Journal of Statistics and Mathematics
ISSN: 0976-8807 & E-ISSN: 0976-8815, Volume 3, Issue 2, 2012, pp.-103-108.
Available online at http://www.bioinfo.in/contents.php?id=85

THERMOELASTIC PROBLEM OF AN ISOTROPIC CIRCULAR ANNULAR FIN

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Received: June 18, 2012; Accepted: June 25, 2012

Abstract- In this paper, an attempt has been made to determine the temperature distribution, displacement, and thermal stresses of a thin annular fin when the boundary conditions are known. Integral transform techniques have been utilized to obtain the solution of the problem. The results are obtained in the form of infinite series in terms of Bessel's function.
Keywords- Integral transform, Transient problem Thermal stresses, Circular annular fin,

Citation: Hiranwar P.C. and Khobragade N.W. (2012) Thermoelastic Problem of an Isotropic Circular Annular Fin. Journal of Statistics and Mathematics, ISSN: 0976-8807 & E-ISSN: 0976-8815, Volume 3, Issue 2, pp.-103-108.

Introduction

The transient thermal stresses in an annular fin have been investigated by Wu (1997) and the solution has been obtained with the help of exponential-like solution. Deshmukh (2003) analyzed the transient thermal stresses applying Hankel transform and Fourier transform techniques, where temperature transfer condition was prescribed on the surface of an annular fin. The boundary value problem described by Wu et al. and Deshmukh et al. occurs in design application.

The present paper attempts to generalize the problem considered by Shang-Sheng Wu and obtain the exact solution of the problem. This paper further investigates the transient thermal stresses by the use of finite Marchi- Zgrablich transform and Laplace transform techniques. The results are obtained in the form of infinite series in terms of Bessel's function.

Statement of the Problem

Consider an isotropic circular annular fin occupying the space $D=\{(x,y,z)\in R^3 : a \le (x^2+y^2)^{1/2} \le b, \quad 0 \le z \le l\}$. The material of the fin is isotropic, homogenous and all properties are assumed to be constant. We assume that the fin is of a small thickness and its boundary surfaces remain traction free (as shown in Figure 1). The governing equations and boundary conditions for the stress field [5, 8] consist of:

a non-zero stress strain-displacement equation [7]

$$\varepsilon_r = \frac{\partial u}{\partial r}, \qquad \varepsilon_\phi = \frac{u}{r}, \tag{1}$$

a single equilibrium equation

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\phi}{r} = 0, \tag{2}$$

two equations of stress-strain-temperature relations [9]

$$\sigma_r = \frac{E}{1-\upsilon^2}\left[\varepsilon_r + \upsilon\varepsilon_\phi - (1+\upsilon)\alpha T\right] \tag{3}$$

$$\sigma_\phi = \frac{E}{1-\upsilon^2}\left[\varepsilon_\phi + \upsilon\varepsilon_r + (1+\upsilon)\alpha T\right] \tag{4}$$

and, two boundary conditions

$$\sigma_r = 0 \quad \text{at} \quad r = a \tag{5}$$

$\sigma_r = 0$ at $r = b$ (6)

Combining equations (1)-(4), integrating twice with respect to r, and applying the boundary conditions (5,6), one obtains the stress -displacement relations as

$$\sigma_r = -\frac{\alpha E}{r^2}\int_a^r (T - T_\infty)\eta\, d\eta + \frac{\alpha E}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)\int_a^b (T - T_\infty)\eta\, d\eta \quad (7)$$

$$\sigma_\phi = -\alpha E(T - T_\infty) + \frac{\alpha E}{r^2}\int_a^r (T - T_\infty)\eta\, d\eta + \frac{\alpha E}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)\int_a^b (T - T_\infty)\eta\, d\eta \quad (8)$$

Substituting these expressions, for the radial & tangential stresses into the stress equilibrium equation (2), leads to the following governing equation for the thermoelastic equilibrium of the circular annular fin in qualitative agreement with equations found earlier [3 and 6]:

$$k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) - \frac{2h}{l}\left(T - T_\infty\right) = \rho c \frac{\partial T}{\partial t}$$

