



AN INVESTIGATION INTO THE DISSIPATIVE STOCHASTIC MECHANICS BASED NEURON MODEL UNDER NOISY INPUT CURRENTS

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Abstract- In this paper, it is attempted to look into the dissipative stochastic mechanics based neuron model under noisy input currents. Specifically, it is concentrated on the role of input noise with reference to the renormalization terms in the model. The investigation shows that the use of noise in the inputs can improve the spiking rates and the spike coherence values, especially in the presence of the renormalization terms.

Keywords- Ion Channel Noise, Stochastic Ion Channels, Neuronal Dynamic, Hind marsh-Rose Model, Dissipative Stochastic Mechanism Model

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Introduction

Guided by the existence of a multiple number of gates in each ion channel, it was recently expected that the activity equations of the neuronal dynamics obtain a number of renormalization terms, which play important role in the membranes that are small in size [1-3].

Neurons exhibit electrical action which is in nature known to be stochastic [4]. The external noise from the synapses is the main cause for stochastic. Still the interior noise, which participates to the gating probabilistic nature of the ion channel, also it can have important effects on the neuron's dynamic performance as displayed by the experimental studies [5,6] and by the numerical simulations or theoretical researches [7,8].

Neuronal dynamics under the effect of channel fluctuation is usually modeled with stochastic differential equations acquired by using some vanishing white-noise conditions into the fundamental deterministic equations [7]. The dissipative stochastic mechanics (DSM neuron) based neuron model raised by Güler [3], is a special case of this. The DSM model has some forms of functionality named the renormalization terms, as well as some vanishing white-noise conditions in the activity equations. The DSM model has been studied in numerical detail for its time independent input current's dynamics [1,3]; it was established that the corrections of renormalization increases the changes in behavior from quiescence to spiking and from tonic firing to bursting. It was further established that the existence of renormalization corrections can result in faster temporal synchronization of the electric coupled consecutive discharges of two neuronal units [2,9].

In this paper, the DSM model is investigated in the situation of noise fluctuating input currents and concentrates on what role the renormalization terms and noise could have on the spiking rates and the spike coherence values.

Modeling

A brief summary on Hodgkin-Huxley (HH) and Hind marsh-Rose's (HR) models are presented. Following that, the dissipative stochastic mechanic (DSM) based neuron model will be elaborated that yields the dynamics of Hind marsh-Rose model in a deterministic condition on which the present study and experiments were conducted.

The Hodgkin-Huxley Model

In this model, section of nerve membrane had an electrical feature that can be sculptured by an equivalent circuit in such a way that current passing through the membrane has two major elements, the first one related with charging the membrane capacitance and the other one related to specific types of ion's movement through membrane. After that, the ionic current is also subdivided into three recognizable currents, sodium I_{Na} , potassium I_K , and small leakage I_L that are mostly conveyed by chloride ions.

The differential equation that corresponds to the electrical circuit is shown below:

$$C_m \frac{dV_m}{dt} + I_{ion} = I_{ext}$$

where C_m is membrane capacitance, V_m is membrane potential, and

I_{ext} is the current that externally applied. I_{ion} is ionic current passing through the membrane and can be calculated from the next equation:

$$I_{ion} = \sum_i I_i$$

$$I_i = g_i(V_m - E_i)$$

where I_i indicates every ionic element that having related conductance g_i and reversal potential E_i .

In the model of a giant squid axon, it has three kinds of currents (I_i): sodium I_{Na} , potassium I_K , and leakage I_L and that will give us this equation:

$$I_{ion} = I_{Na} + I_K + I_L$$

$$= g_{Na}(V_m - E_{Na}) + g_K(V_m - E_K) + g_L(V_m - E_L)$$

The macroscopic g_i (g_{Na} , g_K , g_L) conductance starts from the united influence of a great amount of membrane microscopic ion channels. Ion channel can be considered as physical gates in a small number that manage the ions flow across the channel. When all the gates in an ion channel are in the permissive condition, ions can flow through the channel, and the channel is open.

The Hind Marsh-Rose Model

Fitz Hugh [10] and Nagumo [11] noticed separately in the Hodgkin-Huxley equations, that in equivalent time-scales the membrane potential $V(t)$ and sodium activation $m(t)$ developed during an action potential, where the change of sodium inactivation $h(t)$ and potassium activation $n(t)$ are similar, even though that's happened in slower time scales. Consequently, now the simulation spiking response of a model can be represented in the following equations:

$$\dot{x} = a(y - f_1(x) + I)$$

$$\dot{y} = b(g_1(x) - y)$$

where x indicates membrane potential and y denotes the recovery parameter. $f_1(x)$ is represented with cubic function, $g_1(x)$ with linear function, variables a and b are time constants and $I(t)$ is the external current applied or clamping as time function t .

Hind marsh-Rose benefited from the Fitz Hugh-Nagumo model to enhance their model, which was a simplified version of the Hodgkin-Huxley equations and substituted the linear function $g(x)$ with a quadratic function so that the model in a long interspaces interval can accomplish rapid firing. [Fig-1] displays the diagram of null-cline of the model of Hind marsh-Rose in 1984 [12].

