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MODIFIABLE AND NON-MODIFIABLE RISK FACTORS OF CABG PATIENTS AND PARAMETRIC ESTIMATION OF SURVIVAL PROPORTIONS OF COMPLETE POPULATIONS

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Abstract- In this paper, modifiable and non-modifiable risk factors of Coronary Artery Bypass Graft Surgery (CABG) patients are considered. A new approach of complete population from its incomplete population is used. In the complete population, censored patients have been proportionally incorporated into the known died and survived patients respectively. The availability of a complete population may represent better behavior of lifetimes / survival proportions for medical investigations. Survival proportions of the CABG patients of complete populations, with respect to modifiable and non-modifiable risk factors, are obtained from suitable parametric models (Weibull and Exponential). Maximum likelihood method, in-conjunction with Davidon-Fletcher-Powell (DFP) optimization method and Cubic Interpolation method is used in estimation of survivor's proportions of the parametric models.
Keywords- CABG Patients, Complete & Incomplete Populations, Modifiable & Non-Modifiable Risk Factors, Parametric models (Weibull and Exponential), Maximum likelihood method, Davidon-Fletcher-Powell optimization method and Survivor's Proportions.

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Introduction

The Coronary Artery Disease (CAD) is due to accumulation of cholesterol and other material, called plaque, within inner walls of the coronary arteries (the arteries that supply blood with oxygen and nutrients to heart muscles). As this buildup grows (in medical term, Arteriosclerosis), only less blood can flow through the arteries. Over the time heart weakens. This leads to chest pain (Angina) which is a symptom of Myocardial Infarction (MI). When the clot (thrombus) completely cuts off the hearts blood supply, this leads to permanent heart damage or heart attack (MI). Heart failure means the heart can't pump blood well to the rest of the body [10]. CAD is the leading cause of death worldwide (see William, Stephen, Van -Thomas and Robert) [36], John [18], Hansson [13], Axel, Yiwen, Dalit, Veena, Elaine, Catia, Matthew, Jonathan, Edward & Len [3] and Sun & Hoong [33]). The symptoms and signs of coronary artery disease are noted in the advanced stage of disease, most individuals with CAD show no evidence of disease for decades as the disease progresses before the first onset of symptoms, often there is a sudden heart. The disease is the most common cause of sudden death of men and women over 20 years of age [30]. The medical scientists; William, Ellis, Josef, Ralph and Robert [36], Heymann [15], Goldstein, Adams, Alberts, Appel, Brass, Bushnell, Culebras, DeGraba, Gorelick & Guyton [11] and Jennifer [17] are of the opinion that CABG is an effective treatment option for CAD patients. The medical research organizations like Heart and Stroke Foundation Canada [14] and American Heart Association [2] have classified risk factors of CABG patients as modifiable and non-modifiable. Modifiable risk factors are those factors, which may be controlled by changing life style or taking medications to reduce cardiovascular risk. These risk factors include high blood pressure (hypertension), diabetes, smoking, high cholesterol, sedentary lifestyle and obesity. Non-modifiable risk factors are those factors, which may not be changed. These factors include age, gender and family history (Genetic predisposition). William, Ellis, Josef, Ralph and Robert [35] in 1995 carried out the survival study on incomplete population (progressive censoring of type 1) of CABG patients comprising 2011 patients using Kaplan Meier method [19]. The patients were grouped with respect to Male, Female, Age, Hypertension, Diabetes, and Ejection Fraction, Vessels, Congestive Heart Failure, Elective and Emergency Surgery. The patients were undergone through a first re-operation at Emory University hospitals from 1975 to 1993. This study also comprises the same data set of 2011 patients. The details of patients are given in the article [35].

In this paper we present survival study of the CABG patients with respect to some modifiable risk factors, Hypertension & Diabetes and non-modifiable risk factors. Gender and Age. Khan, Saleem and Mahmud in the article [21] proposed a procedure, to make an incomplete population (*IP*) a complete population (*CP*). The differences between the means of survival proportions of the CABG patients, obtained by using different survival models (Weibull, Expontial etc) are statistically at 5% level of significance for details see the article [21].

