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SQUEEZING AND SUB-POISSONIAN EFFECT IN EIGHTH HARMONIC GENERATION

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Abstract- The quantum effect of squeezing of electromagnetic field is investigated in fundamental mode in eighth harmonic generation under the short time approximation. Squeezing is found to be dependent on coupling constant g and phase of the field amplitude. The effect of photon number on squeezing and signal to noise ratio in field amplitude in fundamental mode has also been investigated.
Keywords- Harmonic generation, Nonlinear optics, sub-Poissonian

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Introduction

Squeezing particularly of quantized electromagnetic field has attracted considerable attention in the last few years. Quantum systems are uncertain by nature. The quantum limit can be circumvented by using squeezed states of light, where fluctuations are reduced below the symmetric quantum limit in one quadrature at the expense of increased fluctuations in the canonically conjugate quadrature, while preserving Heisenberg uncertainty principle. The first experimental demonstration of squeezed light succeeded in 1985 [1]. These states have non-classical noise statistics and their predicted generation schemes include as harmonic generation [2-3], multi-wave mixing processes [4], optical parametric oscillation [5-6], and nonlinear polarisation rotation [7].

Squeezed states of light constitute an important non-classical resource in the field of high precision quantum measurements. They have been used to improve the sensitivity of interferometers for the detection of gravitational waves [8-9]. Recent work has highlighted the potential applications of squeezed states to demonstrate several quantum information protocols, for example, quantum teleportation [10], quantum cryptography [11] and quantum computation [12]. Further, squeezed states of light have been used to prepare macroscopic quantum superposition states for quantum information networks [13-14].

The aim of the present work is to study the quantum squeezing in eighth harmonic generation in the field amplitude in fundamental mode. We have observed that eighth harmonic generation in fundamental mode is one of such distinguished example where light exhibits both squeezing and sub-Poissonian photon statistics at the same time as presented in this paper.

Definition of Squeezing and Higher Order Squeezing

Squeezing is defined in various ways. Hong and Mandel [15-16] and Hillery [17] have introduced the notion of higher order squeezing of quantized electromagnetic field as generalization of normal squeezing. Normal squeezing is defined in terms of the operators

$$X_1 = \frac{1}{2}\left(A + A^\dagger\right) \quad \text{and} \quad X_2 = \frac{1}{2i}\left(A - A^\dagger\right)$$

Where X_1 and X_2 are the real and imaginary parts of the field amplitude, respectively. A and $A^\dagger$ are slowly varying operators defined by

$$A = ae^{i\omega t} \quad \text{and} \quad A^{\dagger} = a^{\dagger}e^{-i\omega t}$$

The operators X_1 and X_2 obey the commutation relation

$$[X_1, X_2] = \frac{i}{2}$$

which leads to uncertainty relation $(\hbar = 1)$

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

A quantum state is squeezed in X_i variable if

$$\Delta X_i < \frac{1}{2} \quad \text{for } i = 1 \text{ or } 2$$

Squeezing of Fundamental Mode in Eighth Harmonic Generation

Eighth harmonic generation model is shown in "Fig. (**1**)". In this model, the interaction is looked upon as a process which involves the absorption of eight photons, each having a frequency ω_1 going from state $|1\rangle$ to state $|2\rangle$ and emission of one photon of frequency ω_2 where $\omega_2 = 8\omega_1$.

The Hamiltonian for this process is given as follows ($\hbar = 1$)

$$H = \omega_1 a^{\dagger}a + \omega_2 b^{\dagger}b + g\left(a^8 b^{\dagger} + a^{\dagger 8} b\right) \quad (1)(1)$$

in which g is a coupling constant for eighth harmonic generation. $A = a \exp(i\omega_1 t)$ and $B = b \exp(i\omega_2 t)$ are the slowly varying operators at frequencies ω_1 and ω_2, $a(a^{\dagger})$ and $b(b^{\dagger})$ are the usual annihilation (creation) operators, respectively.

The Heisenberg equation of motion for fundamental mode A is given as ($\hbar = 1$)

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + i\left[H, A\right] \quad (2)$$

Using "Eq. (1)" in "Eq. (2)", we obtain

$$\dot{A} = -8igA^{\dagger 7}B \quad (3)$$

Similarly, equation of motion for harmonic mode B is

$$\dot{B} = -igA^8 \quad (4)$$

By assuming the short time interaction of waves with the medium and expanding *A(t)* by using Taylors series expansion and retaining the terms up to g^2t^2 as

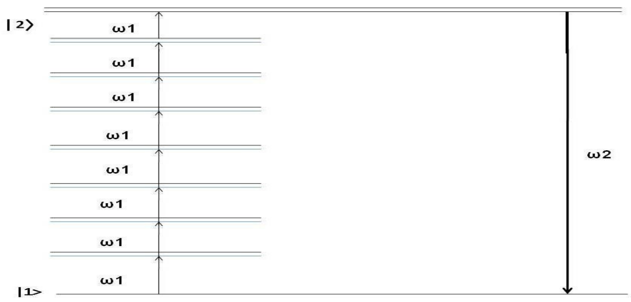

Fig.1- Eighth harmonic generation model.

$$A(t) = A - 8igtA^{\dagger 7}B + 4g^2t^2\Big[56\left(A^{\dagger 6}A^7 + 21A^{\dagger 5}A^6 + 210A^{\dagger 4}A^5 + 1050A^{\dagger 3}A^4 + 2520A^{\dagger 2}A^3 + 2520A^{\dagger}A^2 + 720A\right)B^{\dagger}B - A^{\dagger 7}A^8\Big]$$

