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TRANSIENT THERMOELASTIC PROBLEM OF A SEMI INFINITE CYLINDER WITH HEAT SOURCES

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Abstract- We apply integral transformation techniques to study thermoelastic response of a semi infinite hollow cylinder, in general in which sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. The results are obtained as series of Bessel functions. Numerical calculations are carried out for a particular case of a cylinder made of Aluminum metal and the results are depicted in figures.
Keywords- Transient response, cylinder, temperature distribution, thermal stress, integral transform

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Introduction

Nowacki [1] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede [9] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there aren't many investigations on transient state. Roy Choudhuri [8] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In a recent work, some problems have been solved by Noda, et al. [5] and Deshmukh, et al. [1]. In all aforementioned investigations, an axisymmetrically heated plate has been considered. Nasser [6,7] proposed the concept of heat sources in generalized thermoelasticity and applied to a thick plate problem. They have not however considered any thermoelastic problem with boundary conditions of radiation type, in which sources are generated according to the linear function of the temperatures, which satisfies the time-dependent heat conduction equation.

This paper is concerned with the transient thermoelastic problem of a finite length hollow cylinder in which sources are generated according to the linear function of temperature, occupying the space $D = \{(x, y, z) \in R^3 : \quad a \le (x^2 + y^2)^{1/2} \le b,$ $-h \le z \le h\}$, where $r = (x^2 + y^2)^{1/2}$ with radiation type boundary conditions.

The success of this novel research mainly lies in the new mathematical procedures with much simpler approach for optimization for the design in terms of material usage and performance in engineering problem, particularly in the determination of thermoelastic behaviour in cylinder engaged as the foundation of pressure vessels, furnaces, etc.

Statement of the Problem

Consider a semi infinite hollow cylinder in which sources are generated according to the linear function of temperature. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type, the quasi-static thermal stresses are required to be determined. The equation for heat conduction is $\theta(r,z,t),$ the temperature, in cylindrical coordinates, is:

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2}\right]+\Theta(r,z,t,\theta)=\frac{\partial\theta}{\partial t} \tag{2.1}$$

Where $\Theta(r,z,t,\theta)$ is the internal source function and $\kappa=\lambda/\rho C,\quad \lambda$ being the thermal conductivity of the material, ρ is the density and C is the calorific capacity, assumed to be constant. For convenience, we consider the undergiven functions as the superposition of the simpler function [2]:

$$\Theta(r,z,t,\theta)=\Phi(r,z,t)+\psi(t)\,\theta(r,z,t) \tag{2.2}$$

and

$$T(r,z,t)=\theta(r,z,t)\exp\left[-\int_0^t\psi(\zeta)d\zeta\right], \tag{2.3}$$

$$\chi(r,z,t)=\Phi(r,z,t)\exp\left[-\int_0^t\psi(\zeta)d\zeta\right] \tag{2.4}$$

or for the sake of simplicity, we consider

$$\chi(r,z,t)=\frac{\delta(r-r_0)\,\delta(z-z_0)}{2\pi r_0}\exp(-\omega t),\qquad a\le r_0\le b,$$

$$0\le z_0\le\infty,\qquad \omega>0$$

Substituting equations (2.2) and (2.3) to the heat conduction equation (2.1), one obtains

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \tag{2.5}$$

where κ is the thermal diffusivity of the material of the cylinder (which is assumed to be constant), subject to the initial and boundary conditions

$$M_t(T,1,0,0)=T_0 \quad \text{for} \quad a\le r\le b,\quad 0\le z<\infty \tag{2.6}$$

$$M_r(T,1,k_1,a)=0, \quad \text{for} \quad 0\le z<\infty,\quad t>0 \tag{2.7}$$

$$M_r(T,1,k_2,b)=0 \quad \text{for} \quad 0\le z<\infty,\quad t>0 \tag{2.8}$$

$$M_z(T,1,1,\infty)=0 \quad \text{for} \quad a\le r\le b,\quad t>0 \tag{2.9}$$

$$M_z(T,1,1,0)=f(r,t) \quad \text{for} \quad a\le r\le b,\quad t>0 \tag{2.10}$$

The most general expression for these conditions can be given by

$$M_\vartheta(f,\bar{k},\bar{\bar{k}},\$)=(\bar{k}\,f+\bar{\bar{k}}\,\hat{f})_{\vartheta=\$}$$

where the prime (^) denotes differentiation with respect to ϑ; $\delta(r-r_0)$ is the Dirac Delta function with $a\le r_0\le b$; $\omega>0$ is a constant; $\exp(-\omega t)\delta(r-r_0)$ is the additional sectional heat available on its surface at *z* = *h*; T_0 is the reference temperature; $\bar{k}$ and $\bar{\bar{k}}$ are radiation coefficients of the cylinder, respectively.

