

## MPC AND RTDA CONTROLLER FOR FOPDT & SOPDT PROCESS

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**Abstract-** This paper provides a comparative study between the model predictive control (MPC) and robust set point tracking disturbance rejection -Aggressiveness (RTDA) control schemes. The MPC presented here follows the optimization problem which ultimately leads to the description of the Dynamic Matrix Control(DMC). Recently a new SISO digital control scheme utilizing modern digital technology has been proposed called the RTDA-which directly addresses the controller parameters of robustness, set point tracking and disturbance rejection. The new control scheme features explicit tuning parameters for performance attributes such as robustness, set point tracking and disturbance rejection. The objective of this paper is to characterize the performance of the new control scheme in comparison to the popular MPC family of controllers for First order plus dead time(FOPDT) and Second order plus dead time (SOPDT) processes

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## Introduction

## **RTD-A Control Scheme**

The key attributes of the overall performance of any controller are robustness, set-point tracking, and disturbance rejection. The primary objective of control system design therefore is the design and implementation of controllers that attain a reasonable degree of success in each of these three attributes. Thus, for all control schemes - from regulatory control using the classical PID algorithm to the more advanced supervisory control using model predictive control - the objective is to select tuning parameters that result in good set-point tracking and disturbance rejection without sacrificing robustness. In PID controllers, the tuning parameters are not related to these critical attributes directly; designing the controller to achieve desired performance in each of these attributes is therefore not straightforward. Furthermore the PID controller has weaknesses that limit its achievable performance especially on dead-time dominant, inverse response and nonlinear processes. The proposed RTD-A controller, consists of a judiciously simplified linear model predictive control scheme that makes use of precisely the same process reaction curve information required for tuning PID controllers. It is capable of offering performance equal or even superior to a well-tuned PID controller. In addition, the main advantage of the RTD-A controller lies in the transparency of its tuning parameters. In particular, the tuning for set-point tracking and disturbance rejection are separate. The tuning requirements are the same as that of a PID controller that is a FOPDT model of the process. [2]

## Literature Review

B.A. Ogunnaike and K. Mukati (2005) proposed an alternative control scheme that combines the simplicity of PID controller with the versatility of model predictive controller. Michael Rasch et al. (2008) have presented robust stability results for proposed controller for any given plant/model mismatch, the results of which are used to generate simple tuning rules for the controller. Bergman R.N. et al. (1986) have explained about the minimal model for blood glucose relationship. Luisella Balbis et al.(1996) presents two solutions to solve the optimization problem: either the optimal

predictive controller replaces the regulatory level PID controllers, or the predictive controller is implemented at the supervisory level. Balaquer P. et al. (2008) proposed a tuning methodology based on a PID controller matching to a figure of merit and the PID tuning parameters are selected to minimize certain error metric when compared with an optimal controller. Reza Katebi et al. (2010) implemented multivariable predictive PID controller on a multiinputs control problem i.e., quadruple tank system, in comparison with a simple multi loop PI controller. Qing-Guo Wang et al (1999) proposed. a simple PID controller design method that achieves high performance for a wide range of linear self-regulating processes. Truong Nguyen Luan Vu and Jietate Lee (2007) proposed a new method a designing multi-loop PID controllers by using the generalized IMC-PID method for multi-loop systems.Zheng Zhi and Morari Manfred (1993) gave a new design technique for a robust model predictive controller. Qin and Badgwell(2003) proposed a study on survey of industrial model predictive control technology. Dong and Brosilow(1997) proposed a new method for design of robust multivariable PID controllers using IMC.

The paper is organized as follows: Chapter II describes design of MPC and RTDA controller. Implementation of MPC and RTDA controller for FOPDT and SOPDT processes is discussed in chapter III. Chapter IV culminates the conclusion.

## Design Of Mpc And Rtda Controller MPC based control

The basic idea behind the MPC has been discussed below, which consists of basic algorithm of MPC followed by the idea of receding horizon, optimization problem and the Dynamic Matrix Control.



Fig. 1- Basic block diagram of MPC

Figure.1 shows the basic structure of a MPC. The models takes data from the past inputs and outputs and combines it with the predicted future inputs, and give a predicted output for the time step. This predicted output is combined with the difference trajectory giving the predicted future error of the signal. These errors are fed into the optimizer, which enforce the constraint of the system on the predicted output and minimize the operating cost function. This gives the future inputs, which are fed back into the main model, restarting the cycle.

## **MPC** algorithm

- Development of a process model:
- At time t, previous process input and output are used, along with process model.
- Predict future process outputs over a "predictive horizon".
- The control signal that produces the most desired behavior is

selected.

- The control signal is implemented over a predefined time interval.
- Time advance to the next interval and the procedure is repeated from step 2.

The detailed implementation of Dynamic matrix control based MPC is available in many literature[1,9,11].

The MPC based control discussed in detail in process control, modeling and design by Wayne Bequette.

