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AN INFINITE THERMOELASTIC SOLID DEFORMATION PROBLEM OF A PENNY-SHAPE CRACKED

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Abstract- This work is to study an infinite thermoelastic solid that is assumed to be homogeneous and isotropic is subjected to temperature and stress distributions. A cylindrical system of coordinates is used, in which the plane is that of the crack and the z-axis is normal to it at the centre. The corresponding set of the homogeneous thermoelastic equilibrium differential equation is solved by the Hankel transforms method. The mixed boundary conditions reduce the problem to the solution of two pairs of dual integral equations. The both dual integral equations for the thermal and thermoelastic parts are effectively reduced. We deduce, by using the inverse integral transform. These quantities of physical interest are given analytically and represented graphically. A numerical application is considered with some concluding results with discussions. All the definite integrals involved were calculated using Romberg technique of numerical integration with the aid of a Fortran Program compiled with Visual Fortran v.6.1 on a Pentium-IV pc with processor speed 2GHz.

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Introduction

According to the theory of generalized thermoelasticity with one relaxation time, a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law of heat conduction [1]. The hat equation associated with this theory is hyperbolic, eliminating automatically the paradox of infinite speeds of propagation inherent in previous uncoupled and the coupled theories of thermoelasticity. Such a theory was extended to include both the effects of anisotropy and the presence of heat sources [5].

Cracking takes into account the existence of manufacturing defects such as inclusions or voids in the material of areas of damage from which cracks will propagate and learn to reach a size where the structure reaches the ruin. The linear fracture mechanics used to solve many practical problems of engineering. Such as the ruin of the structure, material selection, predicting the lifespan of structures and definition of criteria acceptance of defects.

The failure of materials used in industry is almost always an adverse event, and for several reasons; it can endanger lives, cause economic losses and hinder the production of goods and services. The interest of researchers was first raised several studies on the problems dealing with the homogeneous and linear axisymmetric elastic problems. Study of such failure mechanics helps to maintain the structural integrity due to cracks. The thermoelastic problem of an infinite elastic medium containing a penny-shaped crack [6].The both cases of the flux of heat being a constant and in the form of a Fourier- Bessel series expansion were considered. The problem was transformed to a Fredholm integral equation.

The thermoelasticity problem for a half-space when the temperature is prescribed over a circular region. The corresponding dual integral equations were also reduce to a Fredholm integral and were treated with the aid of Cooke results. The stresses and the

displacements have been expressed in the form of double integrals which converge rapidly but have not been evaluated numerically [7].
The steady state thermal stresses in an elastic layer where a heat flux was imposed over a circular area discussed in[8]. The solution was achieved by the same the method mentioned above for the determination of the temperature field. A bidimensional analogous problem was studied by means of the Fourier transform method [9]. A similar study commissioned by the European Union concluded that billions of ECU per year could be saved using fracture mechanics technology.

Formulation of the problem

The cylindrical system of coordinates will be used, in which the plane , is the plane of the crack and the z-axis is normal to the crack at its centre. The crack occupies the region, and is subjected to prescribed distributions that vary with the radial distance , where is the radius of the crack, as shown in figure table (1). The solid is assumed to be homogeneous, isotropic and elastic. Since the Geometry of the region is symmetric about the crack plane, the problem is reduced to a mixed boundary value problem of thermoelasticity for the region, . All considered functions will depend on and only.
The displacement vector, thus, has the form

$$\underline{u} = (u, 0, w)$$

where $'u'$ and $'w'$ represent the components of the displacement vector, $\underline{u}$ in the radial and axial directions, respectively.
The governing equations can be written as [10]The displacement vector, thus, has the form

$$\underline{u} = (u, 0, w)$$

where $'u'$ and $'w'$ represent the components of the displacement vector, $\underline{u}$ in the radial and axial directions, respectively.
The governing equations can be written as **[10]**

$$\mu\nabla^2 u - \frac{\mu}{r^2}u + (\lambda+\mu)\frac{\partial e}{\partial r} - \gamma\frac{\partial T}{\partial r} = 0 \tag{1}$$

$$\mu\nabla^2 w + (\lambda+\mu)\frac{\partial e}{\partial z} - \gamma\frac{\partial T}{\partial z} = 0 \tag{2}$$

where λ and μ are Lame's elastic constant, γ is a material constant.