$a \le r \le b$, $0 \le z \le l$, $t > 0$ (9)

Introducing the following dimensionless parameters as defined in the nomenclature (Appendix A):

$$\theta = \frac{k(T - T_\infty)}{(q_b a)},\quad \xi = \frac{r}{a},\quad \zeta = \frac{z}{a},\quad L = \frac{l}{a}$$

$$\tau = \frac{(kt)}{(\rho c a^2)},\quad R = \frac{b}{a},\quad N^2 = \frac{2ha^2}{kl}$$

The equation (9) can be written in the dimensionless form as:

$$\frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial \theta}{\partial \xi} + \frac{\partial^2 \theta}{\partial \zeta^2} - N^2\theta = \frac{\partial \theta}{\partial \tau}$$

$1 \le \xi \le R$, $0 \le \zeta \le L$, $\tau > 0$ (10)

The dimensionless radial and tangential stresses S_r and S_f in terms of the dimensionless displacement function are,

$$S_r = -\left(\frac{1}{\xi^2}\right)\int_1^\xi \theta\xi\, d\xi + \left(\frac{1}{\xi^2}\frac{\xi^2 - 1}{R^2 - 1}\right)\int_1^R \theta\xi\, d\xi \quad (11)$$

$$S_\phi = -\theta + \left(\frac{1}{\xi^2}\right)\int_1^\xi \theta\xi\, d\xi + \left(\frac{1}{\xi^2}\frac{\xi^2 + 1}{R^2 - 1}\right)\int_1^R \theta\xi\, d\xi \quad (12)$$

The various dimensionless boundary conditions are defined to determine the influence of the thermal boundary conditions on the thermal stresses as:

The initial condition

$\theta(\xi, \zeta, \tau) = 0$, for all $1 \le \xi \le R$, $0 \le \zeta \le L$ and $\tau = 0$ (13)

the boundary conditions

$$\left[\theta(\xi, \zeta, \tau) + k_1 \frac{\partial \theta(\xi, \zeta, \tau)}{\partial \xi}\right]_{\xi = 1} = F_1(\zeta, \tau)$$

, for all $0 \le \zeta \le L$ and $\tau > 0$ (14)

$$\left[\theta(\xi, \zeta, \tau) + k_2 \frac{\partial \theta(\xi, \zeta, \tau)}{\partial \xi}\right]_{\xi = R} = F_R(\zeta, \tau)$$

, for all $0 \le \zeta \le L$ and $\tau > 0$ (15)

where k_1 and k_2 are the radiation constants on the two annular fin surfaces

$$\left[\theta(\xi, \zeta, \tau) + c \frac{\partial \theta(\xi, \zeta, \tau)}{\partial \zeta}\right]_{\zeta = 0} = f(\xi, \tau)$$

, for all $1 \le \xi \le R$ and $\tau > 0$ (16)

$$\left[\theta(\xi, \zeta, \tau) + c \frac{\partial \theta(\xi, \zeta, \tau)}{\partial \zeta}\right]_{\zeta = L} = g(\xi, \tau)$$

, for all $1 \le \xi \le R$ and $\tau > 0$ (17)

where $F_1(\zeta, \tau)$ and $F_R(\zeta, \tau)$ are known constants and here it is set to be zero, as this assumption, commonly made in the literatures [6] and [8], leads to considerable mathematical simplification and the function $f(\xi, \tau)$ and $g(\xi, \tau)$ are assumed to be known.

The equations (10) to (17) constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

Applying finite Marchi-Zgrablich integral transform defined in [8] to the equations (10) to (13), (16) and (17) one obtains

$$\frac{d^2\bar{\theta}(n, \zeta, \tau)}{d\zeta^2} - (\mu_n^2 + N^2)\bar{\theta}(n, \zeta, \tau) = \frac{d\bar{\theta}(n, \zeta, \tau)}{d\tau} \quad (18)$$

where m_n are the positive roots of equation $J_0(k_1, \mu a).Y_0(k_2, \mu b) - J_0(k_2, \mu b).Y_0(k_1, \mu a) = 0$.