The HR model needed more than one equilibrium point to generate burst firing reaction. Basically, the state of Sub-threshold stable resting will have one point and one point inside the cycle of firing limit. To make the null-clines meet and bring additional points of equilibrium, a minor deformation was necessary. The controlling equations were altered to satisfy the requirements as shown in the following equations:

$$\dot{x} = y - f(x) + I$$

$$\dot{y} = g(x) - y$$

where in the simple image of HR model $f(x)=x^3-3x^2$ and $g(x)=1-5x^2$. Analysis of the phase plane of the granted equations is shown next page in [Fig-2].

The steady point in the [Fig-2] is the node A that corresponds to the neuron's resting state. By using current pulse de-polarizing that is large enough, $\dot{x} = 0$ null-cline is to be lowered so that the nodes A

and B meets and vanishes. Ending firing is impossible by just terminating the stimulus and the state will get out of the limit cycle only after applying a suitable hyper-polarizing pulse. Therefore, to terminate the firing state of the model the term z was inserted. The variable that has been additive stands for a slowly changed current, changing the inserted current I to the effective input $I - z$. When the neuron is in a firing state, the z value is required to be raised. After this modification, the general set of equations for HR model is as shown below:

$$\dot{x} = -x^3 + 3x^2 + y + I - z$$

$$\dot{y} = 1 - 5x^2 - y$$

$$\dot{z} = r(h(x) - z)$$

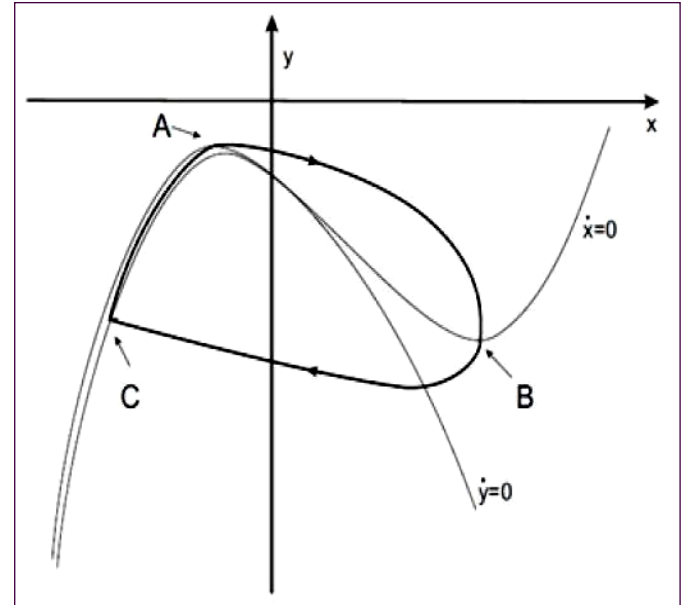


Fig. 1- The 1984 HR model phase plane representation. Null-clines $\dot{x} = 0, \dot{y} = 0$ (thin lines) and firing limit-cycle (thick line). The model has one equilibrium node [13].

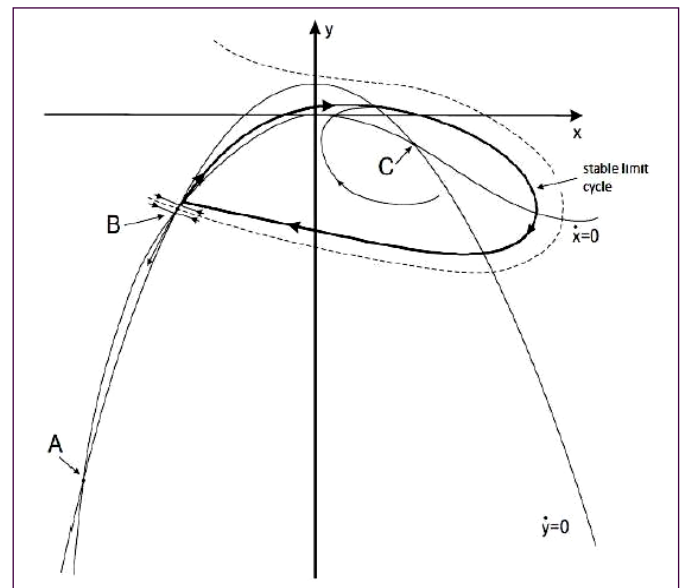


Fig. 2- Hind marsh-Rose model phase plane description. The equilibrium points A, B, and C are a stable node, an unstable saddle, and an unstable spiral, correspondingly, a humble form of $f(x)$ is used in this equation as is indicated null-cline shows [13].

It should be noted that the $f(x)$ and $g(x)$ are removed and substituted by their equivalents. Where x indicates membrane potential, y denotes the recovery parameter, and z stands for the current adaptation with time constant r . Parameter z rises up through fire state and goes down through the non-fire state what made the model able to show bursting, chaotic bursting and post-inhibitory rebound are variables h and r . [12,13]. [Fig-3] display the analysis of phase plane of the equation below applying more complex form of $f(x)$ as suggested in [12].

