partial W}{\partial z}\right) \tag{15}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial W}{\partial z} + \frac{\partial U}{\partial r}\right) \tag{17}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \tag{18}$$

Where $\lambda = \frac{2Gv}{1-2v}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components.

The equations (1) to (18) constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

In order to solve fundamental differential equation (5) under the boundary conditions (7) and (8), we firstly introduce the integral transform of order 0 over the variable r. Let n be the parameter of the transform, then the integral transform and its inversion theorem is written as

$$\overline{f}(n) = \int_a^b r\, f(r)\, S_0(k_1, k_2, \mu_n r)\, dr,$$

$$f(r) = \sum_{n=1}^{\infty} \frac{\overline{f}(n)}{C_n} S_0(k_1, k_2, \mu_n r) \tag{19}$$

where $\overline{f}(n)$ is the transformation of $f(r)$ with respect to nucleus $S_0(k_1, k_2, \mu_n r)$.

The kernel function $S_0(k_1, k_2, \mu_n r)$ can be defined as

$$S_0(k_1, k_2, \mu_n r) = [J_0(k_1, \mu_n a) Y_0(k_1, \mu_n b)] + [Y_0(k_2, \mu_n a)\, J_0(k_2, \mu_n b)]$$

with

$$J_0(k_i, \mu_n r) = J_0(\mu_0 r) + k_i\, \mu_n\, J_0'(\mu_n r)$$

$$Y_0(k_1, \mu_n r) = Y_0(\mu_n r) + k_i \mu Y_0'(\mu_n r) \quad \text{for} \quad i = 1, 2$$

and

$$C_n = \int_a^b r[S_0(k_1, k_2, \mu_n b)]^2\, dr$$

in which $J_0(\mu_n r)$ and $Y_0(\mu_n r)$ are Bessel's functions of the first and second kind of order $p = 0$ respectively.

Applying the transformation defined in equation (19) to the equations (5) (6) and (9) and using equation (7) and (8) one obtains

$$\left[-\mu_n^2 \overline{T}(n,z,t) + \frac{k\, \partial^2 \overline{T}(n,z,t)}{\partial z^2} \right] + \frac{\delta(z - z_0)}{2\pi r_0} \exp(-\omega t)$$

$$\times r_0\, S_0(k_1, k_2, \mu_n r_0)] = X + \frac{1}{\alpha} \frac{\partial \overline{T}(n,z,t)}{\partial t} \tag{20}$$

Where X =

$$M_t(\overline{T}, 1, 0, 0) = \overline{F} \tag{21}$$

$$M_z(\overline{T}, 1, k_3, h) = \exp(\omega t)\, r_0\, S_0(k_1, k_2, \mu_n r) \tag{22}$$

$$M_r(\overline{T}, 1, k_4, -h) = \exp(-\omega t)\, r_0\, S_0(k_1, k_2, \mu_n r) \tag{23}$$

Where $\overline{T}$ is the transformed is function of T and n is the transformed parameter. The Eigen values μ_n are the positive roots of the characteristic equation.

$$S_0(k_1, k_2, \mu_n b) - S_0(k_1, k_2, \mu_n a) = 0$$

We introduce another integral transformation that respond to the radiation type boundary conditions

$$\bar{f}(m,t) = \int_{-h}^{h} f(z,t)\, P_m(z)\, dz \qquad f(z,t) = \sum_{m=1}^{\infty} \frac{\bar{f}(m,t)}{\lambda_m} P_m(z), \tag{24}$$

Further apply the transformation defined in equation (24) to the equation (20) and using (22) and (23) one obtains.

$$\left[-(\mu_n^2 + \xi_m^2)\, \overline{T}^*(n,m,t) + \left[\frac{P_m(h)}{k_3} - \frac{P_m(-h)}{k_4} \right] \exp(-\omega t)\, r S_0(k_1, \mu_2, \mu_n r) \right]$$

$$+ \frac{\exp(-\omega t)}{2\pi} S_0(k_1, k_2, \mu_n, r)\, P_m(z_0) \quad = \frac{1}{\alpha} \frac{d\overline{T}^*(n,m,t)}{dt} \tag{25}$$

where $M_t(\overline{T}^*, 1, 0, 0) = \overline{F}$ (26)

where $\overline{T}^*$ is the transformed function of $\overline{T}$ and m is the transformed parameter. The symbol (*) means a function in the transform domain and the nucleus is given by the orthogonal functions in the interval $-h \le z \le h$ as

$$P_m(z) = Q_m \cos(\xi_m z) - W_m \sin(\xi_m z)$$

where

$$Q_m = \xi_m (k_3 + k_4) \cos(\xi_m h)$$

$$W_m = 2\cos(\xi_m h) + (k_3 - k_4)\, \xi_m \sin(\xi_m h)$$

$$\lambda_m = \int_{-h}^{h} p_m^2(z)\, dz = h[Q_m^2 + W_m^2] + \sin\frac{(2\xi_m h)}{2\xi_m}[Q_m{}^2 - W_m^2]$$