Applying Laplace transform to the equation (18), one obtains

$$\frac{d^2\bar{\theta}^*(n, \zeta, s)}{d\zeta^2} - q^2\bar{\theta}^*(n, \zeta, s) = 0 \quad (19)$$

Equation (19) is a second order differential equation whose solution is given by

$$\bar{\theta}^*(n, \zeta, s) = Ae^{q\zeta} + Be^{-q\zeta} \quad (20)$$

where $q = (\mu_n^2 + N^2 + s)^{1/2}$, s is the Laplace transform parameter, A and B are two arbitrary constants.

Using equations (16) and (17) in (20), we obtain the values of A and B. Substituting these values in equation (20), we get

$$\bar{\theta}^*(\xi, \zeta, s) = \left(\frac{\bar{g}^*(n, s)}{1 - c^2q^2}\right)\left(\frac{\sinh(q\zeta) - cq\cosh(q\zeta)}{\sinh(qL)}\right) - \left(\frac{\bar{f}^*(n, s)}{1 - c^2q^2}\right)\left(\frac{\sinh q(\zeta - L) - cq\cosh q(\zeta - L)}{\sinh(qL)}\right) \quad (21)$$

Applying inversion of Laplace transform and finite Marchi-Zgrablich transform to the equation (21), one obtains

$$\theta(\xi,\zeta,\tau)=\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\left(\frac{1}{c_n}\right)\sum_{m=0}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi\zeta}{L}\right)-\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times S_0(k_1,k_2,\mu_n r)\times\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$-\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\left(\frac{1}{c_n}\right)\sum_{m=0}^{\infty}m(-1)^{m+1}\left[\sin m\pi\left(\frac{\zeta-L}{L}\right)-\left(\frac{m\pi}{L}\right)\cos m\pi\left(\frac{\zeta-L}{L}\right)\right]$$

$$\times S_0(k_1,k_2,\mu_n r)\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt' \quad (22)$$

where

$$C_n=\frac{b^2}{2}\left\{S_p^2(k_1,k_2,\ \mu_n b)-J_{p-1}(k_1,k_2,\ \mu_n b)J_{p+1}(k_1,k_2,\ \mu_n b)\right\}$$

$$-\frac{a^2}{2}\left\{S_p^2(k_1,k_2,\ \mu_n a)-J_{p-1}(k_1,k_2,\ \mu_n a)J_{p+1}(k_1,k_2,\ \mu_n a)\right\}$$

and

$$S_p(k_1,k_2,\ \mu_n\xi)=J_p(\mu_n\xi)\left\{Y_p(k_1,\mu_n a)+Y_p(k_2,\mu_n b)\right\}$$

$$-Y_p(\mu_n\xi)\left\{Y_p(k_1,\mu_n a)+Y_p(k_2,\mu_n b)\right\}$$

being

$$J_p(k_i,\mu\xi)=J_p(\mu\xi)+k_i\ \mu J'_p(\mu\xi)$$

and for $i=1,2$

$$Y_p(k_i,\mu\xi)=Y_p(\mu\xi)+k_i\ \mu Y'_p(\mu\xi)$$

Here $J_p(mx)$ and $Y_p(mx)$ $=\frac{1}{2}\pi\operatorname{cosec}(p\pi)[J_{-p}(\mu x)-e^{-ip\pi}J_p(\mu x)]$ are Bessel's functions of first and second kind respectively of order p.

Substituting the value of temperature distribution function in the given thermal stresses equations (11) and (12), one obtains

$$S_r=\left(\frac{1}{\xi^2}\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\left(\frac{1}{c_n}\right)\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)-\sin\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times\int_1^{\xi}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{1}{\xi^2}\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\left(\frac{1}{c_n}\right)\sum_{m=0}^{\infty}m(-1)^{m+1}\left[\sin m\pi\left(\frac{\zeta-L}{L}\right)-\left(\frac{m\pi}{L}\right)\cos m\pi\left(\frac{\zeta-L}{L}\right)\right]$$