The importance of parametric models for analysis of lifetime date has been indicated by Mann, Schefer and Singpurwala [28], Nelson [29], Cyrus [8], Lawless [25], Klein & Moeschberger [22] and Sridhar and Mun Choon Chan [32]. The Exponential distribution model has been used by Lee, Kim and Jung [27] in medical research for survival data of patients. The Weibull distribution model has been used for survival analysis by Cohen [6], Gross and Clark [12], Bunday [5], Crow [7], Klein & Moeschberger [22], Lawrencce [26], Abrenthy [1], Hisada & Arizino [16], Lawless [25], David & Mitchel [9] and Lang [23]. In particular, the survival study of chronic diseases, such as AIDS and Cancer, has been carried out by Bain and Englehardt [4], Khan & Mahmud [20], Klein & Moeschberger [22], Lawless [25] and Swaminathan and Brenner [34] using Exponential and Weibull distributions. Lanju & William [24] used Weibull distribution to human survival data of patients with plasma cell and in response-adaptive randomization for survival trials respectively. Lee, Kim and Jung [27] used exponential in medical research for survival data of the patients. Khan, Saleem and Mohmud [21] concluded that the survival data of the CABG patients has been best modeled by the Weibull and Exponential distributions. In this paper, the survivor proportions of the CABG patients are obtained for complete population of the CABG patients by parametric models (Weibull and Exponential), using data of CABG patients groups: Gender, Age, Hypertension and Diabetes. Maximum likelihood method, in-conjunction with DFP optimization method and Cubic Interpolation method is used. A subroutine for maximizing log-likelihood function of each model is developed in FORTRAN program to obtain the estimates of the parameters of the model. The survival proportions of *CP* of the CABG patients with respect to modifiable and non-modifiable risk factors risk factors are presented, discussed and concluded.

Methodology

Khan, Saleem and Mahmud [21] mentioned that the method proposed by Kaplan Meier [19] and latter discussed by William [35] in 1995 and Lawless [25] is: $S(t)=\prod_{j:t_j<t}(1-\frac{d_j}{n_j})$, where d_j and n_j are the number of items (individuals / patients) failed (died individuals) and number of individuals at risk at time t_j, that is, the number of individuals survived and uncensored at time t_{j-1}. This method does not take into account the censored individuals c_j completely and thus the analysis is performed on incomplete population (IP). Further, Khan, Saleem and Mahmud [21] proposed that the censored individuals c_j could be taken into account. The inclusion of splitted-censored individuals, c_j proportionally $\left[(1-\frac{d_j}{n_{j-1}-c_j})\times c_j \;\; and \; (\frac{d_j}{n_{j-1}-c_j})\times c_j\right]$ into known survived, n_j and died individual's d_j respectively makes the population complete. Thus the survival analysis may be performed on the complete population (CP). Saleem, Mahmud and Khan [31] also mentioned the form of likelihood function proposed by Klein & Moeschberger [22] and Lawless [25], for a survival model, in the presence of censored data. The maximum likelihood method works by developing a likelihood function based on the available data and finding the estimates of parameters of a probability distribution that maximizes the likelihood function. This may be achieved by using iterative method: see Bunday & Al-Mutwali [5] and Khan & Mahmud [20] The likelihood function for all observed died and censored individuals is of the form: $L(t;\underline{\theta})=\prod_{i=1}^{n}\left[f(t_i;\underline{\theta})\right]^{f_{t_i}}\prod_{i=1}^{n}\left[S(t_i;\underline{\theta})\right]^{c_{t_i}}$, where f_{t_i} & c_{t_i} are the number of died & censored individuals in interval *i* each of length *t,* $f(t;\underline{\theta})$ is *pdf* in a parametric model with survivor function, $S(t;\underline{\theta})$ & hazard function, $h(t;\underline{\theta})$ *and* $\underline{\theta}$ is vector of parameters say $\underline{\theta}=(\alpha,\beta)$ of the model. To obtain maximum likelihood estimates of parameters of a parametric model using DFP optimization method, we take negative log on both the sides of above equation and therefore by setting $l=-\ln(L(t;\underline{\theta}))$, we get:

$$l=-\sum_{i=1}^{n}f_{t_i}\ln f(t_i;\underline{\theta})-\sum_{i=1}^{n}c_{t_i}\ln\left(S(t_i;\underline{\theta})\right)$$

or

$$=-\sum_{i=1}^{n}f_{t_i}\ln h(t_i;\underline{\theta})-\sum_{i=1}^{n}(f_{t_i}+c_{t_i})\ln\left(S(t_i;\underline{\theta})\right)$$

(As $f(t_i;\underline{\theta})=h(t_i;\underline{\theta})\,S(t_i;\underline{\theta})$)

Where, the first sum is for failure and the second sum is for all censored individuals.