$$m \dot{X} = \Pi + S_5 I$$

$$\dot{\Pi} = \left(\frac{3a}{m} X^2 - \frac{2b}{m} X + S_0 \right) (\Pi + S_5 I) - S_1 a X^2 + S_2 X^2 + S_6 X - S_7 X_{eq}(I) + S_1 I + S_7 - (1-r) \left[k \left(1 - \frac{\xi y}{m} \right) z + (1-k) \left(1 - \frac{\xi y}{m} \right) y \right]$$

$$\dot{y} = -y - dX^2 + c + n^y$$

$$\dot{z} = -rz + rh(X - x_s) + n^z$$

$$\Pi(t_0) = y(t_0) - z(t_0) - a(X(t_0))^3 + b(X(t_0))^2 + (1 - S_5)I(t_0)$$

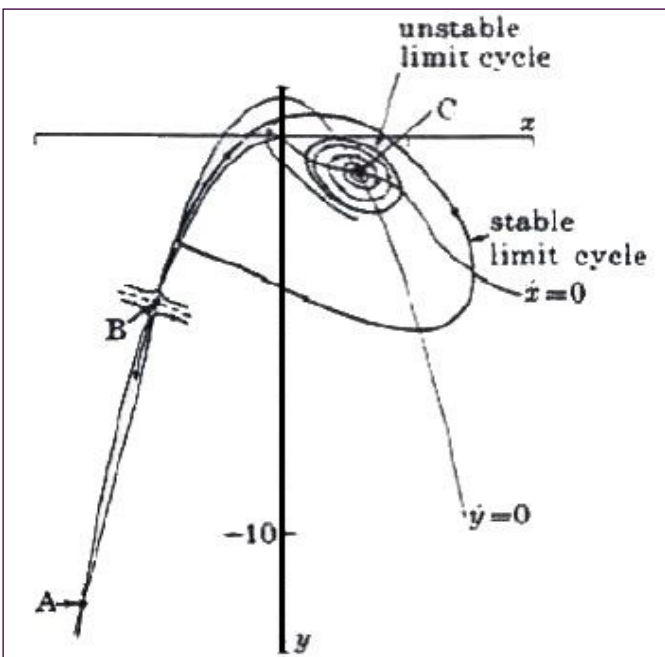


Fig. 3- Hind marsh-Rose model phase plane analysis with the use of more complex form of $f(x)$. The equilibrium nodes A, B, and C are a stable node, an unstable saddle, and an unstable spiral, correspondingly, unstable limit cycle is defined here [12].

The Dissipative Stochastic Mechanics (DSM) Neuron Model

The DSM based neuron special formulation comes from a point of view that ion channels conformational fluctuation are subjected to two distinct types of noise. These two noise types were formulated as the intrinsic noise and topological noise. The first one is the intrinsic noise which starts from gating particles voltage dependent movement between inner and outer of the membrane surfaces which is stochastic in nature. Accordingly, gates open and close in a probabilistic manner, this is the average number, not the precise number. Open gates in the membrane are defined by the voltage.

The second one is the topological noise that comes from multiple numbers of gates existences in the channels and contributes to the changes in the open gates topology, instead of the changes in the open gates number.

Curiously, as gating particles during the dynamics do not follow a specific order for the occupation of the available closed gates, and the evacuation of the open gates, the membrane at two distinct times could have an equivalent number of gates being open but two various conductance values. The topological noise is contributed to the suspicion in the open channels numbers that occurs even if open gates numbers are precisely known. Therefore, in defining the dynamics of the voltage, all permits from the gates open topologies which should be well thought of. DSM neuron formula was developed based on Hind marsh-Rose model [12] and benefit from the Nelson's stochastic mechanics [14], in the dissipation existence, to model the ion channel noise impacts on the membrane voltage dynamics. The topological noise impact on the neuron dynamics gets to be more important in membranes that are small in size. Accordingly, the DSM neuron functions like the Hind marsh-Rose model when the membrane size is too large.

It was shown that the representation of intrinsic noise will get to be more important in small size membranes and it's the same in case of fewer channels in DSM Neuron [1]. The intrinsic noise can be the source of spiking activity in quiet deterministic model and in large input current values bursting can be caused. In [Fig-4] and [Fig-5] the DSM Neuron dynamics in a small size membrane is demonstrated. Notice that renormalization corrections are equal to zero so that the result is studied regardless of the topological noise influence.

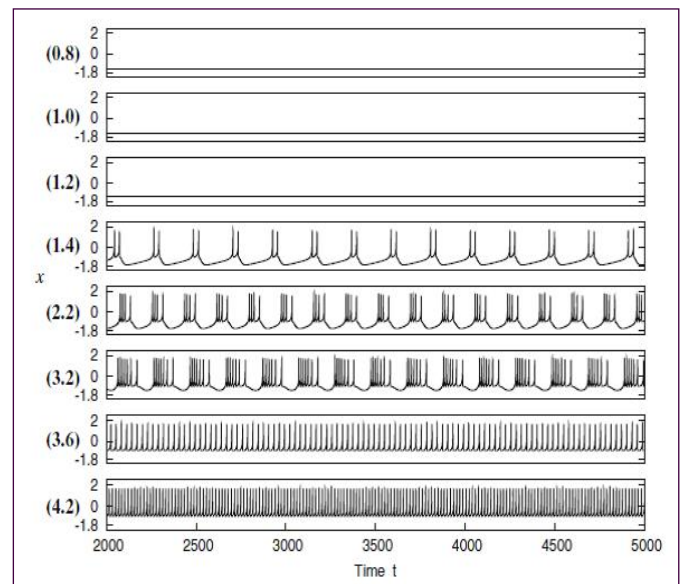


Fig. 4- Membrane voltage time series of the deterministic Hind marsh-Rose model applying the parameter values $m = 1, a = 1, b = 3, c = 1, d = 5, h = 4, r = 0.004$ and $x_s = -1.6$; for different constant inputs current values I , indicated in a parenthesis on the left of each plot [1].