$$\times\int_1^{\xi}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{1}{\xi^2}\frac{(\xi^2-1)}{(R^2-1)}\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\left(\frac{1}{c_n}\right)\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi\zeta}{L}\right)-\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times\int_1^{R}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{2\pi(1-c)}{\xi^2L^2}\frac{(\xi^2-1)}{(R^2-1)}\right)\sum_{m=1}^{\infty}\left(\frac{1}{c_n}\right)m(-1)^{m+1}\left[\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi(\zeta-L)}{L}\right)-\sin\left(\frac{m\pi(\zeta-L)}{L}\right)\right]$$

$$\times\int_1^{R}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt' \quad (23)$$

$$S_\phi=\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\left(\frac{1}{c_n}\right)\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)-\sin\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times S_0(k_1,k_2,\mu_n\xi)\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{m=1}^{\infty}\left(\frac{1}{c_n}\right)m(-1)^{m+1}\left[\sin\left(\frac{m\pi(\zeta-L)}{L}\right)-\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi(\zeta-L)}{L}\right)\right]$$

$$\times S_0(k_1,k_2,\mu_n\xi)\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{2\pi}{\xi^2}\frac{(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\frac{1}{c_n}\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi\zeta}{L}\right)-\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times\int_1^{\xi}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{2\pi(1-c)}{\xi^2L^2}\right)\sum_{m=1}^{\infty}\frac{1}{c_n}m(-1)^{m+1}\left[\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi(\zeta-L)}{L}\right)-\sin\left(\frac{m\pi(\zeta-L)}{L}\right)\right]$$

$$\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'\times\int_1^{\xi}\xi S_0(k_1,k_2,\mu_n\xi)d\xi$$

$$+\left(\frac{1}{\xi^2}\frac{(\xi^2+1)}{(R^2-1)}\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\frac{1}{c_n}\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi\zeta}{L}\right)-\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times\int_1^{R}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{1}{\xi^2}\frac{(\xi^2+1)}{(R^2-1)}\frac{2\pi(1-c)}{L^2}\right)\sum_{m=1}^{\infty}\frac{1}{c_n}m(-1)^{m+1}\left[\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi(\zeta-L)}{L}\right)-\sin\left(\frac{m\pi(\zeta-L)}{L}\right)\right]$$

$$\times\int_1^{R}\xi S_0(k_1,k_2,\mu_n\xi)d\xi\times\int_0^t\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt' \quad (24)$$

Convergence of the Series Solution

In order for the solution to be meaningful, the series expressed in equation (23) should converge for all $D: \ 1 \le \xi \le R$, $0 \le \zeta \le L$ and should further investigate the conditions which has to be imposed on the functions $F_1(\zeta,\tau)$, $F_R(\zeta,\tau)$, $A(\xi,\tau)$, $B(\xi,\tau)$, and $f(\xi,\tau)$, so that the convergence of the series expansion for $\theta(\xi,\zeta,\tau)$ is valid. The expression for temperature (23) for $\theta(\xi,\zeta,\tau)$ in dimensionless parameters may be expressed as

$$\theta(\xi,\zeta,\tau)=\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\frac{1}{c_n}\sum_{m=0}^{M_0}m(-1)^{m+1}\left[\sin\left(\frac{m\pi\zeta}{L}\right)-\left(\frac{m\pi}{L}\right)\cos\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times S_0(k_1,k_2,\mu_n\xi)\times\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{n=1}^{\infty}\frac{1}{c_n}\sum_{m=0}^{M_0}m(-1)^{m+1}\left[\frac{m\pi}{L}\cos m\pi\left(\frac{\zeta-L}{L}\right)-\sin m\pi\left(\frac{\zeta-L}{L}\right)\right]$$

$$\times S_0(k_1,k_2,\mu_n\xi)\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$=\sum_{n=1}^{\infty}(Q_nC_{1n})S_nJ_0(\mu_n\xi)-\sum_{n=1}^{\infty}(Q_nC_{2n})S_nY_0(\mu_n\xi) \tag{25}$$