For an isotropic body, $\gamma = (3\lambda + 2\mu)\alpha_t$ being the coefficient of linear thermal expansion.
Taking the heat equation in the form:

$$\nabla^2 T = 0 \tag{3}$$

In the above equations, T is the absolute temperature. A reference temperature T_o is assumed to be such that

$$\left|\frac{(T-T_o)}{T}\right| < 1$$

and e , the cubical dilatation, is given by the following relation [10]:

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma(T - T_0)$$

$$\begin{aligned} e &= \frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} \\ &= \frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} \end{aligned} \tag{4}$$

∇^2 is the two-dimensional Laplacian operator in a cylindrical coordinates system that takes the form:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The stress components expressed by the following constitutive relations that supplement the above equations:

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma(T - T_0) \tag{5}$$

$$\sigma_{\theta\theta} = 2\mu\frac{\partial w}{\partial r} + \lambda e - \gamma(T - T_0) \tag{6}$$

$$\sigma_{rz} = \mu\left(\frac{\partial z}{\partial z} + \frac{\partial w}{\partial r}\right) \tag{7}$$

Making use of the following non-dimensional variables:

$r' = c_1\eta r$, $z' = c_1\eta z$, $u' = c_1\eta u$, $w' = c_1\eta w$,

$\sigma_{ij}' = \frac{\sigma_{ij}}{\mu}$ and $\theta' = \frac{\gamma(T-T_0)}{(\alpha+2\mu)}$,

where $\eta = \frac{\rho c_E}{k}$, c_1 is the speed of propagation of isothermal elastic waves given by: $c_1 = \sqrt{\frac{\alpha+2\mu}{\rho}}$, in which ρ is the density and k is the material's thermal conductivity.

Using the above non-dimensional variables, the governing equations take the following form (dropping the primes for convenience):

$$\nabla^2 u - \frac{u}{r^2} + (\beta^2 - 1)\frac{\partial e}{\partial r} - \beta^2\frac{\partial \theta}{\partial r} = 0 \tag{8}$$

$$\nabla^2 w + (\beta^2 - 1)\frac{\partial e}{\partial z} - \beta^2\frac{\partial \theta}{\partial z} = 0 \tag{9}$$

$$\nabla^2 \theta = 0 \tag{10}$$

while the stress components (5)-(7) are reformulated to become:

$$\sigma_{rr} = 2\frac{\partial u}{\partial r} + (\beta^2 - 2)e - \beta^2\theta \tag{11}$$

$$\sigma_{zz} = 2\frac{\partial w}{\partial z} + (\beta^2 - 2)e - \beta^2\theta \tag{12}$$

$$\sigma_{rz} = \left(\frac{\partial z}{\partial z} + \frac{\partial w}{\partial r}\right) \tag{13}$$

In the above equations, note that $\lambda^2 = \dfrac{\alpha + 2\mu}{\mu}$

Combining equations (8) and (9), regarding equations (4) and (10) to get:

$$\nabla^2 e = 0$$

The boundary conditions for the problem at $z = 0$ may be taken as:

$$\theta(r,0) = f(r) \quad , \quad 0 < r < a \tag{14}$$

$$\frac{\partial\theta(r,0)}{\partial z} = 0 \quad , \quad 0 < r < \infty \tag{15}$$

$$w(r,0) = 0 \quad , \quad a < r < \infty \tag{16}$$

$$\frac{\partial\sigma_{zz}}{\partial r} = 0 \quad , \quad a < r < \infty \tag{17}$$

$$\frac{\partial\sigma_{rz}}{\partial r} = -P(r) \quad , \quad 0 < r < \infty \tag{18}$$

where $f(r)$ and $p(r)$ are known functions of temperature and mechanical stress, respectively.

Analytical solution of the problem

The Hankel transform with parameter ξ of a function $f(r,z)$ denoted by $\overline{f}(\xi,z)$ is given by the relation[11]

$$\overline{f}(\xi,z) = H\left[f(r,z)\right] = \int_0^\infty f(r,z)\, rJ_n(\xi r)\, dr$$

where $J_n(\xi r)$ is the Bessel function of the first kind of order n .