Setting $N_{t_i} = (f_{t_i} + c_{t_i})$, where N_{t_i} represents total no of individuals at time t_i we get:

$$l = -\sum_{i=1}^{n} f_{t_i} \ln h(t_i;\underline{\theta}) - \sum_{i=1}^{n} (N_{t_i}) \ln\left(S(t_i;\underline{\theta})\right) \quad (1)$$

In this study time is partitioned into intervals, which are of unit length t starting from zero. Moreover, failures and censoring of the patients occur in each interval i of equal length of time t, $i = 1,2,...,12.$

For complete population the term for censored observations is dropped from the likelihood function.

3. Application

Khan, Saleem and Mahmud [21] presented detail application of above methodology for parametric model (Weibull distribution). Same procedure is followed for second parametric model (Exponential distribution) considered in this article. The methodology is reproduced here. The probability density function (*pdf*) of Weibull distribution is: $f(t;\underline{\theta}) = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$, where $\underline{\theta}$ is vector of parameters α and β ; α is a scale parameter and β is a shape parameter; α, β and $t > 0$.

The survival and hazard functions of Weibull distribution are:

$$S(t;\underline{\theta}) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \quad \text{and} \quad h(t;\underline{\theta}) = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1}$$

For incomplete population replacing values of the survival and hazard functions of Weibull distribution in equation (1), we get

$$l = -\sum_{i=1}^{n} f_{t_i} \ln\left[\left(\frac{\beta}{\alpha}\right)\left(\frac{t_i}{\alpha}\right)^{\beta-1}\right] - \sum_{i=1}^{n} N_{t_i} \ln\left(e^{-\left(\frac{t_i}{\alpha}\right)^{\beta}}\right) \quad \text{or}$$

$$= -(F)\ln\left(\frac{\beta}{\alpha}\right) - (\beta-1)\sum_{i=1}^{n} f_{t_i} \ln\left(\frac{t_i}{\alpha}\right) + \sum_{i=1}^{n} N_{t_i}\left(\frac{t_i}{\alpha}\right)^{\beta} \quad (2)$$

where, $F = \sum_{i=1}^{n} f_{t_i}$ is the total number of failures in a given time.

Differentiating (2) with respect to α and β and simplifying we get

$$\frac{\partial l}{\partial \alpha} = F\left(\frac{\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha}\right)\sum_{i=1}^{n} N_{t_i}\left(\frac{t_i}{\alpha}\right)^{\beta} \quad (3)$$

$$\frac{\partial l}{\partial \beta} = -\frac{F}{\beta} - \sum_{i=1}^{n} f_{t_i} \ln\left(\frac{t_i}{\alpha}\right) + \sum_{i=1}^{n} N_{t_i}\left(\frac{t_i}{\alpha}\right)^{\beta} \ln\left(\frac{t_i}{\alpha}\right) \quad (4)$$

By using (2), (3) and (4) in the DFP optimization method, we find the parameters estimates for which value of the likelihood function is maximum. For complete population we drop the term for censored observations from likelihood function. Same procedure is followed for Exponential model. FORTRAN program for the parameters estimation of both the models is developed. The optimal estimates of the scale and shape / location parameters (α and β respectively) of Weibull and Exponential distributions distribution using *CP* of groups: Hypertension (Absent & Present), Diabetes (Absent & Present), Male & Female and Age groups (Less than 50 years (I), 50 to 59 years (II), 60 to 59 years (III) and 70 & above years (IV)) are obtained by maximizing the log-likelihood function. The *t*-ratios of the parameters are given in parenthesis. The values of parameters estimates, *t*-ratios, log-likelihood function and variance-covariance matrix are given below: -

Weibull Distribution

a] Non-Modifiable factors of CABG Patients (Gender and Age)

Male &Female and Age Groups I, II, III & IV (Male and Female CABG Patients)

The survival proportions of male & female and age groups I, II, III & IV of CABG patients are obtained using Weibull distribution (two parametric; α and β) as explained earlier. The optimal estimates of the parameters obtained by maximizing the log-likelihood function are given below in table 1.

The estimated values of scale parameter $\alpha > 0$ and shape parameter $\beta > 0$ for male & female and age groups I, II, III & IV of CABG patients are given in the table 1 along with *t*-ratios in the parenthesis, indicating that the estimates of scale and shape parameters are significant at 5% level of significance. The estimated value of β is greater than 1 for the male & female and age groups. I, II, III & IV, of CABG patients, indicate the increasing failure rate with time. The positive or negative values of co-variances indicates that the movements of $\hat{\alpha}$ and $\hat{\beta}$ are in the same or opposite directions respectively.