(5)

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode *A* and the vacuum state for the harmonic mode *B* i.e.

$$|\psi\rangle = |\alpha\rangle|0\rangle \quad (6)$$

For field amplitude squeezing, the real quadrature component for the fundamental mode is given as

$$X_{1A}(t) = \frac{1}{2}\left[A(t) + A(t)\right] \quad (7)$$

Using "Eqs.(5)", "(6)" and "(7)", we get the expectation values as

$$\left\langle \psi \left| X_{1A}^2(t) \right| \psi \right\rangle = \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 + 1 - 4g^2t^2(2\alpha^2|\alpha|^{14} + 7\alpha^2|\alpha|^{12} + 2\alpha^{*2}|\alpha|^{14} + 7\alpha^{*2}|\alpha|^{12} + 168\alpha^{*4}|\alpha|^{10} + 210\alpha^{*4}|\alpha|^{8} + 8|\alpha|^{18} + 36|\alpha|^{16})\Big] \quad (8)$$

and

$$\left\langle \psi \left| X_{1A}(t) \right| \psi \right\rangle^2 = \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 - 4g^2t^2(2\alpha^2|\alpha|^{14} + 2\alpha^{*2}|\alpha|^{12} + 4|\alpha|^{16})\Big] \quad (9)$$

Therefore,

$$\left[\Delta X_{1A}(t)\right]^2 = \frac{1}{4}\Big[1 - 4g^2t^2(7\alpha^2|\alpha|^{12} + 7\alpha^{*2}|\alpha|^{12}\Big]$$

and

$$\left[\Delta X_{1A}(t)\right]^2 - \frac{1}{4} = -14g^2t^2|\alpha|^{14}\cos 4\theta \quad (10)$$

The right hand side of "Eq. (10)" is negative and thus shows the existence of squeezing in field amplitude in fundamental mode for which $\cos 4\theta > 0$

Using "Eqs. (5)" and "(6)", number of photons in mode A may be expressed as

$$N_{1A}(t) = A^{+}(t)A(t) = A^{+}A - 8g^2t^2A^{+8}A^8 \quad (11)$$

The Photon statistics in fundamental mode is given by

$$\left[\Delta N_{1A}(t)\right]^2 - \left\langle N_{1A}(t)\right\rangle = -56g^2t^2|\alpha|^{16} \quad (12)$$

Equation "(12)," shows the sub-Poissonian behavior in field amplitude of the electromagnetic wave.

Signal-to-Noise Ratio

Signal to noise ratio is defined as ratio of the magnitude of the signal to the magnitude of the noise. With the approximations $\theta = 0$ and $|gt|^2 \approx 10^{-4}$, the maximum signal to noise ratio (in decibels) in field amplitude, is given below.

$$SNR_1 = 20^* \log_{10} 0.6|\alpha|^2 \qquad (13)$$

Results

The results show the presence of squeezing in fundamental mode in eighth harmonic generation. We denote right hand side of "Eq. (10)" by S_X which shows the presence of squeezing in fundamental mode, in field amplitude. Taking $|gt|^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing, the variations of S_X is shown in "Fig. (2)". Degree of squeezing is shown as a function of $|\alpha|^2$.

It is clear from "Fig. (2)" that the squeezing increases non-linearly with $|\alpha|^2$. This confirms that the squeezed states are associated with the photon number in fundamental mode. The variation of SNR in field amplitude for a squeezed state with photon number has also been shown in "Fig. (3)".

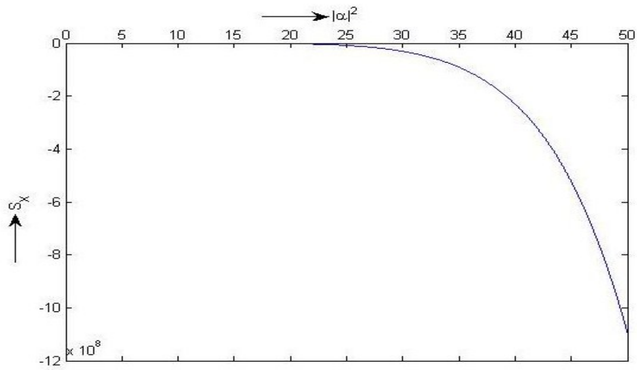

Fig. 2- Dependence of field amplitude squeezing on

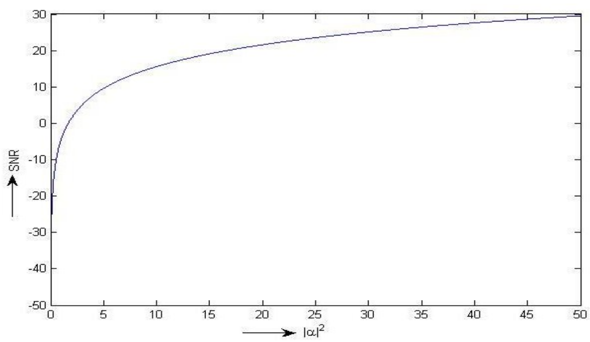

Fig.3- Signal to noise ratio for field amplitude squeezing.

Conclusion

Results show the occurrence of squeezing in field amplitude in fundamental mode in eighth harmonic generation. The steady increase in curve in "Fig. (3)" shows the reduction in quantum noise with increase in the number of photons . Thus noise reduces and the degree of squeezing increases with increasing photon numbers in the fundamental mode. Further, the results of this paper show the simultaneous squeezing and sub-Poissonian photon statistics at the same time which can be used in reducing noise in the output of certain non-linear devices.

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