Renormalization corrections are caused by the interaction between the topological and intrinsic noises. The existence of correction's parameters further increases the shift in behavior from quiescence to spiking and from tonic firing to bursting to a significant degree and with evidence to this; it causes the bursting activity to occur in a wider domain of input currents. Hence, in the existence of the correction terms, the spiking activity begins to occur at smaller input current values and the bursting activity is extended for higher input current values. The DSM neuron manner under the effect of corrections is displayed next page in [Fig-6].

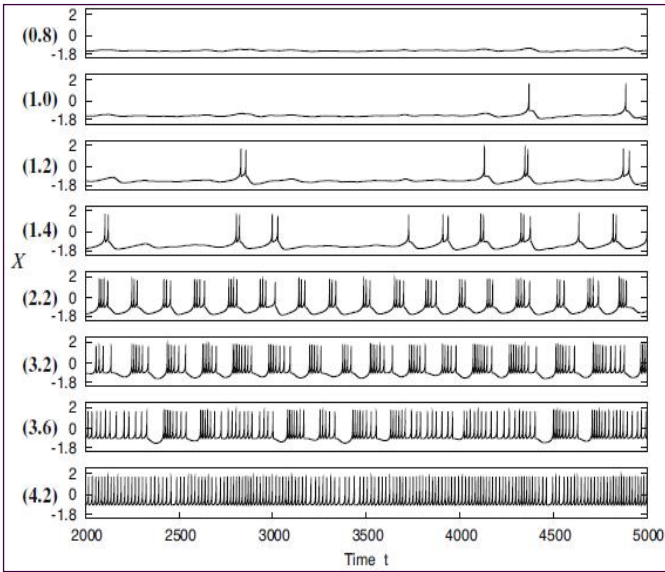


Fig. 5- Time series of X when the DSM neuron is exposed just to the intrinsic noise applying the Hind marsh-Rose $m = 0.25$, $a = 0.25$, $b = 0.75$, $c = 0.25$, $d = 1.25$, $h = 1$, $r = 0.004$ and $X_s = -1.6$ with the temperature $T = 0.008$. Schemes for different constant inputs current values $4 I$ (scaled by the factor of four) [1].

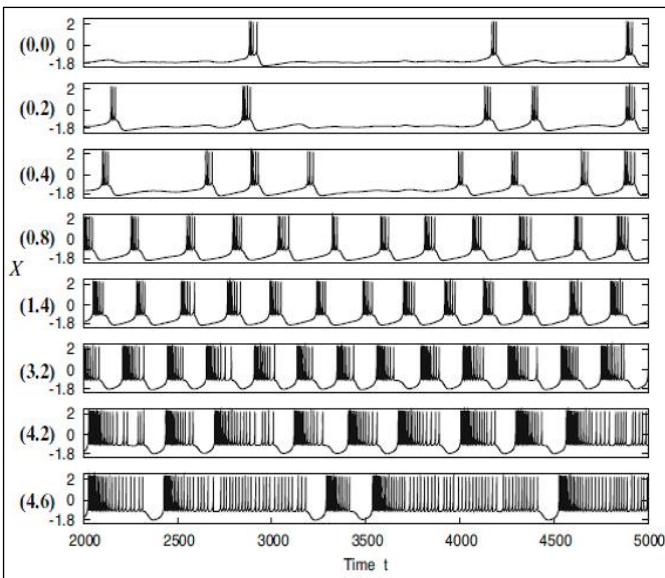


Fig. 6- Time series of X using the correction coefficients $\epsilon_m^y = 0.1$, $\epsilon_u^y = 1.0$, $\epsilon_m^z = 0.001$ and $\epsilon_m^y = 0.005$ with the temperature $T = 0.008$. The Hind marsh-Rose parameter are $m = 1$, $a = 1$, $b = 3$, $c = 1$, $d = 5$, $h = 4$, $r = 0.004$ and $x_s = -1.6$ [1].

Noise in Neuronal Information Processing

Noise can enhance neuronal systems from signal transmission properties point of view under certain conditions. Sub-threshold oscillations in a neuron may have an important effect on the data coding in neurons when magnified by noise [15]. The perfect noise amount existence in the neuron system may have association with the input signal to enhance signal observation [16].

There are two types of noise; the internal and external which have been explained within the DSM neuron approach model in the third chapter. In this study the noise is a white Gaussian noise and considered to be one variable containing both the internal and the external noise.

Gaussian noise is statistical noise that has its probability density function equal to that of the normal distribution, which is also known as the Gaussian distribution. In other words, the values that the noise can take on are Gaussian-distributed. A special case is white Gaussian noise, in which the values at any pairs of times are statistically independent (and uncorrelated). In applications, Gaussian noise is most commonly used as additive white noise to yield additive white Gaussian noise.

Experiment and Results

Rather than investigating the role of the correction coefficients separately, the standard values of epsilons (renormalization terms) has been taken as follows ($Y_m = 0.1$, $Y_u = 0.5$, $Z_m = 0.001$, and $Z_u = 0.005$) and scale them to zero to have a benchmark of various sets of correction coefficients. We use the following periodic input current for the neuron:

$$I = I_{base} + gwn$$

where I_{base} indicates the current and gwn are Gaussian white noise.