The Navier's equations without the body forces for axisymmetric two-dimensional thermoelastic problem can be expressed as [**5**]

$$\nabla^2 u_r-\frac{u_r}{r^2}+\frac{1}{1-2\upsilon}\frac{\partial e}{\partial r}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial\theta}{\partial r}=0 \tag{2.11}$$

$$\nabla^2 u_z-\frac{1}{1-2\upsilon}\frac{\partial e}{\partial z}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial\theta}{\partial z}=0 \tag{2.12}$$

Where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation *e* as

$$e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}$$

The displacement functions in the cylindrical coordinate system are represented by the Goodier's thermoelastic displacement potential *f* and Michell's function *M* as [6]

$$u_r=\frac{\partial\phi}{\partial r}-\frac{\partial^2 M}{\partial r\partial z}, \tag{2.13}$$

$$u_z=\frac{\partial\phi}{\partial z}+2(1-\upsilon)\nabla^2 M-\frac{\partial^2 M}{\partial^2 z} \tag{2.14}$$

In which Goodier's thermoelastic potential must satisfy the equation

$$\nabla^2\phi=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\theta \tag{2.15}$$

and the Michell's function *M* must satisfy the equation

$$\nabla^2(\nabla^2 M)=0 \tag{2.16}$$

Where $\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}$

The component of the stresses are represented by the use of the potential ϕ and Michell's function *M* as

$$\sigma_{rr}=2G\left\{\left(\frac{\partial^2\phi}{\partial r^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\upsilon\nabla^2 M-\frac{\partial^2 M}{\partial r^2}\right)\right\}, \tag{2.17}$$

$$\sigma_{\theta\theta}=2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\upsilon\nabla^2 M-\frac{1}{r}\frac{\partial M}{\partial r}\right)\right\}, \tag{2.18}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}, \quad (2.19)$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\} \quad (2.20)$$

Where G and υ are the shear modulus and Poisson's ratio respectively.

The equations (2.1) to (2.20) constitute the mathematical formulation of the problem under consideration.

Solution of the Problem

In order to solve fundamental differential equation (2.4) under the boundary condition (2.6), we firstly introduce the integral transform [3] of order *n* over the variable *r*. Let *n* be the parameter of the transform, then the integral transform and its inversion theorem are written as

$$\bar{g}_p(n) = \int_a^b r\, g(r)\, S_p(k_1, k_2, \mu_n r)\, dr,$$

$$g(r) = \sum_{n=1}^{\infty} (\bar{g}_p(n)/C_n)\, S_p(k_1, k_2, \mu_n r) \quad (3.1)$$

Where $\bar{g}_p(n)$ is the transform of $g(r)$ with respect to nucleus $S_p(k_1, k_2, \mu_n r)$.

Applying the transform defined in equation (3.1) to the equations (2.3) to (2.5) and (2.7) and taking into account equation (2.6), one obtains

$$\kappa\left[-\mu_n^2 \bar{T}(n,z,t) + \frac{\partial^2 \bar{T}(n,z,t)}{\partial z^2}\right] + \bar{\chi}(n,z,t) = \frac{\partial \bar{T}(n,z,t)}{\partial t} \quad (3.2)$$

$$M_t(\bar{T}, 1, 0, 0) = \bar{T}_0 \quad (3.3)$$

$$M_z(\bar{T}, 1, 1, \infty) = 0, \quad (3.4)$$

$$M_z(\bar{T}, 1, 1, 0) = \bar{f}(n,t) \quad (3.5)$$

$$\bar{\chi}(n,z,t) = r_0\, S_0(k_1, k_2, \mu_n r_0)\, \delta(z - z_0)\exp(-\omega t) \quad (3.6)$$