where

$$C_{1n}=Y_0(k_1,\mu_na)+Y_0(k_2,\mu_nb)$$

$$C_{2n}=J_0(k_1,\mu_na)+J_0(k_2,\mu_nb) \qquad Q_n=\frac{2a}{\eta C_n},$$

$$Y_0(k_i,\mu_n\xi)=Y_0(\mu\xi)+k_i\mu Y_0'(\mu\xi),$$

$$J_0(k_i,\mu_n\xi)=J_0(\mu\xi)+k_i\mu J_0'(\mu\xi),$$

and

$$S_r=-\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{m=0}^{M_0}m(-1)^{m+1}\left[\sin\left(\frac{m\pi\zeta}{L}\right)-\frac{m\pi}{L}\cos\left(\frac{m\pi\zeta}{L}\right)\right]$$

$$\times\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'$$

$$+\left(\frac{2\pi(1-c)}{L^2}\right)\sum_{m=0}^{M_0}m(-1)^{m+1}\left[\left(\frac{m\pi}{L}\right)\cos m\pi\left(\frac{\zeta-L}{L}\right)-\sin m\pi\left(\frac{\zeta-L}{L}\right)\right]$$

$$\times\int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt' \tag{26}$$

After taking in to account the asymptotic behaviour of μ_n, $S_0(k_1,k_2,\mu_n\xi)$ and C_n given in [8], it is observed that the series expansion (26) for $\theta(\xi,\zeta,\tau)$ will be convergent, if

$$\left.\begin{array}{l}\int_0^t\left(\frac{\overline{g}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt' \\ \int_0^t\left(\frac{\overline{f}(n,t')}{1-c^2\mu_n^2}\right)\times e^{-\left[\left(\frac{m\pi}{L}\right)^2+\mu_n^2+N^2\right](t-t')}dt'\end{array}\right\}=0\left(\frac{1}{\mu_n^k}\right),\ k>0 \tag{27}$$

Here, $\overline{g}(n,t')$, $\overline{f}(n,t')$ in equation (27) can be chosen as one of the following functions or the combinations thereof with addition or multiplication or both, as the laws of combination: *Constant, Sin (wt), Cos(wt),* e^{-kt} *or polynomials in x,* S_n, $J_0(m_nx)$, $Y_0(m_nx)$, Q_nC_{1n}, Q_nC_{2n} are convergent and thus, $\theta(\xi,\zeta,\tau)$ is convergent to a limit $\{\theta(\xi,\zeta,\tau)\}_{\xi=R,\zeta=L}$. Here, we consider that the convergence of a series for $\xi=R$ implies to the convergence for all $\xi\le R$, and $\zeta=L$ implies to the convergence for all $\zeta\le L$.

Special Case and Numerical Results

Set $f(\xi,\tau)=(1-e^{-t})\,e^{\xi}(1+c)$, $g(\xi,\tau)=(1-e^{-t})\,e^{\xi}e^{L}(1+c)$, a = 0.5, b = 1, c=1, R = 2, $k=0.375$, $k_1=0.25$, $k_2=0.25$, τ = 1 sec. and L = 2, l = 1, $\eta=1$ in the equation (24) to obtain

$$\theta(\xi,\tau)=\sum_{n=1}^{\infty}\frac{1}{c_n}\left\{\frac{2\pi ka^2}{\eta^2}\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\left(\frac{m\pi a}{\eta}\right)\cos\left(\frac{m\pi aL}{\eta}\right)-\sin\left(\frac{m\pi aL}{\eta}\right)\right]\right.$$

$$\left.\times\int_0^{\tau}(1-e^{-t})\,e^{\xi}(1+c),\ e^{-\left[\left(\frac{m\pi a}{\eta}\right)^2+\mu_n^2+N^2\right](\tau-t)}dt\right\}$$

$$\times S_0(k_1,k_2,\mu_n\xi)r_0\ \ S_0(k_1,k_2,\mu_nr_0)$$

$$+\frac{2\pi ka^2}{\eta^2}\sum_{m=1}^{\infty}(2m-1)(-1)^{m+1/2}$$

$$\times\left[\sin\left(\frac{(m-1/2)\pi aL}{\eta}\right)-\left(\frac{(m-1/2)\pi a}{\eta}\right)\cos\left(\frac{(m-1/2)\pi aL}{\eta}\right)\right]$$

$$\left.\times\int_0^{\tau}(1-e^{-t})\,e^{\xi}e^{L}(1+c)\ e^{-\left[\left(\frac{(m-1/2)\pi a}{\eta}\right)^2+\mu_n^2+N^2\right](\tau-t)}dt\right\}$$

$$\times S_0(k_1,k_2,\mu_n,\xi)\quad r_0\ \ S_0(k_1,k_2,\mu_nr_0) \tag{28}$$

Conclusion

The temperature, displacements and thermal stresses of a thin annular fin have been obtained, using finite Integral transform techniques, when the boundary conditions are known. The results are obtained in terms of Bessel's function in the form of infinite series.