The inverse Hankel transform is given by the relation[11], [12]

$$f(r,z) = H^{-1}\left[\overline{f}(\xi,z)\right] = \int_0^\infty \overline{f}(\xi,z)\,\xi J_0(\xi r)\, d\xi$$

Similarly,

$$H\left(\frac{\partial^2 f(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial f(r,z)}{\partial r}\right) = -\xi^2\overline{f}(\xi,z) \tag{19}$$

Taking the Hankel transform with parameter ξ of both sides of equation (10) and using the operational relation of the Hankel transform [11] which is given in the equation (19), one obtain

$$(D^2 - \xi^2)\overline{\theta} = 0 \text{ , where } D = \frac{\partial}{\partial z}$$

The solution of the above equation, which is bounded at infinity, can be written as,

$$\overline{\theta}(\xi,z) = A(\xi)e^{-\xi|z|}$$

where $A(\xi)$ is a parameter depending on ξ only.

Due to symmetry, only the case where $z > 0$ will be considered, accordingly.

$$\overline{\theta}(\xi,z) = A(\xi)e^{-\xi z} \tag{20}$$

Taking the inverse Hankel transform of both sides of Eq. (20), gives:

$$\theta(r,z) = \int_0^\infty A(\xi)\, e^{-\xi z}\xi J_0(\xi r)\, d\xi \tag{21}$$

Similarly, since e satisfies the same differential equation as ξ , $\overline{e}(\xi)$ can be written in the form.

$$\overline{e}(\xi,z) = \frac{\lambda^2 A(\xi) - 2B(\xi)}{\lambda^2 - 1}e^{-\xi z} \tag{22}$$

Where $B(\xi)$ is a parameter depending on ξ only.

Applying the inverse Henkel transform for Eq. (22), its reads. We get

$$e(r,z) = \int_0^\infty \frac{\lambda^2 A(\xi) - 2B(\xi)}{\lambda^2 - 1}e^{-\xi z}\xi J_0(\xi r)d\xi \tag{23}$$

Again applying the Hankel transform to both sides of Eq. (9) and regarding equation (20) and (22), the latter became:

$$(D^2 - \xi^2)\overline{w} = -2\xi B(\xi)e^{-\xi z} \tag{24}$$

The solutions of Eq. (24), for $z > 0$, which is bounded at infinity is given by,

$$\overline{w}(\xi,z) = [C(\xi) + B(\xi)]\, e^{-\xi z} \tag{25}$$

Where $C(\xi)$ is a parameter depending on ξ only.

Applying the inverse Hankel transform to Eq. (25), we obtain

$$w(r,z) = \int_0^\infty [C(\xi) + B(\xi)]\, e^{-\xi z}\xi J_0(\xi r)d\xi \tag{26}$$

Taking the Hankel transform of both sides of Eq. (4) and considering the Eq. (22) and (25).

$$H\left(\frac{1}{r}\frac{\partial(ru)}{\partial r}\right)= \left[\frac{\lambda^2 A(\xi)-(\lambda^2+1)B(\xi)}{\lambda^2-1}\right.$$

$$\left.+\xi C(\xi)+\xi B(\xi)z\right]e^{-\xi z}$$

After taking the inverse Hankel transform to the Eq. (27) is given by

$$u=\int_0^\infty\left[\frac{\lambda^2 A(\xi)-(\lambda^2+1)B(\xi)}{\lambda^2-1}+\xi C(\xi)+\xi B(\xi)z\right]$$

$$\times e^{-\xi z}J_1(\xi r)d\xi \tag{28}$$

The stress tensor components will take the forms:

$$\sigma_{zz}=-\int_0^\infty\left[\frac{\lambda^2 A(\xi)-2B(\xi)}{\lambda^2-1}+2\xi C(\xi)+2\xi B(\xi)z\right]$$

$$\times e^{-\xi z}\xi J_0(\xi r)d\xi \tag{29}$$

$$\sigma_{rz}=-\int_0^\infty\left[\frac{\lambda^2[A(\xi)-2B(\xi)]}{\lambda^2-1}+2\xi C(\xi)+2\xi B(\xi)z\right]$$

$$\times e^{-\xi z}\xi J_1(\xi r)d\xi \tag{30}$$

Substituting from equations (21), (28), (29) and (30) in to the boundary conditions (14)- (18) the following relations are obtained:

$$\int_0^\infty A(\xi)\,\xi J_0(\xi r)d\xi=f(r)\;,\quad 0<r<a \tag{31}$$

$$\int_0^\infty\left[\frac{\lambda^2[A(\xi)-2B(\xi)]}{\lambda^2-1}+2\xi C(\xi)\right]$$