Where $\bar{T}$ is the transformed function of T and n is the transform parameter. The eigenvalues μ_n are the positive roots of the characteristic equation

$$J_0(k_1, \mu a)\, Y_0(k_2, \mu b) - J_0(k_2, \mu b)\, Y_0(k_1, \mu a) = 0$$

The kernel function $S_0(k_1, k_2, \mu_n r)$ can be defined as

$$S_0(k_1,k_2,\mu_n r) = J_0(\mu_n r)[Y_0(k_1,\mu_n a) + Y_0(k_2,\mu_n b)] - Y_0(\mu_n r)[J_0(k_1,\mu_n a) + J_0(k_2,\mu_n b)]$$

With

$$J_0(k_i, \mu r) = J_0(\mu r) + k_i\, \mu J_0'(\mu r)$$

$$Y_0(k_i, \mu r) = Y_0(\mu r) + k_i\, \mu Y_0'(\mu r) \quad \text{for } i = 1, 2$$

and

$$C_n = \int_a^b r[S_0(k_1, k_2, \mu_n b)]^2\, dr$$

in which $J_0(\mu r)$ and $Y_0(\mu r)$ are Bessel functions of first and second kind of order $p = 0$ respectively.

We introduce another integral transform [2] that responds to the boundary conditions of type (2.7):

$$\bar{f}(m,t) = \int_0^{\infty} f(z,t)\cos pz\, dz,$$

$$f(z,t) = \frac{2}{\pi}\sum_{m=1}^{\infty} \bar{f}(m,t)\cos pz \quad (3.7)$$

Further applying the transform defined in equation (3.7) to the equations (3.2), (3.3) and (3.5) and using equation (3.4) one obtains

$$\kappa\left[-\mu_n^2 \bar{T}^*(n,m,t) - a_m^2 \bar{T}^*(n,m,t)\right] + \bar{\chi}^*(n,m,t) = \frac{d\bar{T}^*(n,m,t)}{dt} \quad (3.8)$$

$$M_t(\bar{T}^*, 1, 0, 0) = \bar{T}_0^* \quad (3.9)$$

$$\bar{\chi}^*(n,m,t) = r_0\, S_0(k_1, k_2, \mu_n r_0)\, P_m(z_0)\exp(-\omega t) \quad (3.10)$$

where $\bar{T}^*$ is the transformed function of $\bar{T}$ and m is the transform parameter.

After performing some calculations on equation (3.8) and using equation (3.5), the reduction is made to linear first order differential equation as

$$\frac{d\bar{T}^*}{dt} + \kappa\, \Lambda_{n,m}\, \bar{T}^* = H(\mu_n, a_m) \quad (3.11)$$

Where $a_m^2 = \frac{p^2}{2}$, $\Lambda_{n,m} = \mu_n^2 + a_m^2$

and

$$H(\mu_n, a_m) = r_0\, S_0(k_1, k_2, \mu_n r_0)\exp(-\omega t)$$

The general solution of equation (26) is a function

$$\bar{T}^*(n,m,t)\exp(\kappa\Lambda_{n,m} t) = \frac{H(\mu_n, a_m)}{\kappa\Lambda_{n,m} - \omega}\exp(-\kappa\Lambda_{n,m} t) + C \quad (3.12)$$

Using equations (3.4) in equation (3.7), we obtain the values of arbitrary constants *C*. Substituting these values in (3.7) one obtains the transformed temperature solution as

$$\bar{T}^*(n,m,t) = \frac{H(\mu_n, a_m)}{\kappa\Lambda_{n,m} - \omega}\exp(-\omega t) + \left[\bar{T}_0^* - \frac{H(\mu_n, a_m)}{\kappa\Lambda_{n,m} - \omega}\right]\exp(-\kappa\Lambda_{n,m} t) \quad (3.13)$$