The series of solutions converge, provided we take sufficient number of terms in the series. Since the thickness of annular fin is very small, the series of solution given here will be definitely convergent. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the series of expressions. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

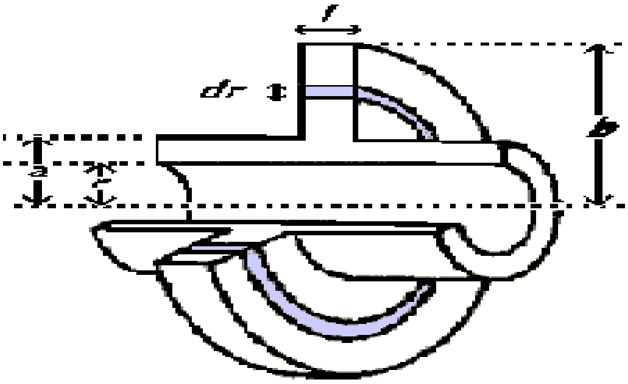

Fig. 1- Cross-sectional view of an isotropic circular annular fin

Fig. (2) and (3) shows that as time and radius increases, temperature and radial stresses goes on increasing respectively,

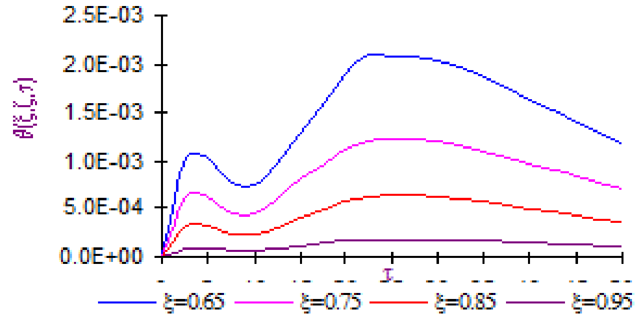

Fig. 2-

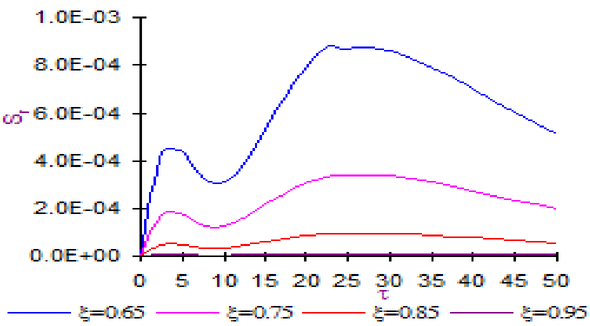

Fig. 3-

but in Fig. (4) as time and radius increases, the Tangential stresses goes on decreasing. Fig. (5) and (6) shows that as time and radius increases, temperature and radial stresses goes on decreases respectively, but in Fig. (7) as time and radius increases, the Tangential stresses goes on increasing.