The estimated survival proportions of male & female and age groups I, II, III & IV of CABG patients and respective graphs are given in table 2.

Discussion

The graph of survival proportions obtained by using Weibull distribution for male and female groups of CABG patients indicates that for the difference between the survival proportions is small at the start, continuously but slowly increasing, whereas the survival proportions of female group of the CABG patients are lower than those of male group of the CABG patients. The graph of survival proportions obtained by using Weibull distribution for age groups I, II, III and IV of CABG patients indicates that for the age groups I and II, the difference between survival proportions of is small, whereas for the age groups III and IV the difference between survival proportions of is small at the start, and increasing continuously. The survival proportions of the age group IV are comparatively lowest in four age groups.

Conclusion

The survival proportions of female group of the CABG patients are lower than those of female group of the CABG patients. The survival proportions of the age group IV are comparatively lowest in four age groups, which indicates increasing failure rate with the increase in age.

b] Modifiable factors of CABG Patients (Hypertension and Diabetes

Hypertension Absent ($\hat{y}_t$) $_{HYa}$ & Present ($\hat{y}_t$)$_{HYp}$ and Diabetes Absent ($\hat{y}_t$)$_{Da}$ & Present ($\hat{y}_t$)$_{Dp}$

The survival proportions ($\hat{y}_t$)$_{HYa}$ & ($\hat{y}_t$)$_{HYp}$ and ($\hat{y}_t$)$_{Da}$ & ($\hat{y}_t$) $_{Dp}$ groups of the CABG patients are obtained using Weibull distribution as explained earlier. The optimal estimates of parameters obtained by maximizing the log-likelihood function are given in table 3.

The estimated values of scale parameter α > 0 and shape parameter β> 0 for HY_a & HY_p and D_a & D_p groups of CABG patients are given in the table 3 along with *t*-ratios in the parenthesis, indicating that the estimates of scale and shape parameters are significant at 5% level of significance. The estimated value of β is greater than 1 for HY_a & HY_p and D_a & D_p groups of CABG patients, which indicates the increasing failure rate with time. The positive or negative values of co-variances indicates that the movements of $\hat{\alpha}$ and $\hat{\beta}$ are in the same or opposite directions respectively.

The estimated survival proportions of HY_a & HY_p and D_a & D_p groups of CABG patients and respective graphs are given in table 4.

Discussion

The graph of survival proportions (obtained by using Weibull distribution) for HY_a & HY_p and D_a & D_p groups of CABG patients shows that the difference between the survival proportions or HY_a & HY_p and D_a & D_p is small at the start and continuously increasing.

Conclusion

The survival proportions (obtained by using Weibull distribution) of the CABG patients in the presence of hypertension and diabetes are lower than in the absence of hypertension and diabetes.

Finally, as a whole the survival proportions (obtained by using Weibull distribution), of the CABG patients groups (HY_a & HY_p and D_a & D_p) in the presence of diabetes are lowest.

Exponential Distribution

a] Non-Modifiable factors of CABG Patients (Gender and Age)

Male &Female and Age Groups (Male and Female CABG Patients)

Survival Proportions ($\hat{y}_t$)$_M$ & ($\hat{y}_t$)$_F$ of Male & Female and ($\hat{y}_t$)$_I$, ($\hat{y}_t$)$_{II}$, ($\hat{y}_t$)$_{III}$ & ($\hat{y}_t$)$_{IV}$ of Age Groups I, II, III &IV respectively of the CABG Patients from exponential distribution (as parametric,*a ;* keeping β = 1) as explained earlier. The optimal estimates of the parameters obtained by maximizing the log-likelihood function are given in table 5.

The estimated values of scale parameter α > 0 for male & female and age groups I, II, III IV of CABG patients are given in the table 5 along with *t*-ratios in the parenthesis, indicating that the estimates of scale are significant at 5% level of significance.

The estimated survival proportions of male & female and age groups I, II, III & IV of CABG patients and respective graphs are given in table 6.

Discussion

The graph of survival proportions obtained by using exponential distribution for male and female groups of CABG patients indicates that the difference between the survival proportions is small at the start and increasing continuously, whereas the survival proportions of female group of the CABG patients are lower than those of male group of the CABG patients. The graph of survival proportions obtained by using exponential distribution for age groups I, II, III and IV of CABG patients indicates that for the age groups I and II, the difference between survival proportions of is small, whereas for the age groups III and IV the difference between survival proportions of is small at the start, and increasing continuously. The survival proportions of the age group IV are comparatively lowest in four age groups.