Applying inversion theorems of transformation rules defined in equations (2.17) to the equation (3.8), there results

$$\bar{T}(n,z,t)=\frac{2}{\pi}\sum_{m=1}^{\infty}\bar{f}(m,t)[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\cos pz \tag{3.14}$$

and then accomplishing inversion theorems of transformation rules defined in equations (3.2) on equation (3.9), the temperature solution is shown as follows:

$$T(r,z,t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\bar{f}(m,t)[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\cos pz\right\}$$

$$\times S_0(k_1,k_2,\mu_n r) \tag{3.15}$$

$$\wp_{n,m}=\frac{H(\mu_n,a_m)}{\kappa\Lambda_{n,m}-\omega}$$

Where

Taking into account the first equation of equation (2.3), the temperature distribution is finally represented by

$$\theta(r,z,t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\bar{f}(m,t)[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\cos pz\right\}$$

$$\times S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{3.16}$$

The function given in equation (3.11) represents the temperature at every instant and at all points of finite hollow cylinder of finite height when there are conditions of radiation type.

Determination of Thermoelastic Displacement

Referring to the fundamental equation (2.1) and its solution (3.16) for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermoelastic displacement potential ϕ governed by equation (2.11) are represented by

$$\phi(r,z,t)=\frac{2}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\bar{f}(m,t)}{\Lambda_{n,m}}[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\cos pz\right\}$$

$$\times S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.1}$$

Similarly, the solution for Michell's function M are assumed so as to satisfy the governed condition of equation (2.12) as

$$M(r,z,t)=\frac{2}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\bar{f}(m,t)}{\Lambda_{n,m}}[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times[\sinh(\mu_n z)+z\cosh(\mu_n z)]S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.2}$$

In this manner two displacement functions in the cylindrical coordinate system f and M are fully formulated. Now, in order to obtain the displacement components, we substitute the values of thermoelastic displacement potential ϕ and Michell's function M in equations (2.9) and (2.10), one obtains

$$u_r=\frac{2}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\bar{f}(m,t)}{\Lambda_{n,m}}[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times[\cos pz-(\mu_n+1)\cosh(\mu_n z)-\mu_n z\sinh(\mu_n z)]S_0'(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.3}$$

$$u_z=\frac{2}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\bar{f}(m,t)}{\Lambda_{n,m}}[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times[-a_m(\cos pz)-4\upsilon\mu_n\sin(\mu_n z)-\mu_n^2(\sinh(\mu_n z)+z\cosh(\mu_n z))]$$

$$\times S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.4}$$

Thus, making use of the two displacement components, the dilation is established as

Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ϕ from equation (3.12) and Michell's function M from equation (3.13) in equations

$$e=\frac{2}{\lambda}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\bar{f}(m,t)}{\Lambda_{n,m}}[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times[-\cos pz+(\mu_n+1)\cosh(\mu_n z)+\mu_n z\sinh(\mu_n z)-a_m^2\cos pz-(4\upsilon+1)\mu_n^2\cosh(\mu_n z)$$

$$-\mu_n^3(\cosh(\mu_n z)+z\sinh(\mu_n z))]S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.5}$$

(2.13) to (2.16), one obtains

$$\sigma_{rr}=\frac{4G}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\bar{f}(m,t)[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times\{-\cos pz[\Lambda_{n,m}^{-1}S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]-\Lambda_{n,m}^{-1}[2\upsilon\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)$$

$$-[(\mu_n+1)\cosh(\mu_n z)+z\mu_n\sinh(\mu_n z)]S_0''(k_1,k_2,\mu_n r)]\}\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.6}$$

$$\sigma_{\theta\theta}=\frac{4G}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\bar{f}(m,t)[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times\{-\cos pz[r\Lambda_{n,m}^{-1}S_0'(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]-\Lambda_{n,m}^{-1}[2\upsilon\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)$$

$$-[(\mu_n+1)\cosh(\mu_n z)+z\mu_n\sinh(\mu_n z)]r^{-1}S_0'(k_1,k_2,\mu_n r)]\}\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.7}$$

$$\sigma_{zz}=\frac{4G}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\bar{f}(m,t)[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times\{-\cos pz[\Lambda_{n,m}^{-1}S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]+[(2\upsilon+\mu_n)\mu_n^2\cosh(\mu_n z)$$

$$+\mu_n^3 z\sinh(\mu_n z)]S_0(k_1,k_2,\mu_n r)\}\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.8}$$

$$\sigma_{rz}=\frac{4G}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)a_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\bar{f}(m,t)}{\Lambda_{n,m}}[\wp_{n,m}\exp(-\omega t)+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa\Lambda_{n,m}t)]\right\}$$

$$\times\{a_m[\cos pz)]S_0(k_1,k_2,\mu_n r)+[2\upsilon\mu_n\sinh(\mu_n z)$$

$$+\mu_n^2[\sinh(\mu_n z)+z\cosh(\mu_n z)]]S_0'(k_1,k_2,\mu_n r)\}\exp\left[\int_0^t\psi(\zeta)d\zeta\right] \tag{4.9}$$