$$\xi^2 J_1(\xi r)d\xi=0\;,\quad 0<r<\infty \tag{32}$$

$$\int_0^\infty\left[\frac{\lambda^2 A(\xi)-2B(\xi)}{\lambda^2-1}+2\xi C(\xi)\right]$$

$$\xi^2 J_1^{'}(\xi r)d\xi=\frac{P(r)}{2}\;,\quad 0<r<a \tag{33}$$

$$\int_0^\infty A(\xi)\,\xi^2 J_0(\xi r)d\xi=0\;,\quad a<r<\infty \tag{34}$$

$$\int_0^\infty C(\xi)\,\xi J_0(\xi r)d\xi=0\;,\quad a<r<\infty \tag{35}$$

Since Eq.(32) is valid for all values of r , $B(\xi)$ is obtained in the form,

$$B(\xi)=\left[\frac{\lambda^2 A(\xi)+2(\lambda^2-1)\xi C(\xi)}{(\lambda^2-1)}\right] \tag{36}$$

Substituting for $B(\xi)$ from Eq. (36) and using Eq. (31), Eq. (33) reduces to,

$$\int_0^\infty C(\xi)\,\xi^3 J_1^{'}(\xi r)d\xi=\left[\frac{\lambda^2[P(r)-f(r)]}{2(\lambda^2-1)}\right]\;,\quad 0<r<a \tag{37}$$

Eqs. (31) and (34) are set of dual integral equations whose solution gives the unknown variable $A(\xi)$, also Eq.(35) and (37) are a set of a dual integral equations, the solution of which gives the unknown variable $C(\xi)$ .The solution of the dual integral equations (31) and (34) is given by [2]

$$A(\xi)=\frac{2}{\pi\xi^2}\int_0^a\cos(\xi u)\left\{\frac{d}{du}\int_0^u\frac{rf(r)}{\sqrt{u^2-r^2}}dr\right\}du \tag{38}$$

The solution of the dual integral Eq. (35) and (37) has the form

$$C(\xi)=\frac{\beta^2}{\pi\xi^2\left(\lambda^2-1\right)}\int_0^t t\sin(\xi t)\left(\int_0^{\pi/2}\sin\theta\right.$$

$$\left.\times\left[p(t\sin\theta)-f(t\sin\theta)d\theta\right]\right)dt \tag{39}$$

Numerical Analysis:

In what follow we shall take,

$$f(r)=1\;,\quad 0<r<a\;,\quad p(r)=r^2\;,\quad 0<r<a$$

Substituting these values into Eq. (38) and (39), and after some manipulations,

$$A(\xi)=\frac{2\sin(\xi a)}{\pi\xi^3} \tag{40}$$

$$C(\xi)=\frac{\beta^2}{\pi\xi^2(\lambda^2-1)}\left[\frac{(3a-2a^3)\cos(\xi a)}{3\xi}+\right.$$

$$\left.\frac{(2a^2-1)\sin(\xi a)}{\xi^2}+\frac{4a\cos(\xi a)}{\xi^3}-\frac{4\sin(\xi a)}{\xi^4}\right] \tag{41}$$

Substituting from Eq. (40) and (41) into relation (36), once obtained,

$$B(\xi)=\frac{1}{\pi\xi}\left[\frac{(3a-2a^3)\cos(\xi a)}{3\xi}+\frac{(2a^2)\sin(\xi a)}{\xi^2}\right.$$

$$\left.+\frac{4a\cos(\xi a)}{\xi^3}-\frac{4\sin(\xi a)}{\xi^4}\right] \tag{42}$$

Substituting the values of $A(\xi)$ from Eq. (33) into Eq. (14), We obtain,

$$\theta(r,z)=\frac{2}{\pi\xi}\int_0^\infty\frac{\sin(\xi a)}{\xi}e^{-\xi z}J_0(\xi r)d\xi \tag{43}$$

For $z=0$.

$$\theta(r,0)=\frac{2}{\pi\xi}\int_0^\infty \frac{\sin(\xi a)}{\xi}J_0(\xi r)d\xi \tag{44}$$

In order to evaluate the integral on the right hand side of Eq. (44) for $r>a$,
The following into integral formulae of the Bessel's functions [3], [4] are to be used.