The eigen values ξ_m are the positive roots of the characteristic equation

$$[k_3 a \cos(ah) + \sin(ah)][\cos(ah) + k_4 a \sin(ah)]$$

$$= [k_4 a \cos(ah) - \sin(ah)][\cos(ah) - k_3 a \sin(ah)]$$

After performing some calculations on the equation (25), the reduction is made to linear first order differential equation as

$$\frac{d\overline{T}^*}{dt} + \alpha(\mu_n^2 + a_m^2)\, \overline{T}^* = -(\Lambda + \Psi)\alpha \tag{27}$$

The transformed temperature solution is

$$\overline{T}^*(n,m,t) = -Z + C \exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \tag{28}$$

Where $\delta = \dfrac{(\Lambda + \Psi)}{(\mu_n{}^2 + a_m{}^2)}$ (29)

Applying the inversion of transformation rules defined in equations (19) and (24) the temperature solution is shown as follows

$$T(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right\}$$

$$\times \frac{P_m(z)}{\lambda_m} S_0(k_1, k_2, \mu_n r) \tag{30}$$

Taking into account of the first equation of equation (3), the temperature distribution is finally represented by

$$\Theta(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \frac{P_m(z)}{\lambda_m} \right\}$$

$$\times S_0(k_1, k_2, \mu_n r) \quad \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \tag{31}$$

The equation (31) represents the temperature at any instant and at all points of the hollow cylinder when there are radiation type boundary conditions.

Determination of Thermoelastic Solution:

Substituting value of $\theta(r,z,t)$ from (31) in (14) one obtains the thermoelastic displacement function $\Omega(r,z,t)$ as

$$\Omega(r,z,t) = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right.$$

$$\times \quad \frac{P_m(z)}{\lambda_m(\mu_n{}^2 + a_m{}^2)} \quad S_0(k_1, k_2, \mu_n r) \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \Bigg] \tag{32}$$

Substituting the value of $\Omega(r,z,t)$ from equation (32) in (12) and (13) one obtains.

$$U = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right.$$

$$\times \quad \frac{P_m(z)}{\lambda_m(\mu_n{}^2 + a_m{}^2)} \quad \mu_n S_0'(k_1, k_2, \mu_n r) \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \tag{33}$$

$$W = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right.$$

$$\times \quad (-\xi_m)\,[Q_m \sin(\xi_m z) + W_m \cos(\xi_m z)] \quad \}$$

$$S_0(k_1, k_2, \mu_n r) \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \tag{34}$$

Now making use of two displacement components the volume dilatation is established as

$$e = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right. \quad P_m(z)$$

$$\left[\mu_n^2 S_0''(k_1,k_2,\mu_n r) + \frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r} - \xi_m^2 S_0(k_1,k_2,\mu_n r) \right] \quad \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \tag{35}$$

Determination of Stress Functions

The stress components can be evaluated by substituting the values of thermoelastic displacement from equations (33) and (34) in equations (15), (16), (17) and (18) one obtains.

$$\sigma_r = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right. \quad P_m(z)$$

$$\times \left[(\lambda + 2G)\, \mu_n S_0''(k_1, k_2, \mu_n r) + \lambda \left(\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r} - \xi_m^2 S_0(k_1,k_2,\mu_n r) \right) \right.$$

$$\left. \times \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \right] \tag{36}$$

$$\sigma_z = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right. \quad P_m(z)$$

$$\times \left[(\lambda + 2G)\, \xi_m^2 S_0(k_1, k_2, \mu_n r) + \lambda \left(\mu_n S_0''(k_1, k_2, \mu_n r) + \frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r} \right) \right.$$

$$\left. \times \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \right] \tag{37}$$

$$\sigma_\theta = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right. \quad P_m(z)$$

$$\times \left[(\lambda + 2G)\, \frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r} + \lambda(-\xi_m^2 S_0(k_1, k_2, \mu_n r) + \mu_n^2 S_0''(k_1,k_2,\mu_n r)) \right.$$

$$\left. \times \exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \right] \tag{38}$$

$$\tau_{rz} = -2G\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \right.$$

$$[(-\xi_m)\,(Q_m \sin(\xi_m z) + W_m \cos(\xi_m r))\, \mu_n S_0'(k_1, k_2, \mu_n r)\}$$

$$\exp\left(\int_0^t \chi(\zeta)\, d\zeta \right) \tag{39}$$

Special Case and Numerical Results

Set $f(z,t) = \delta(r - r_0) a^2 e^{z+t}$, $g(z,t) = \delta(r - r_0) b^2 e^{z+t}$, F=0 (40)