Conclusion

The survival proportions of female group of the CABG patients are lower than those of female group of the CABG patients. The survival proportions of the age group IV are comparatively lowest in four age groups, which indicates increasing failure rate with the increase in age.

b] Modifiable factors of CABG Patients (Hypertension and Diabetes)

The survival proportions ($\hat{y}_t$)$_{HYa}$ & ($\hat{y}_t$)$_{HYp}$ and ($\hat{y}_t$)$_{Da}$ & ($\hat{y}_t$) $_{Dp}$ of the HY_a & HY_p and D_a & D_p groups of CABG patients are obtained using Exponential distribution as explained earlier.

The optimal estimates of parameters obtained by maximizing the log-likelihood function are given below in table 6.

The estimated values of scale parameter α > 0 for HY_a & HY_p and D_a & D_p groups of CABG patients are given in the table 7 along with *t*-ratios in the parenthesis, indicating that the estimates of scale and shape parameters are significant at 5% level of significance.

The estimated survival proportions of HY_a & HY_p and D_a & D_p groups of CABG patients and respective graphs are given in table 8.

Discussion

The graph of survival proportions (obtained by using exponential distribution) for HY_a & HY_p and D_a & D_p groups of CABG patients shows that the difference between the survival proportions or HY_a & HY_p and D_a & D_p is small at the start and increasing continuously.

Conclusion

The survival proportions (obtained by using Exponential distribution) of the CABG patients in the presence of Hypertension and Diabetes are lower than in the absence of Hypertension and diabetes.

Finally, as a whole the survival proportions (obtained by using Exponential distribution), of the CABG patients groups (HY_a & HY_p and D_a & D_p) in the presence of Diabetes are lowest.

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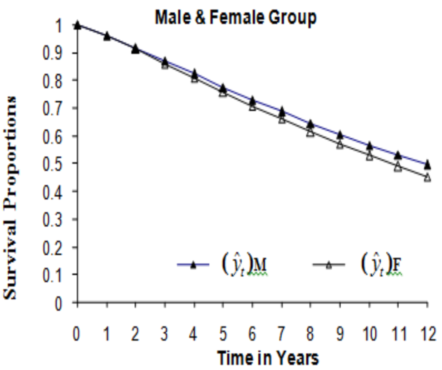
Table 1- Estimates of Parameters of Weibull Distribution Using Data of Male & Female and Age Groups (I, II, III & IV) of CABG Patients

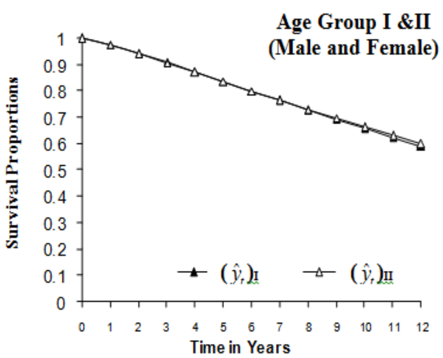
Parameters	Male Group		Female Group	
	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients
α	16.2415 (30.64)	-7.24×10^{-11}	13.3967 (16.03)	2.67×10^{-9}
β	1.1598 (31.68)	-2.71×10^{-10}	1.1689 (15.41)	-4.47×10^{-8}
Log-Likelihood	3302.079		728.6315	
Variance-Covariance Matrix	$\begin{pmatrix} 0.281 & -8.4\times10^{-3} \\ -8.4\times10^{-3} & 1.34\times10^{-3} \end{pmatrix}$		$\begin{pmatrix} 0.698 & -1.62\times10^{-2} \\ -1.62\times10^{-2} & 5.75\times10^{-3} \end{pmatrix}$	