The model's behavior is studied in the context of spiking rate and coherence, within the following ranges of the parameters: the time will be measured mS. voltage will be measured in mV. Noise variances values have been used between 0 and 2 and will be measured in $\mu A/cm^2$ and I_{base} values between 0.8 and 2.2 and will be measured in $\mu A/cm^2$. The spiking rate when the I_{base} values under 0.8 is small and after the I_{base} pass the values of 2.5 it become too large so in both cases we didn't use that results in this thesis for comparison. Only the optimum result was taken in case of the lowest and highest spiking rate.

In the [Fig-7] the experiment was done by fixing the renormalization terms values and the noise variance values to zero. And by changing the I_{base} which lead to the result shown in the [Fig-6]. When the value of I_{base} is small there is no spiking action and after increasing the value of I_{base} the neuron spiking rate start to increase in a rapid manner and the experiments have a low coherence.

[Fig-8] the result is gotten by fixing the renormalization terms values to $\epsilon_m^y = 0.1$, $\epsilon_u^y = 1.0$, $\epsilon_m^z = 0.001$ and $\epsilon_m^y = 0.005$ and the noise variance values to zero and when changing the I_{base} the number of spikes in the experiments will increase in slow manner instead of the fast increasing as in the experiments shown in the [Fig-7] and the coherence will be high in this experiments contrary to the experiments in the [Fig-7].

In the [Fig-9], the experiment was done by fixing the renormalization terms values to zero and the noise variance values to 0.8. And by changing the I_{base} which also lead to the result shown in the [Fig-8]. When the value of I_{base} is changing the neuron will start spiking from the beginning and the increase will be much better than the result gotten when the noise was zero as in [Fig-7]. The experiments have a much better coherence compare to the result in the [Fig-7] but it still low.

In the [Fig-10], the result is gotten by fixing the renormalization terms values to $\epsilon_m^y = 0.1$, $\epsilon_u^y = 1.0$, $\epsilon_m^z = 0.001$ and $\epsilon_m^y = 0.005$ the noise variance values to 0.8 and when changing the I_{base} the number of spikes in the experiments will increase in much better and slower manner instead of the fast increasing as in the experiments done before that and the coherence will be higher in this experiments contrary to the other experiments as in [Fig-7], [Fig-8], [Fig-9].

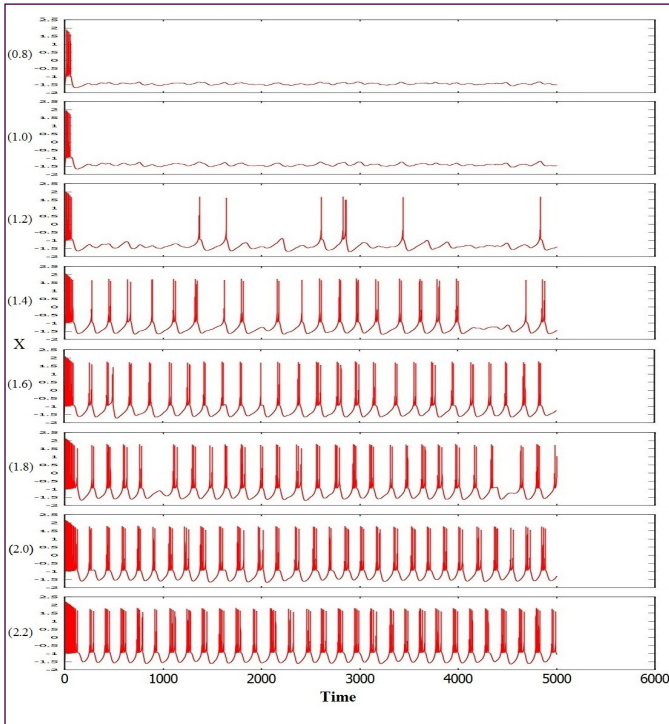


Fig. 7- Voltage time series of the membrane deterministic using DSM model for the parameter values $m=1$, $a=1$, $b=3$, $c=1$, $d=5$, $x_s = -1.6$, $r=0.004$, $h=4$, $T=0.04$ and the epsilon values and noise variances are set to be zero using various constant input current values as shown between the parentheses in the left side of the figure.