## RTDA based Control (Ogunnaike and Mukati, 2006)

The RTD-A control strategy utilizes digital technology to implement a simplified model prediction with transparent tuning parameters. The name of the control scheme stands for the four tuning parameters of the controller Robustness, Set-point Tracking, Disturbance Rejection and Aggressiveness. The main highlight of the controller is the direct relation between tuning objectives and tuning parameters.

## **Model Prediction**

FOPDT models are frequently used to give good approximations for the actual dynamics of industrial processes. In the RTD-A controller, a discretized form of the FOPDT model is used in the model predictive scheme:

$$y(s) = G(s)^* u(s)$$
 (1)

Where equivalent discretized model is given by  $\hat{Y}(k+1)=a\hat{y}(k)+u$  (*k-m*)

Restricting the control action u (k) to remain the same for the entire prediction horizon, the series of N equations is given by:  $\hat{y}(k+m+1)=a^{m+1}\hat{y}(k)+b\mu(k,m)+bu(k)$  (3)

(2)

where  $\mu(k,m) = \sum_{i=1}^{m} a^{i} u(k-i)$ (0)

(4) This prediction must be updated to include the effect of unmeasured disturbances, and other sources of modeling error.

## Error Update

The use of FOPDT model invariable results in errors between the actual process output and the model predicted output. Hence, the prediction requires updating. The plant/ model mismatch given by:

$$e(k) = y(k) - \hat{y}(k)$$
 (5)

contains several components, which can be grouped into two types,  $e_m(k)$  and  $e_d(k)$ , as follows:

$$e(k) = {}^{e_m}(k) + {}^{e_d}(k)$$
(6)

The error is separated into the effects of inherent modeling uncertainties represented by  $e_m(k)$  and the effects of unmodeled

disturbances represented by  $e_d(k)$ 

By using Bayesian principles for estimation,  $e_d(k)$  is estimated in the following way:

$$e_d(k) = \theta_r e_d(k+1) + (1 - \theta_r)e(k)$$
 (7)

(9)

The equation for the estimate of future error is given as:

$$\hat{e}_{D}(k+j) = e_{D}(k) + \frac{\alpha}{1-\alpha} [1-\alpha^{j}] \nabla e_{D}(k)$$
(8)

m+1 ≤ j ≤ m+N where

$$\nabla e_D(k) = e_D(k) - e_D(k-1)$$

The parameter  $\alpha$  is now replaced with a tuning parameter  $(1 - \theta_D)$ to give:

$$e_D(k+1) = e_D(k) + \frac{1-\theta_D}{\theta_D} \left[1 - (1-\theta_D)^j\right] \nabla (1-\theta_D)$$
  
m+1 \le m+N (9)

Here  $\theta_D$  specifies the control response to disturbances. By the convergence condition required by the equation,  $\theta_D$  is scaled between 0 and 1. Using the above stated error estimation, the future prediction of y(k+ m + i) over the N-step prediction horizon is given by

$$\hat{y}(k+m+i) = \hat{y}(k+mk+i) + \hat{e}_{D}(k+m+i)$$

$$1 < i < N$$
(10)

#### Set-point Tracking

For the purpose of set-point tracking, a desired trajectory needs to be defined. The final desired process output is given by the setpoint  $y_d(k)$ . Let the desired trajectory  $y^*(k)$ , be given by:  $y^{*}(k) = \theta_{t}y^{*}(k-1) + (1-\theta_{t})y_{d}(k)$ (11)The above equation provides a first order exponential approach for the set-point target for the controller. Here  $\theta t$  serves as the setpoint tuning parameter.

#### **Control Computation**

The optimization problem for a least-squared objective is given by

$$u(k) \sum_{i=1}^{N} \left( y^{*} (k + i) - \hat{y} (k + m + i) \right)$$
(12)

By defining

$$\eta_{i}(k) = y^{*}(k+i) - \hat{y}(k+m+i)$$
(13)

We obtain

$$r_i(k) = \psi_i(k) - b\eta_i u(k) \tag{14}$$

The analytical solution for the optimization problem is \_ N

$$u(k) = \frac{1}{b} \frac{\sum_{i=1}^{N} \eta_{i} \psi_{i}(k)}{\sum_{i=1}^{N} \eta_{i}^{2}}$$
(15)

Where

$$\psi_{i}(k) = y^{*}(k+I) - a^{m+i}\hat{y}(k) - a^{i-1}b\mu(k,m) - (k+m+i|k)$$
(16)

In the above expression, the control horizon N is yet to be specified. Defining N to be

$$N=1-\frac{\Delta t}{\Delta t}\ln(1-\theta_{\rm A}) \tag{17}$$

A choice of N=1 results in dead beat control and N=∞ results in conservative open loop control strategy.

Design of MPC and RTDA control scheme for FOPDT and SOPDT processes.

## Design of MPC scheme for FOPDT process

The first order plus dead time system chosen is:

$$G(s) = \frac{e^{-0.1s}}{2s+1}$$
(18)

The continuous state space form of above equation is A= [-10.5 -5.0; 1.0 0]; B= [1.0; 0]; C= [-0.5 5.0] D=[0]; : The initializing parameters of MPC are; Prediction horizon (p) = 10Model length (n) = 100; Control horizon (m) = 1 ; Weighting factor (w) = 0Sample time = 0.1 Time of set point change = 1 Final simulation time = 25secs.