Parameters	Age Groups							
	I		II		III		IV	
	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients
α	19.918 (10.31)	1.5×10^{-9}	20.966 (15.25)	5.65×10^{-11}	11.547 (29.81)	2.61×10^{-10}	8.758 (25.39)	-1.96×10^{-8}
β	1.253 (10.63)	4.43×10^{-8}	1.197 (16.76)	-4.17×10^{-8}	1.330 (25.00)	9.74×10^{-9}	1.414 (20.47)	1.97×10^{-8}
Log-Likelihood	415.9276		1043.354		1783.982		997.2853	
Variance–Covariance Matrix	$\begin{pmatrix} 3.735 & -0.131 \\ -0.131 & 1.39\times10^{-2} \end{pmatrix}$		$\begin{pmatrix} 1.889 & -5.83\times10^{-2} \\ -5.83\times10^{-2} & 5.1\times10^{-3} \end{pmatrix}$		$\begin{pmatrix} 0.150 & -3.56\times10^{-3} \\ -3.56\times10^{-3} & 2.83\times10^{-3} \end{pmatrix}$		$\begin{pmatrix} 0.119 & 2.73\times10^{-3} \\ 2.73\times10^{-3} & 4.77\times10^{-3} \end{pmatrix}$	

Table 2- Survival Proportions $(\hat{y}_t)_M$ & $(\hat{y}_t)_F$ of Male & Female and $(\hat{y}_t)_I$, $(\hat{y}_t)_{II}$, $(\hat{y}_t)_{III}$ & $(\hat{y}_t)_{IV}$ of Age Groups I, II, III & IV respectively CABG Patients from Weibull Distribution and Respective Graphs are as under.

Years (t)	Male & Female Groups		Age Groups			
			I	II	III	IV
	$(\hat{y}_t)_M$	$(\hat{y}_t)_F$	$(\hat{y}_t)_I$	$(\hat{y}_t)_{II}$	$(\hat{y}_t)_{III}$	$(\hat{y}_t)_{IV}$
0	1	1	1	1	1	1
1	0.961	0.953	0.976	0.974	0.962	0.954
2	0.915	0.897	0.945	0.941	0.907	0.883
3	0.868	0.840	0.911	0.907	0.846	0.802
4	0.821	0.783	0.874	0.871	0.783	0.718
5	0.774	0.729	0.837	0.835	0.720	0.636
6	0.729	0.676	0.800	0.799	0.658	0.556
7	0.686	0.626	0.763	0.764	0.598	0.482
8	0.644	0.578	0.727	0.729	0.541	0.414
9	0.604	0.533	0.691	0.695	0.487	0.353
10	0.565	0.491	0.656	0.662	0.437	0.299
11	0.529	0.451	0.621	0.630	0.391	0.251
12	0.494	0.415	0.588	0.598	0.349	0.209





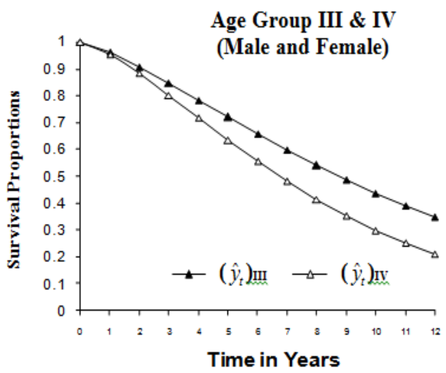
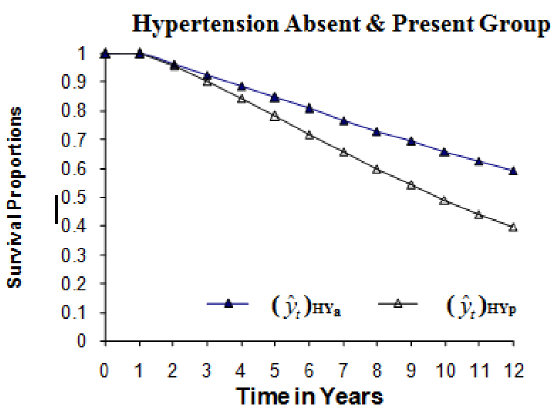


Table 3- Estimates of Parameters of Weibull Distribution Using Data of HY_a & HY_p and D_a & D_p Groups of CABG patients.