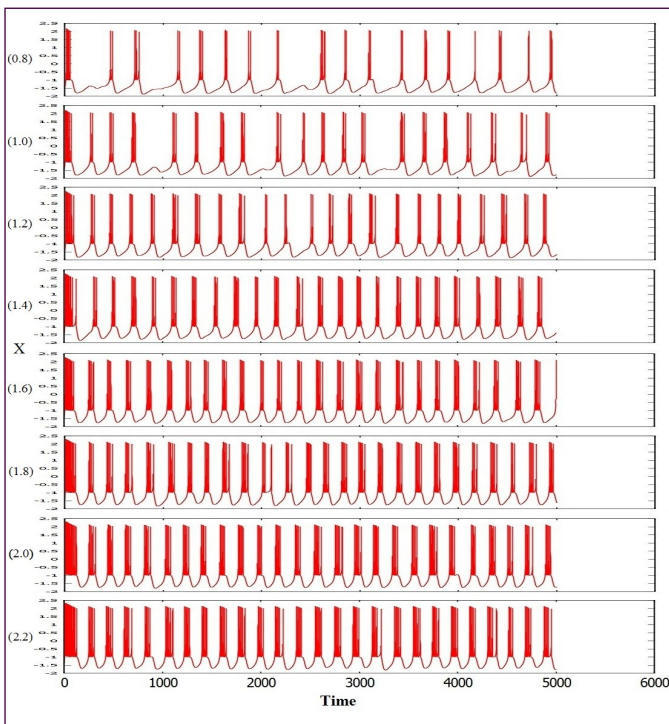


Fig. 8- Voltage time series of the membrane deterministic using DSM model for the parameter values $m=1$, $a=1$, $b=3$, $c=1$, $d=5$, $x_s = -1.6$, $r = 0.004$, $h = 4$, $T = 0.01$ and the epsilon values are $\epsilon_m^x = 0.1$, $\epsilon_u^y = 1.0$, $\epsilon_m^z = 0.001$ and $\epsilon_m^y = 0.005$ using various constant input current values as shown between the parentheses in the left side of the figure, and the noise variances is set to zero in all the experiments.

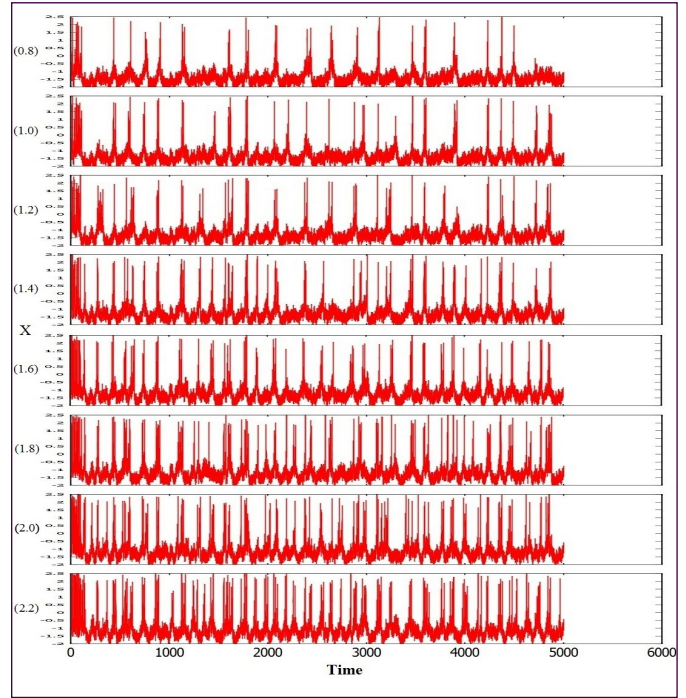


Fig. 9- Voltage time series of the membrane deterministic using DSM model for the parameter values $m=1$, $a=1$, $b=3$, $c=1$, $d=5$, $x_s = -1.6$, $r=0.004$, $h=4$, $T=0.04$ and the epsilon values are equal to zero and the noise variances is 0.8 applying various constant input current values as shown between the parentheses in the left side of the figure.

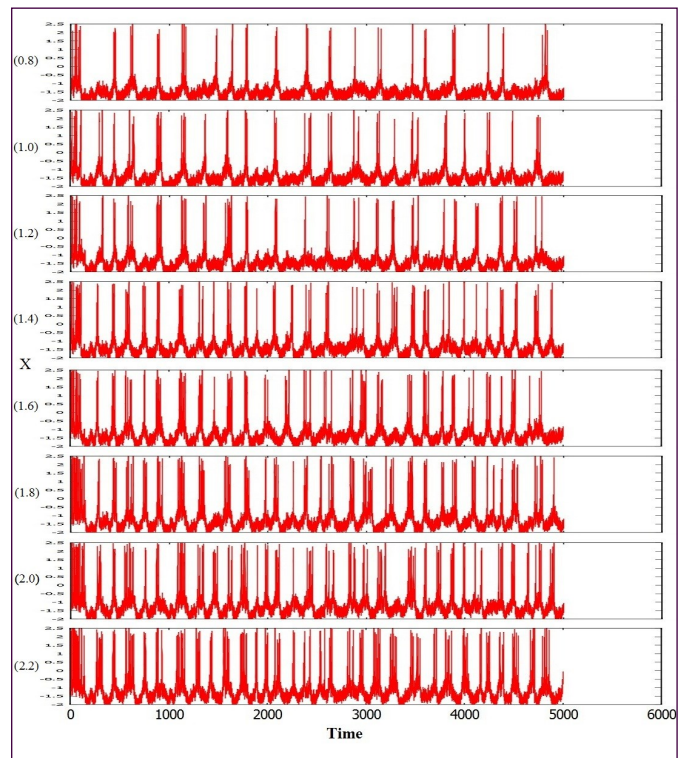


Fig. 10- Voltage time series of the membrane deterministic using DSM model for the parameter values that are the same as in figure 8 and the epsilon values are as in [Fig-7] applying various constant input current values as shown between the parentheses in the left side of the figure, and the noise variances is set to be equal to 0.8 in all the experiments.