Special Case

Set $\psi(\zeta) = -\zeta, \quad T_0 = 0$ (5.1)

$$\Rightarrow \int_0^t \psi(\zeta)\,d\zeta = -t^2/2, \quad \overline{T}_0^* = 0 \tag{5.2}$$

Substituting the value of equation (5.2) into equation (3.15) to (4.9), one obtains the expressions for the temperature and stresses respectively as follows:

$$\theta = \frac{2}{\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\wp_{n,m}\overline{f}(m,t)}{C_n}[\exp(-\omega t)-\exp(-\kappa\Lambda_{n,m}t)]\cos pz\, S_0(k_1,k_2,\mu_n r)\exp[-t^2/2] \tag{5.3}$$

$$\sigma_{rr} = \sigma_0\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\wp_{n,m}\overline{f}(m,t)}{C_n}[\exp(-\omega t)-\exp(-\kappa\Lambda_{n,m}t)]\{-\cos pz[\Lambda_{n,m}^{-1}S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]$$
$$-\Lambda_{n,m}^{-1}[2\upsilon\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)-[(\mu_n+1)\cosh(\mu_n z)+z\mu_n\sinh(\mu_n z)]$$
$$\times S_0'(k_1,k_2,\mu_n r)]\}\exp[-t^2/2] \tag{5.4}$$

$$\sigma_{\theta\theta} = \sigma_0\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\wp_{n,m}\overline{f}(m,t)}{C_n}[\exp(-\omega t)-\exp(-\kappa\Lambda_{n,m}t)]\{-\cos pz[r^{-1}\Lambda_{n,m}^{-1}S_0'(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]$$
$$-\Lambda_{n,m}^{-1}[2\upsilon\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)-[(\mu_n+1)\cosh(\mu_n z)+z\mu_n\sinh(\mu_n z)]r^{-1}$$
$$\times S_0'(k_1,k_2,\mu_n r)]\}\exp[-t^2/2] \tag{5.5}$$

$$\sigma_{zz} = \sigma_0\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\wp_{n,m}\overline{f}(m,t)}{C_n}[\exp(-\omega t)-\exp(-\kappa\Lambda_{n,m}t)]\{-\cos pz[\Lambda_{n,m}^{-1}S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]$$
$$+[(2\upsilon+\mu_n)\mu_n^2\cosh(\mu_n z)+\mu_n^3 z\sinh(\mu_n z)]S_0(k_1,k_2,\mu_n r)\}\exp[-t^2/2] \tag{5.6}$$

$$\sigma_{rz} = \sigma_0\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\wp_{n,m}\overline{f}(m,t)}{C_n}[\exp(-\omega t)-\exp(-\kappa\Lambda_{n,m}t)]\{a_m[\cos pz]S_0(k_1,k_2,\mu_n r)$$
$$+[2\upsilon\mu_n\sinh(\mu_n z)+\mu_n^2[\sinh(\mu_n z)+z\cosh(\mu_n z)]]S_0'(k_1,k_2,\mu_n r)\}\exp[-t^2/2] \tag{5.7}$$

where

$$\sigma_0 = \frac{4G}{\pi}\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t$$

Numerical Results, Discussion and Remarks

To interpret the numerical computations, we consider material properties of Aluminum metal, which can be commonly used in both, wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry and in other transportation fields. Its resistance to corrosion leads to its use in food and chemical handling (cookware, pressure vessels, etc.) and to architectural uses

Table 1- Material properties and parameters used in this study. Property values are nominal.