Parameters	Age Groups							
	HY_a		HY_p		D_a		D_p	
	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients	Estimates/ (*t*-ratio)	Gradients
α	19.476 (19.92)	2.48×10^{-10}	11.659 (31.73)	-1.21×10^{-9}	18.149 (26.01)	-2.48×10^{-10}	9.736 (24.57)	3.28×10^{-10}
β	1.142 (22.14)	-6.22×10^{-10}	1.316 (26.81)	-7.1×10^{-8}	1.154 (27.83)	-1.36×10^{-9}	1.380 (20.07)	-3.91×10^{-8}
Log-Likelihood	1743.991		2054.818		2688.629		1023.245	
Variance–Covariance Matrix	$\begin{pmatrix} 0.956 & -2.74\times10^{-2} \\ -2.74\times10^{-2} & 2.66\times10^{-3} \end{pmatrix}$		$\begin{pmatrix} 0.135 & -2.93\times10^{-3} \\ -2.93\times10^{-3} & 2.41\times10^{-3} \end{pmatrix}$		$\begin{pmatrix} 0.487 & -1.45\times10^{-2} \\ -1.45\times10^{-2} & 1.72\times10^{-3} \end{pmatrix}$		$\begin{pmatrix} 0.157 & 1.01\times10^{-4} \\ 1.01\times10^{-4} & 4.72\times10^{-3} \end{pmatrix}$	

Table 4- Survival proportions ($\hat{y}_t$)$_{HYa}$ & ($\hat{y}_t$)$_{HYp}$ and ($\hat{y}_t$)$_{Da}$ & ($\hat{y}_t$)$_{Dp}$ of the Groups of CABG Patients from Weibull Distribution and Respective Graphs

	HY_a	HY_p	D_a	D_p
Years (t)	$(\hat{y}_t)_{HYa}$	$(\hat{y}_t)_{HYp}$	$(\hat{y}_t)_{Da}$	$(\hat{y}_t)_{Dp}$
0	1	1	1	1
1	0.966	0.961	0.965	0.957
2	0.928	0.906	0.924	0.893
3	0.888	0.845	0.882	0.821
4	0.848	0.783	0.839	0.746
5	0.809	0.720	0.797	0.671
6	0.770	0.659	0.756	0.598
7	0.732	0.600	0.716	0.530
8	0.696	0.543	0.678	0.466
9	0.660	0.491	0.640	0.407
10	0.626	0.441	0.605	0.354
11	0.594	0.396	0.570	0.306
12	0.562	0.354	0.537	0.263



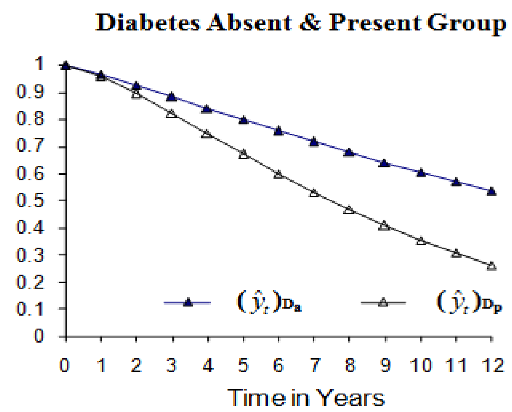


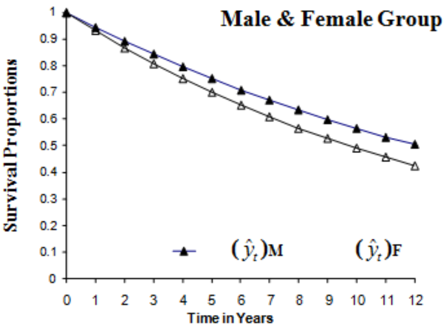
Table 5- Estimates of Parameters of Exponential Distribution Using Data of Male & Female and Age Groups (I, II, III & IV) of CABG Patients

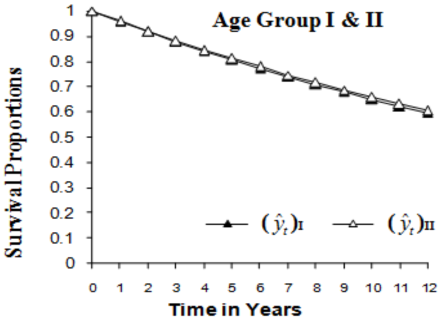
Parameter	Male Group Estimates/ (t-ratio)	Gradients	Female Group Estimates/ (t-ratio)	Gradients
α	5.72×10^{-2} (7.1)	4.07×10^{-7}	7.1×10^{-2} (3.76)	-1.0×10^{-9}
Log-Likelihood	3312.51		731.3791	
Variance	6.49×10^{-5}		3.56×10^{-4}	