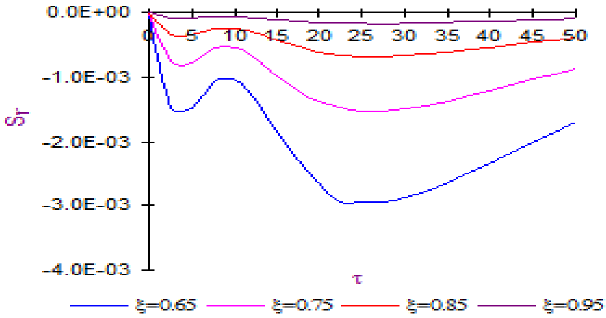

Fig. 4-

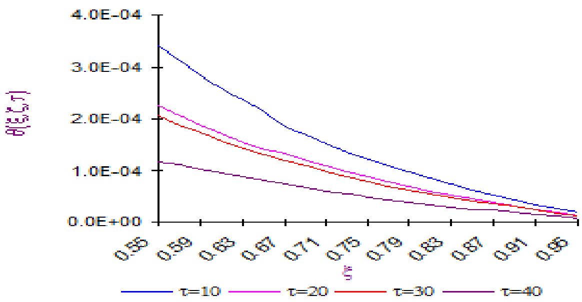

Fig. 5-

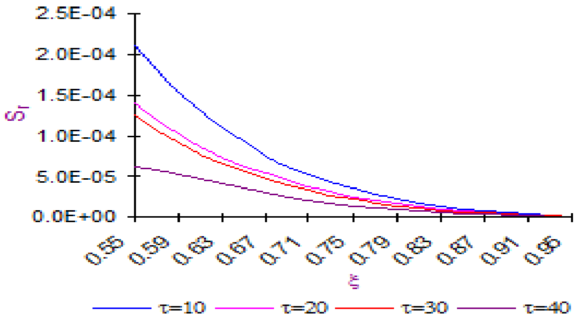

Fig. 6-

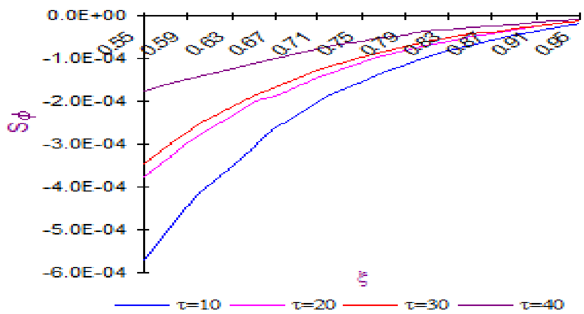

Fig. 7-

Appendix A

NOMENCLATURE	
a, b	Inner and outer radii of the fin
c	Specific heat of material of the fin
C_1, C_2	Constants
E	Young's modulus of material of the fin
h	Heat transfer coefficient
k	Thermal conductivity of material of the fin
N	Dimensionless parameter
q_b	Heat flux from the base of the fin
R	Dimensionless outer radius,
S_r, S_ϕ	Dimensionless radial and tangential stresses
T	Temperature of the fin
T_∞	Ambient temperature
t	Time
u	Radial displacement
α	Linear thermal expansion coefficient of material of the fin
L	Thickness of the fin
ε_r -ε_ϕ	Radial and tangential strains
θ	Dimensionless temperature of the fin
ν	Poisson's ratio of material of the fin
σ_r, σ_ϕ	Radial and tangential stresses
τ	Dimensionless time
r, ϕ	Polar coordinates
ζ	Dimensionless thickness

Acknowledgement
The authors are thankful to University Grant Commission, New Delhi for providing the partial financial assistance under major research project scheme.

References

[1] Crandall S.H., Dahl N.C. and Lardner T.J. (1978) *An introduction to the Mechanics of solids*, McGraw-Hill, New York.
[2] Sneddon I.N. (1972) *The use of integral transform, Mc Graw Hill, New York*.
[3] Shang-Sheng Wu (1997) *Journal of Thermal stresses*, 20, 591 -615.
[4] Timoshenko S.P. and Goodier J.N. (1970) *Theory of Elasticity, McGraw-Hill, New York*.
[5] Boley B.A. and Weiner J.H. (1960) *Theory of thermal stresses, Johan Wiley and Sons, New York*.
[6] Warade R.W. and Deshmukh K.C. (2003) *Acta. Ciencia Indica.*, XXIXM(3), 507-516.
[7] Sokolnikoff I.S. (1956) *Mathematical Theory of Elasticity, 2nd ed, Mc Graw Hill, New York*.
[8] Marchi E. and Zgrablich J. (1964) *Proceeding Edinburg Math. Soc.* 14(11), 159-164.
[9] Carslaw S.H. and Jaeger J.C. (1959) *Conduction of heat in solids, 2nd ed. Oxford University Press, New York*.