In the [Fig-11], the comparison done between the results done earlier which are shown in the [Fig-7], [Fig-8]. Leading to the result as shown below while using the renormalization terms in the first experiments and will take the blue color in the figure and the second experiments will take the red color in the figure and the renormalization terms are set to zero and the noise variance is set zero. The difference is very large between them.

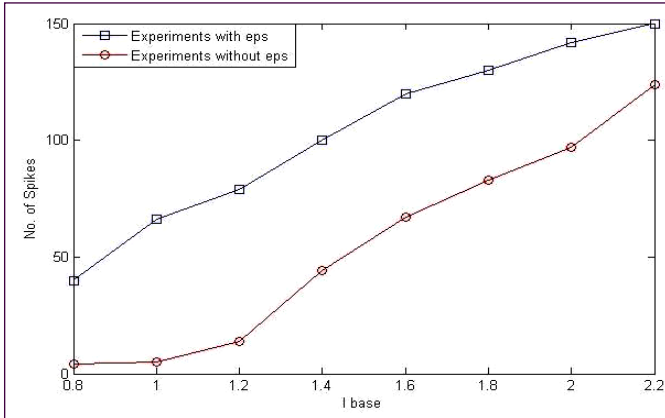


Fig. 11- Shows the difference between the two experiments. In the first experiment epsilon values are set to $\epsilon_m^y = 0.1, \epsilon_u^y = 1.0, \epsilon_m^z = 0.001$ and $\epsilon_m^x = 0.00$. In the second experiment, it is set to 0. I_{base} as shown below in the figure, and the noise variances is set to zero.

In the [Fig-12], the comparison done between the results done earlier which are shown in the [Fig-8], [Fig-10]. Leading to the result as shown below while using the renormalization terms in the first experiments and will take the blue color in the figure and the second experiments will take the red color in the figure and the renormalization terms are set to zero and the noise variance is set 0.8. The difference is very small between them compare to the other results in [Fig-7], [Fig-9] and the compare in [Fig-11]. As the noise variance increases the difference between the existence and the absence of renormalization terms will decrease but until the last experiments it didn't vanish.

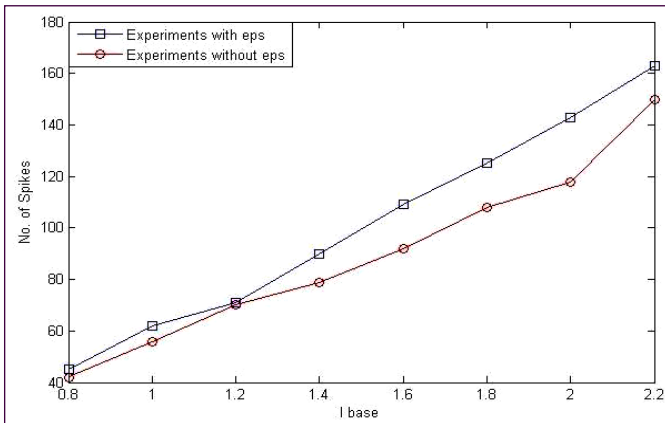


Fig. 12- Shows the difference between the two experiments. In the first experiment epsilon values are set to $\epsilon_m^y = 0.1, \epsilon_u^y = 1.0, \epsilon_m^z = 0.001$ and $\epsilon_m^x = 0.00$. In the second experiment, it is set to 0. I_{base} and the noise variances is set to (0.8).

The renormalization terms was fixed to ($\epsilon_m^y = 0.1, \epsilon_u^y = 1.0, \epsilon_m^z = 0.001$ and $\epsilon_m^x = 0.00$) the I_{base} is fixed to 0.8 and by changing the noise variance as [Fig-12], that led to the result as in the [Fig-13].

In the [Fig-14] the renormalization terms was fixed to ($\epsilon_m^y = 0.1, \epsilon_u^y = 1.0,$

$\epsilon_m^z = 0.001$ and $\epsilon_m^x = 0.005$) and the I_{base} is fixed to 1.2 and by changing the noise variance as shown in the [Fig-13]. Which led to the result as in the [Fig-14] that shows that the renormalization terms have all the effect on the neuron around 0.5 and after the value of the noise variance pass the 0.5 limit the noise variance will have almost all the effect on the neuron and the renormalization terms effect will be much smaller.

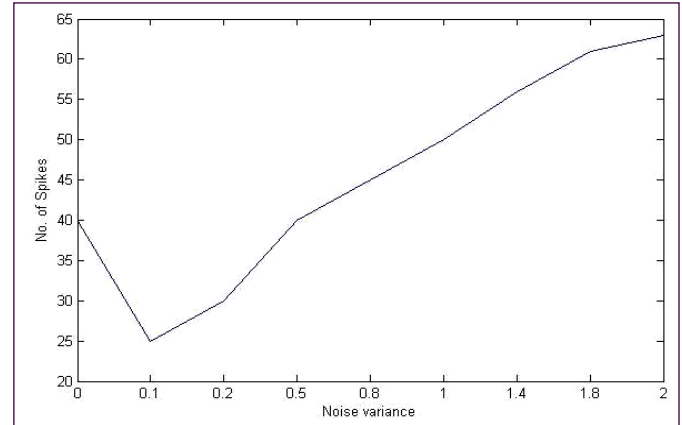


Fig. 13- Shows the change in the number of spikes when the $I_{base} = 0.8$, the epsilons value are set to their default values as in figure (8) and the noise variance is changed as in the figure.