Parameters	Age Groups I Estimates/ (*t*-ratio)	I Gradients	II Estimates/ (*t*-ratio)	II Gradients	III Estimates/ (*t*-ratio)	III Gradients	IV Estimates/ (*t*-ratio)	IV Gradients
α	4.31×10^{-2} (2.37)	2.11×10^{-7}	4.18×10^{-2} (3.53)	1.60×10^{-7}	8.19×10^{-2} (6.3)	5.07×10^{-7}	0.11688 (5.69)	-1.59×10^{-7}
Log-Likelihood	418.5939		1047.648		1807.052		1019.492	
Variance	3.31×10^{-4}		1.40×10^{-4}		1.69×10^{-4}		4.22×10^{-4}	

Table 6- Survival Proportions $(\hat{y}_t)_M$ & $(\hat{y}_t)_F$ of Male & Female and $(\hat{y}_t)_I$, $(\hat{y}_t)_{II}$, $(\hat{y}_t)_{III}$ & $(\hat{y}_t)_{IV}$ of Age Groups I, II, III & IV respectively of the CABG Patients from Exponential Distribution and Respective Graphs are as under.

Years (t)	Male & Female $(\hat{y}_t)_M$	$(\hat{y}_t)_F$	Age Groups I $(\hat{y}_t)_I$	II $(\hat{y}_t)_{II}$	III $(\hat{y}_t)_{III}$	IV $(\hat{y}_t)_{IV}$
0	1	1	1	1	1	1
1	0.944	0.931	0.957	0.959	0.921	0.888
2	0.891	0.866	0.917	0.919	0.848	0.790
3	0.842	0.807	0.878	0.882	0.782	0.702
4	0.795	0.751	0.841	0.845	0.720	0.624
5	0.751	0.699	0.806	0.811	0.663	0.555
6	0.709	0.651	0.772	0.778	0.611	0.493
7	0.669	0.606	0.739	0.746	0.563	0.438
8	0.632	0.564	0.708	0.715	0.519	0.389
9	0.597	0.525	0.678	0.686	0.478	0.346
10	0.564	0.489	0.649	0.658	0.440	0.308
11	0.532	0.455	0.622	0.631	0.406	0.273
12	0.503	0.424	0.596	0.605	0.374	0.243





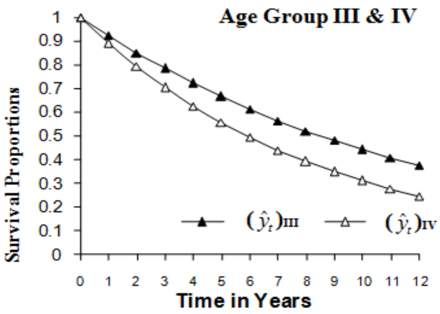


Table 7- Estimates of Parameters of Exponential Distribution Using Data of HY_a & HY_p and D_a & D_p Groups of CABG patients.

Parameter	HY_a Estimates/ (t-ratio)	Gradients	HY_p Estimates/ (t-ratio)	Gradients	D_a Estimates/ (t-ratio)	Gradients	D_p Estimates/ (t-ratio)	Gradients
α	4.71×10^{-2} (6.31)	2.09×10^{-7}	8.15×10^{-2} (5.37)	-8.81×10^{-8}	5.07×10^{-2} (4.89)	1.36×10^{-7}	0.1016 (6.74)	-9.42×10^{-7}
Log-Likelihood	1748.113		1465.947		2696.202		1041.88	
Variance	9.26×10^{-5}		1.46×10^{-4}		6.45×10^{-5}		3.58×10^{-4}	

Table 8- Survival proportions ($\hat{y}_t$)$_{HYa}$ & ($\hat{y}_t$)$_{HYp}$ and ($\hat{y}_t$)$_{Da}$ & ($\hat{y}_t$)$_{Dp}$ of Groups HY_a & HY_p and D_a & D_p of CABG patients from Exponential Distribution and Respective Graphs.

Years (t)	HY_a	HY_p	D_a	D_p
0	($\hat{y}_t$)$_{HYa}$	($\hat{y}_t$)$_{HYp}$	($\hat{y}_t$)$_{Da}$	($\hat{y}_t$)$_{Dp}$
1	0.954	0.921	0.950	0.903
2	0.910	0.849	0.903	0.816
3	0.868	0.783	0.859	0.737
4	0.828	0.721	0.816	0.666
5	0.790	0.665	0.776	0.601
6	0.753	0.613	0.737	0.543
7	0.719	0.565	0.701	0.491
8	0.686	0.520	0.666	0.443
9	0.654	0.480	0.633	0.400
10	0.624	0.442	0.602	0.362
11	0.595	0.407	0.572	0.327
12	0.568	0.375	0.544	0.295