Modulus of Elasticity, E (dynes/cm²)	6.9 X 10^{11}
Shear modulus, G (dynes/cm²)	2.7 X 10^{11}
Poisson ratio, u	0.281
Thermal expansion coefficient, a_t (cm/cm-^{0}C)	25.5 X 10^{-6}
Thermal diffusivity, k (cm²/sec)	0.86
Thermal conductivity, l (cal-cm/^{0}C/sec/ cm²)	0.48
Inner radius, a (cm)	1
Outer radius, b (cm)	4
length, h (cm)	2

The foregoing analysis are performed by setting the radiation coefficients constants, $k_i = 0.86\,(i=1,3)$ and $k_i = 1\,(i=2,4)$, so as to obtain considerable mathematical simplicities. The other parameters considered are r_0 = 2.5, z_0 = 1. The derived numerical results from equation (43) to (47) has been illustrated graphically (refer figs. 2 to 5) with available additional sectional heat on its flat surface at z = 1.

Fig. 1 shows the temperature distribution along the radial and thickness direction of the finite hollow cylinder at t = 0.25. It is observed that due to the thickness of the cylinder, a steep increase in temperature was found at the beginning of the transient period. As expected, temperature drop becomes more and more gradually along thickness direction. It is also observed that, without internal heat source, magnitude of temperature gradient decreases.

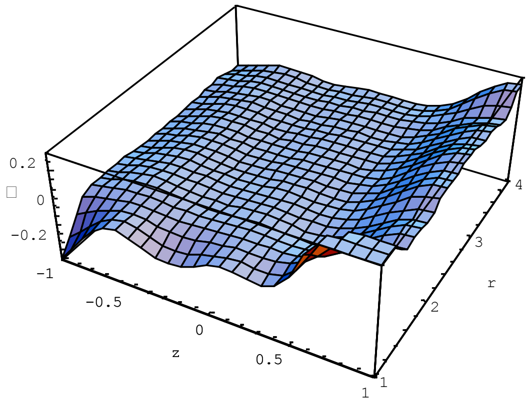

Fig. 1- Temperature distribution along r- and z-direction for t =0.25

Fig. 2 shows the radial stress distribution s_{rr} along the radial and thickness direction of the finite hollow cylinder at t =0.25. From the figure, the location of points of minimum stress occurs at the end points through-the-thickness direction, while the thermal stress response are maximum at the interior and so that outer edges tends to expand more than the inner surface leading inner part being under tensile stress.

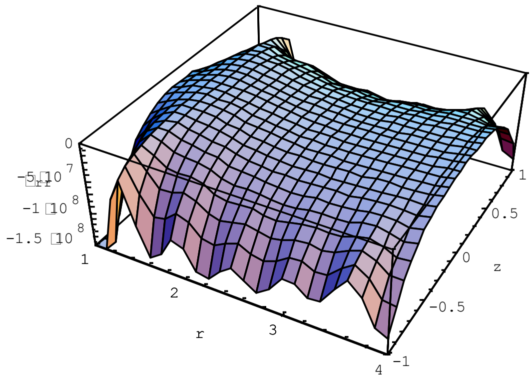

Fig. 2- Radial stress distribution for varying along r-axis and z-axis for t =0.25

Fig. 3 shows the tangential stress distribution s_{qq} along the radial and thickness direction of the finite hollow cylinder at t =0.25. The tangential stress follows a sinusoidal nature with high crest and troughs at both end i.e. r = 1 and r = 4.

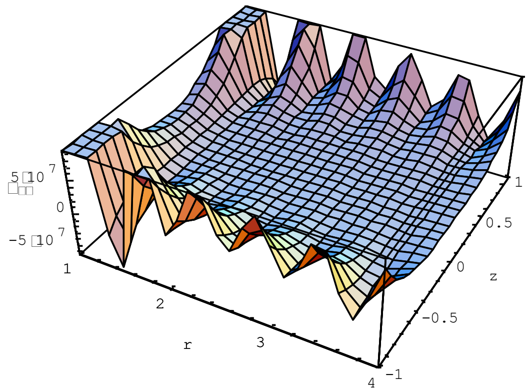

Fig. 3- Tangential stress distribution for varying along *r*-axis and *z* -axis for *t* =0.25

Fig. 4 shows the axial stress distribution s_{zz}, which is similar in nature, but small in magnitude as compared to radial stress component.