#### Servo response

The servo response of the process is shown in the figure (2). From this response, the process output is able to reach the new set point without offset. The manipulated variable response is shown in the figure 2 b.



Fig. 2- Servo response of MPC (a) Process Output (b) Controller output

#### Robustness Analysis by varying prediction horizon

The robustness performance of designed MPC is tested by varying the prediction horizon from the nominal value 10 to 30 as well from 30 to 50 as shown in the figure 3. The response becomes more sluggish after 50. Smaller p requires more control action and is more sensitive to model uncertainty.



Fig. 3- Robustness analysis for MPC by varying Prediction horizon (a) Process output (b) Controller Output.

#### Design of RTDA scheme for FOPDT

The first order plus dead time system chosen is:

$$G(s) = \frac{e^{-0.13}}{2s+1}$$
(19)

The initial parameters for RTDA are: Overall System gain (K) = 1; Dead time ( $\alpha$ ) = 0.11

Time constant ( $\tau$ ) = 2; Sample time = 0.1 sec ; Set point =1

#### Servo Response

The servo response of the process is shown in the figure 4a. From this response, the process output is able to reach the new set point without offset.



Fig. 4- Servo response of RTDA (a) Plant Output (b) Controller output

# Robustness Analysis by variation of Tracking Parameter and prediction Horizon

Figure 5. Depicts that an increase in the value of the tuning parameter  $\theta_T$  results in adoption of a conservative policy for set point tracking by the RTDA controller.



Fig.5- Robustness Analysis of RTD-A for Tracking parameter variation (a)Plant Output (b)Controller Output.





Figure 6.depicts that an increase in the aggressiveness tuning parameter  $\theta_A$  results in increased conservativeness for the output response.

## Implementation of MPC for SOPDT process

The second order plus dead time system chosen is:

$$G(s) = \frac{e^{-0.13}}{.2s^2 + 2s + 1}$$
(20)

The continuous state space form of above equation is: A= [-20 -105 -50; 1 0 0; 0 1 0];B= [1; 0; 0] C= [0 -5 5]; D= [ 0 ]; The initializing parameters of MPC are: Model length (n) = 100; Prediction horizon (p) = 10 Control horizon (m) = 1; Weighting factor (w) = 0 Sample time = 0.1 sec; Time of set point change = 1 Final simulation time =25 sec

#### Servo response

The servo response of the process is shown in the figure (7). From this response; the process output is able to reach the new set point without offset.





## Robustness Analysis by varying prediction horizon

The robustness performance of designed MPC is tested by varying the prediction horizon from the nominal value 10 to 30 as well from 30 to 50 as shown in the figure 8. From this response if the prediction horizon increases the response become sluggish and vice versa



Fig. 8- Robustness analysis of MPC by prediction horizon variation (a)Process Output (b) Controller Output

#### Implementation of RTDA for SOPDT

The transfer function of SOPDT in consideration is

$$G(s) = \frac{e^{-0.1s}}{.2s^2 + 2s + 1}$$
(21)

the approximated FOPDT form of which is given by the transfer function

$$G(s) = \frac{e^{-0.2s}}{1.8s+1}$$
(22)

Hence the initial conditions for the RTDA will be: Overall System gain (K) = 1; Dead time ( $\alpha$ ) = 0.2 Time constant ( $\tau$ ) = 1.8; Sample time = 0.1 sec; Set point =1

## Servo Response

The servo response of the process given in the equation (22) is shown in the figure 9 a. From this response, the process output is able to reach the new set point without offset



Fig. 9- Servo Response for SOPDT model(a) Plant Response (b) Controller Response

## Robustness Analysis by variation of Tracking Parameter and prediction horizon

The value of tuning parameters  $\theta A$ ,  $\theta R$  and  $\theta D$  are kept constant at 0.3 while the value of parameter  $\theta T$  is varied as shown. An optimum response is only obtained for values of  $\theta T$  between 0.78 and 0.83 as depicted in figure 10. Figure 11.depicts that an increase in the tuning parameter  $\theta A$  results in increased conservativeness for the output response The parameters  $\theta T$ ,  $\theta R$  and  $\theta D$ are kept constant at 0.3 while the value of parameter  $\theta A$  is varied as shown. For values of  $\theta A = 0.37$  chattering is observed in the Outputs.



Fig. 10- Robustness Analysis by variation of Tracking Parameter (a) Plant Output (b) Controller Output



Fig. 11- Robustness Analysis by Prediction Horizon Variation (a) Plant Output (b)Controller Output

#### Conclusion

The Model Predictive Control (MPC) and the Robustness Tracking Disturbance rejection- overall Aggressiveness (RTD-A) Controller are studied and implemented for first order plus dead time and second order plus dead time processes. The Robustness analysis has been carried out on both the controllers. The performances of both the controllers are found to be satisfactory.

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