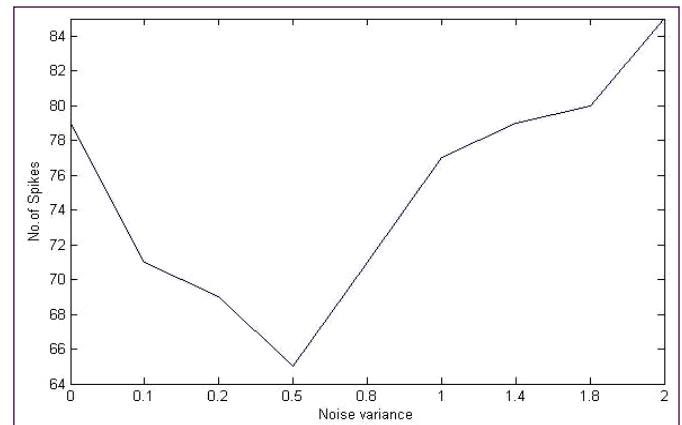


Fig. 14- Shows the change in the number of spikes when the $I_{base} = 1.2$, the epsilons value is set to the default as in [Fig-8] and the noise variance is changed as in the figure.

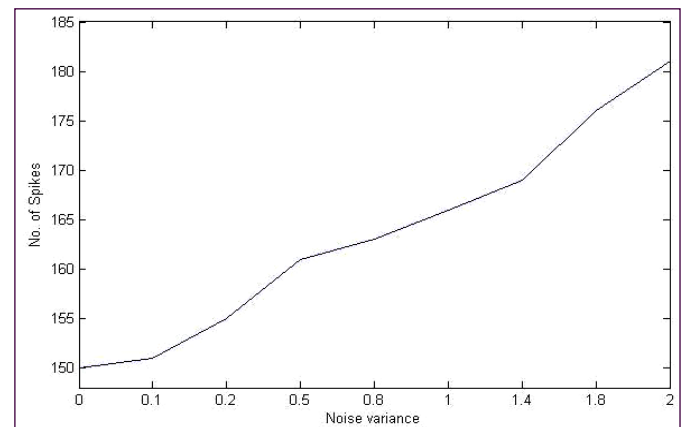


Fig. 15- Shows the change in the number of spikes when the $I_{base} = 2.2$, the epsilons value is set to the default as in [Fig-8] and the noise variance is changed as in the figure.

In [Fig-15], the renormalization terms was fixed to ($\varepsilon_m^y = 0.1$, $\varepsilon_u^y = 1.0$, $\varepsilon_m^z = 0.001$ and $\varepsilon_m^x = 0.005$) and the I_{base} is fixed to 2.2 and by changing the noise variance as shown in the [Fig-14]. Which is led to the result as in the [Fig-15] that shows that the renormalization terms have all the effect on the neuron around 0.5 and after the value of the noise variance pass the 0.5 limit the noise variance will have almost all the effect on the neuron and the renormalization terms effect will be much smaller and as the I_{base} value increase with the noise variance the effect of the renormalization terms will be reduce in much faster manner and almost vanish.

Conclusions

In this paper, the DSM neuron model was investigated from a numerical point of view when exposed to input current that is noisy and periodic in nature. The impacts of both the epsilon values and noise variances on the spiking rates and coherence were checked. Correction coefficients were used as an effective measure of renormalization corrections to the model. It should be considered that these renormalization corrections appear from the dilemma of being in doubt of how many open ion-channel numbers there are, even if we know the exact number of open gates.

DSM model neurons appear to be more complex than other models. It shows quicker synchronizing between two DSM neurons [1,9], dynamics of the models under constant input currents [1] and in addition, its ability in detecting signals under noise varying and periodic input currents, that have been inspected during this study, are all the model benefits that deserve tolerating the complexity of it. Furthermore, it should be taken into consideration that this model is extremely capable of handling the small membrane sizes of the neurons.

The experiments show that the epsilon values play an important role. The absence of the epsilon values makes the neuron generate spikes at the beginning of the experiment in slow manner and after a while the spikes generation will rise in a rapid way as shown in [Fig-8], [Fig-10].

The existence of the epsilon values when the noise variance is under 0.5 makes the neuron spiking smother from the beginning to the end of the experiment without any large differences between any two consecutive experiments and increases the neuron spiking stability and coherence as shown in [Fig-9], [Fig-10], [Fig-11]. The difference between the absence and existence of the epsilon values is also shown in a compared matter in the [Fig-11] and [Fig-12].

But the [Fig-13], [Fig-14], [Fig-15] shows that after the noise variances across the value 0.5, the effect of the epsilons values will be smaller, and the noise variances will have the most significant effects on the neuron spiking behavior and how it reacts. In addition, the noise variance almost takes all the roles played by the epsilon values from smothering the spike rates and coherence and increasing the neuron stability.

The existence of epsilon values increases the coherence in the neuron and the absence of the epsilon values reduces the coherence of the neuron. As for the noise variance using it increases the coherence of the DSM neuron model.

The results reveal that the neurons are extremely able to make a complicated and advantageous use of the channel noise in handling signals. From a technological point of view, the study shows that the DSM model has promising potential for signal detection.

Conflict of Interest: None declared.

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