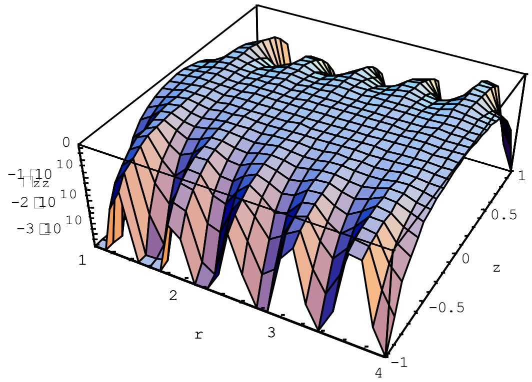

Fig. 4- Axial stress distribution for varying along *r*-axis and *z*-axis for *t* =0.25

Fig. 5 shows the shear stress distribution s_{rz} along the radial and thickness direction of the finite hollow cylinder at *t* =0.25. Shear stress also follows more sine waveform with high pecks and troughs along the radial direction at *r* = 1 and *r* = 4, but minimum at the center part along thickness direction.

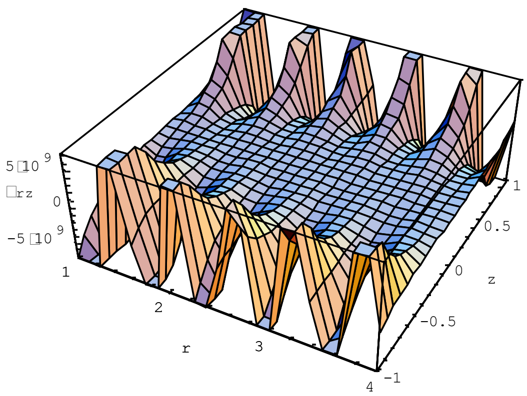

Fig. 5- Shear stress distribution for varying along *r*-axis and *z*-axis for *t* =0.25

In order for the solution to be meaningful the series expressed in equation (43) should converge for all $a \le r \le b$ and $-h \le z \le h$. The temperature equation (43) can be expressed as

$$\theta = \sum_{n=1}^{M} \sum_{m=1}^{M'} \frac{\wp_{n,m}}{C_n \lambda_m} [\exp(-\omega t) - \exp(-\kappa \Lambda_{n,m} t)] P_m(z) S_0(k_1, k_2, \mu_n r) \exp[-t^2/2] \quad (48)$$

We impose conditions so that $\theta(r, z, t)$ converge in some generalized sense to $g(r, s)$ as $t \to 0$ in the transform domain. Taking into account of the asymptotic behaviors of $P_m(z)$, μ_n, $S_0(k_1, k_2, \mu_n r)$ and C_n [2,3], it is observed that the series expansion for $\theta(r, z, t)$ will be theoretically convergent due to the bounded functions.

As convergence of the series for $r = b$ implies convergence for all $r \le b$ at any value of z. An exact solution requires use of infinite number of terms in the equations. The effects of truncating of terms are brought out by the comparison table for solutions of different functions for 5 and 10 terms. For the convergence test of the present method, we calculate the temperature values at the point *r* = 4, z =1 with different time for 5 terms and 10 terms, as shown in the table 2.

Table 2-The temperature under different time and different number of terms at r = 4, z = 1.

Functions	5 terms	10 terms
Time = 0.1	0.252	0.479
Time = 1.0	0.045	0.067
Time = 2.0	0.001	0.002

Conclusion

In this study, we treated the two-dimensional thermoelastic problem of a finite hollow cylinder in which sources are generated according to the linear function of the temperature. We successfully established and obtained the temperature distribution, displacements and stress functions with additional sectional heat, $\exp(-\omega t)\delta(r - r_0)$ available at the edge $z = h$ of the cylinder. Then, in order to examine the validity of two-dimensional thermoelastic boundary value problem, we analyze, as a particular case with mathematical model for $\psi(\zeta) = -\zeta$ and numerical calculations were carried out. Moreover, assigning suitable values to the parameters and functions in the equations of temperature, displacements and stresses respectively, expressions of special interest can be derived for any particular case. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behavior with radiation.

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