Applying Marchi-Fasulo transform to equation (40), we obtain,

$$\overline{f}(m,t) = e^{(t+z)}(0.75)\, a^2 S_0(k_1, k_2, \mu_n a)$$

$\overline{g}(m,t) = e^{(t+z)}(0.75)\, b^2 S_0(k_1, k_2, \mu_n b)$, k=0.86, $h = 2$ cm, $b = 4$ cm, $t = 1\sec$, $r_0 = 1$ cm $\omega = 1$. Substitute this values in (15), one obtains

$$\theta(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \frac{P_m(z)}{\lambda_m} \right\} \right. \times$$

$$S_0(k_1, k_2, \mu_n r] \quad \exp(t^2/2)$$

$$\Omega(r,z,t) = -\left(\frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \left\{ \sum_{m=1}^{\infty} \left\{ \overline{F} + \delta \right] \left[\exp\left\{ -\alpha t(\mu_n{}^2 + a_m{}^2) \right\} \right] - \delta \right\} \frac{P_m(z)}{\lambda_m} \right\} \right. \times$$

$$S_0(k_1, k_2, \mu_n r) \exp(t^2/2) \quad \exp(t^2/2)$$

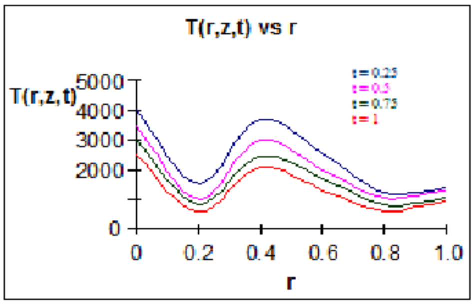


Fig. 1-

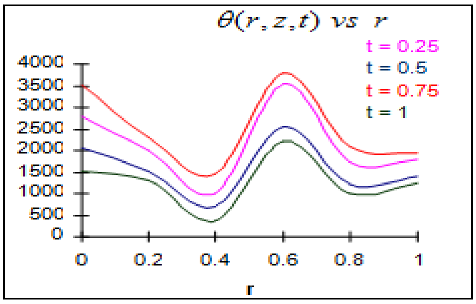


Fig. 2-

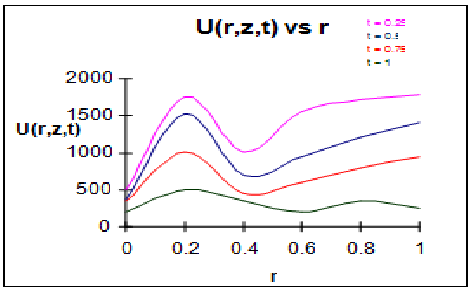
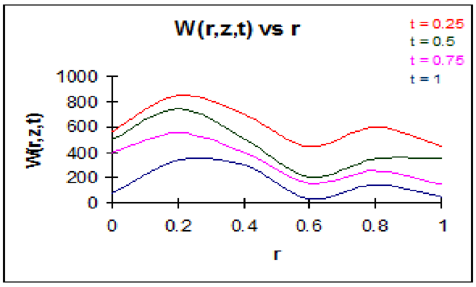


Fig. 3-



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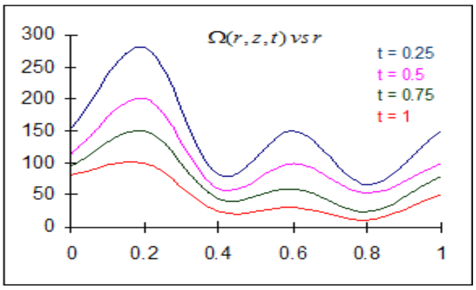

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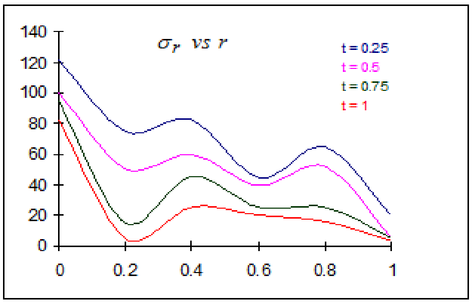

Fig. 6-

Conclusion

In this paper we study thermoelastic response of finite length hollow cylinder in which sources are generated according to linear function of temperature, with boundary conditions of the radiation Marchi - Zgrablich transform and March - Fasulo transform techniques are used to obtain numerical results. The temperature, displacement and stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

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