$$\int_0^\infty \frac{\sin(\xi b)}{\xi}J_0(\xi r)d\xi=\begin{cases}\frac{\pi}{2}, & r<b\\ \sin^{-1}\left(\frac{b}{r}\right), & r>b\end{cases} \tag{45}$$

$$\int_0^\infty \cos(\xi b)J_0(\xi r)d\xi=\begin{cases}0, & r<b\\ \frac{1}{\sqrt{r^2-b^2}}, & r>b\end{cases} \tag{46}$$

Using the integral formula (45), equation (44) becomes,
For $r>a$

$$\theta(r,0)=\frac{2}{\pi\xi}\sin^{-1}(\frac{a}{r}) \tag{47}$$

On substituting the values of $A(\xi)$, $C(\xi)$ and $B(\xi)$ from Eqs. (40), (41) and (42) respectively, into Eq. (28), we obtain,

$$u(r,0)=\frac{-2}{\pi\xi(\lambda^2-1)}\int_0^\infty\left[\frac{(3a-2a^3)\cos(\xi a)}{3\xi}+\frac{(2a^2-1)\sin(\xi a)}{\xi^2}+\frac{4a\cos(\xi a)}{\xi^3}-\frac{4\sin(\xi a)}{\xi^4}\right]J_1(\xi r)d\xi \tag{48}$$

Integrating the resulting relation with respect to b over the range $(0,a)$ to obtain for $r>a$

$$\int_0^\infty\left[\frac{\sin(\xi a)}{\xi^3}-\frac{a\cos(\xi a)}{\xi^2}\right]J_1(\xi r)d\xi=\frac{a}{2}\sin^{-1}\left(\frac{a}{r}\right)-\frac{r^3}{4}\left[\sin^{-1}\left(\frac{a}{r}\right)-\frac{a\sqrt{r^2-a^2}}{r^2}\right] \tag{49}$$

Equation (48) becomes,

$$u(r,0)=\frac{-2}{\pi\xi(\lambda^2-1)}\times\left[\sin^{-1}\left(\frac{a}{r}\right)+\frac{a(3+a^2-3r^2)}{\sqrt{r^2-a^2}}\right] \tag{50}$$

Substituting from Eqs. (33)- (35) into Eq. (23).,

$$\sigma_{rz}=\frac{-2}{\pi\xi}\int_0^\infty\left[\frac{(3a-2a^3)\cos(\xi a)}{3}+\frac{(2a^2)\sin(\xi a)}{\xi}+\frac{4a\cos(\xi a)}{\xi^2}-\frac{4\sin(\xi a)}{\xi^3}\right]\times(1+\xi^2 z)e^{-\xi z}J_1(\xi r)d\xi$$

Putting $z=0$,

$$\sigma_{rz}=\frac{-2}{\pi\xi}\int_0^\infty\left[\frac{(3a-2a^3)\cos(\xi a)}{3}+\frac{(2a^2)\sin(\xi a)}{\xi}+\frac{4a\cos(\xi a)}{\xi^2}-\frac{4\sin(\xi a)}{\xi^3}\right]\times J_1(\xi r)d\xi \tag{51}$$

Integrating Eq. (44), over the range $(0,a)$ given,

$$\int_0^\infty\left[\frac{(-2a^3)\cos(\xi a)}{3\xi}+\frac{(2a^2)\sin(\xi a)}{\xi^2}+\frac{4a\cos(\xi a)}{\xi^3}-\frac{4\sin(\xi a)}{\xi^4}\right]J_1(\xi r)d\xi$$

$$=\frac{r^3}{6}\left(\frac{3}{2}\sin^{-1}(\frac{a}{r})-\frac{2a\sqrt{r^2-a^2}}{r^2}+\frac{a\sqrt{r^2-a^2}}{2r^4}(r^2-2a^3)\right),\ \text{for } r>a \tag{52}$$

Also using the following relation,

$$\int_0^\infty\left[\frac{\sin(\xi a)}{\xi^2}\right]J_1(\xi r)d\xi=\frac{r}{2}\left(\sin^{-1}(\frac{a}{r})-\frac{a\sqrt{r^2-a^2}}{r^2}\right) \tag{53}$$

Substituting Eq. (53) into Eq. (51) to get,
for $r>a$

$$\sigma_{rz}=\frac{-1}{\pi\xi}\left(\frac{r(r^2-2\lambda^2)}{4}\sin^{-1}\left(\frac{a}{r}\right)+\frac{a[12+6\lambda^2+r^2(r^2-2a^2-4)]\sqrt{r^2-a^2}}{12r}\right). \tag{54}$$

Table1- Thermal and elastic constants for copper.

$\alpha_t=1.78(10)^{-5}K^{-1}$	$C_E=383.1J/kgK$
$\lambda^2=3,86(10)^{10}N/M$	$C_1=4.158(10)^3 m/s$
$\mu=3.86(10)^{10}N/M$	$\alpha=7.76(10)^{10}N/M^2$
$\rho=8954kg/m^3$	$T_0=293K$
$a=0.01$	

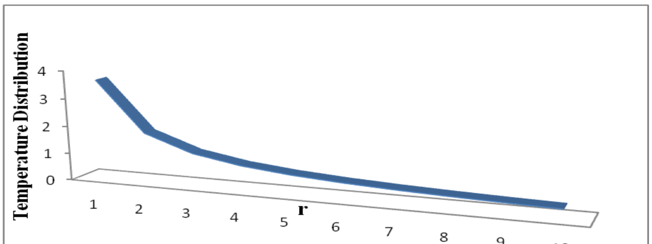
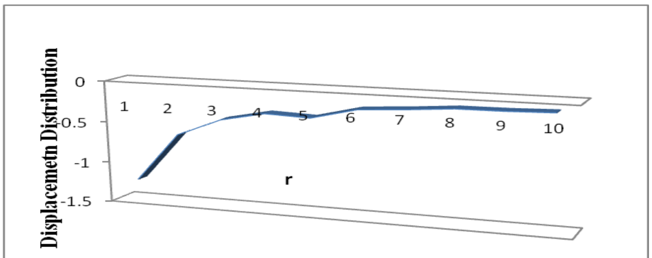


Fig.1 Temperature Distribution



Fi.g.2 Radial Displacement Distribution

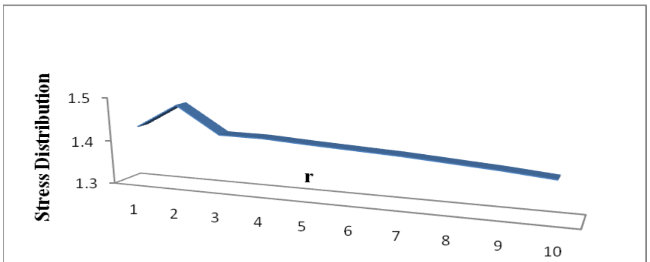


Fig.3 Radial Stress Distriburtion

The above evaluations are applied to copper material, whose constants are shown in table 1.

The computations were performed for different values of z as shown in figure (1-3). All the definite integrals involved were calculated using Romberg technique of numerical integration with variable step size; upon using a computer program compiled with Visual Fortran v.6.1 on a Pentium-IV pc with processor speed 2.0GHZ.

Figure (1) displays the distribution of the temperature T, versus the radial distance r, at value of of the axial distance z. Note that the crack's radius r, is unity or is taken to be the unit of length in the problem so that ,, and that the crack is symmetric with respect to the z-plane . It is clear from the graph that T, has its maximum value at the initial of the crack, it begin to fall just near the crack edge, where it experiences sharp decreases (with maximum negative gradient at the crack's circumference). Graph lines shows slope at crack ends according to z-value. These results obey physical reality for the behavior of copper as a polycrystalline solid.

Figure (2) display a change of volume is attended by a change of the subject of the temperature while the effect of the deformation upon the temperature distribution is the subject of the theory of thermoelasticity. The solid particles radial displacement, u, shows and increase to reach its maximum magnitude just after the crack circumference. Moreover, U rises at a decreasing rate with increasing z, we go vertically farther from the crack. Figure (3) display radial component distributions has its maximum amplitude just at the crack edge and it reaches zero at infinity. Variation of z has a serious effect on the magnitudes of mechanical stresses. Such effect on the radial stress is in opposite manner to that on the axial one. From all figures we have to conclude that to propagate it, the solid need to be subjected to an external stress (tensile, shear…).

Conclusion

- Analytical solutions based upon the integral Hankel transforms for thermoelastic problem in solids have been developed and utilized.
- Temperature, radial and axial distributions were estimated at different distances from the centre of the crack.
- Crack dimensions are significant to elucidate the mechanical structure of the solid.
- Cracks are stationary and external stresess is demanded to propagate such cracks.
- Implement such solutions for a penny-shaped crack in a composite solid.
- Radial and axial stress distributions were evaluated as functions of the distance from the crack centre.

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