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AN HARMONIC POTENTIAL WELL BASED PARTICLE SWARM OPTIMIZATION

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Abstract- Particle swarm optimization (PSO) is preponderantly used to find solution for continuous optimization problems, has the advantage of being cheaper and quicker. This paper introduces a new particle swarm optimization algorithm to solve the complex optimization problems. The availability of the introduced algorithms is validated statistically on several benchmark problems and also compared with the existing versions of PSO.
Keywords -PSO, Quantum PSO, Potential Well

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Introduction

Particle Swarm Optimization (PSO) was proposed by James Kennedy and Russel Eberhart in 1995. From the very first introduction of this algorithm the researchers of stochastic search techniques found it very fascinating and promising. PSO is a population based randomized search technique inspired by birds flocking and fish schooling. In PSO system we generate particles to search the point of interest and these particles change their positions according to few influences to reach towards global optima. This process is done by creating a communication and cooperation system among these particles.

Till date a variety of PSO variants are developed to solve various kinds of optimization problems. One of the basic approaches towards the PSO variants was to use quantum mechanics and its potential wells. The present paper demonstrates a new kind of PSO which is inspired by quantum mechanics. The next section will give the concise mathematical description of PSO and then its newly proposed variant.

Particle Swarm Optimization

For a D-dimensional search space, the i^{th} particle of the swarm at time step t is represented by a *D*- dimensional vector, $x_i^t = \left(x_{i1}^t, x_{i2}^t, \ldots, x_{iD}^t\right)^T$. The velocity of this particle at time step t is represented by another D-dimensional vector $v_i^t = \left(v_{i1}^t, v_{i2}^t, \ldots, v_{iD}^t\right)^T$. The previously best visited position of the i^{th} particle at time step t is denoted as $p_i^t = \left(p_{i1}^t, p_{i2}^t, \ldots, p_{iD}^t\right)^T$. '*g*' is the index of the best particle in the swarm. The velocity of the i^{th} particle is updated using the velocity update equation given by

$$v_{id}^{t+1} = v_{id}^t + c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (p_{gd}^t - x_{id}^t),$$

the position is updated using position update equation given by

$$x_{id}^{t+1} = x_{id}^t + v_{id}^t$$

where *d = 1, 2… D* represents the dimension and *i = 1, 2,…, S* represents the particle index. *S* is the size of the swarm and c_1 and c_2 are constants, called cognitive and social scaling parameters respectively or simply acceleration coefficients. r_1, r_2 are random numbers in the range [0, 1] drawn from a uniform distribution. Equations (1.3) and (1.4) define the classical version of PSO algorithm.

In the velocity update equation (1.3), the new velocity v_{id}^{t+1} can be seen as the sum of three terms:

a. The previous velocity, v_{id}^{t}, which can be thought of as a momentum term and serves as a memory of the previous direction of movement. This term prevents the particle from drastically changing direction.

b. The second term $c_1 r_1 (p_{id}^t - x_{id}^t)$ associated with local search, is proportional to the vector $(p_{id}^t - x_{id}^t)$ and points from the particle's current position back towards its personal best position. This term is referred as the cognitive component of the velocity update equation.

The third term $c_2 r_2 (p_{gd}^t - x_{id}^t)$ is called social component and is associated with a global search. This term is proportional to $(p_{gd}^t - x_{id}^t)$ and points from the particle's current position towards the neighborhood best position.

by keeping the basic philosophy of the PSO instead of controlling the particles by velocity vector, apply potential
wells of Quantum mechanics to control and update the particles positions.

An an Harmonic Potential Well

The mathematical definition of the Harmonic Potential well is given by

$$V(x) = -\lambda (x-p)^4$$

The classical Schrodinger wave equation is given by

$$\nabla^2 \Psi + \frac{2m}{\hbar}(E + V(x))\Psi = 0$$

We will solve the above differential equation for one dimensional system. The solution of this with boundary conditions over a square of length 'a' will be of the form

$$y = p \pm \frac{1}{d}\cosh^{-1}(\sqrt{ue}\ )$$

Where d and R are the constants. Based on this solution we will propose a new algorithm in which particles working as a quantum behaved particles. The outline of the algorithm follows:

Initiallize population : random x_i,
Do
For $i = 1$ to population size of M
If $f(x_i) < f(p_i)$ then $p_i = x_i, P_{ig} = \min(p_i)$
For $d = 1$ to dimension D,
$\phi_1 = rand(0,1), \phi_2 = rand(0,1)$
$P = (\phi_1 * p_{id} + \phi_2 * p_{gd}) / (\phi_1 + \phi_2)$
$u = rand(0,1);$
$d = \frac{1}{R}(\ln(\frac{0.25}{2\beta(|x_k - p|)}))$
if $rand(0,1) > 0.5,$
$x_{id} = p - \cosh^{-1}(\sqrt{ue^{dR}})$
else
$x_{id} = p - \cosh^{-1}(\sqrt{ue^{dR}})$

The major difference between basic PSO and this quantified PSO is that: in basic PSO the particles are controlled by the velocity update equation while in the proposed algorithm the particles are controlled by the potential well and their position will be updated by the term which is evaluated by solving the Schrodinger wave equation in a square with an Harmonic potential well.
This developed algorithm has been tested on 6 benchmark problems listed in Table 1.

Table 1- Bench Mark Functions

Function [Range]	Optimum
$f_1(x) = \sum_{i=0}^{n}(x_i^2 - 10\cos(2\pi x_i) + 10)$ [2.56,5.12]	0
$f_2(x) = \frac{1}{4000}\sum_{i=0}^{n-1} x_i^2 - \prod \cos(\frac{x_i}{\sqrt{i+1}}) + 1$ [300,600]	0
$f_3(x) = \sum_{i=0}^{n-1} 100(x_{i+1} - x_i^2)^2 + x_{i-1}^2$ [15,30]	0
$f_4(x) = -\sum_{i=1}^{n} x_i \sin(\sqrt{\lvert x_i \rvert})$ [-500,500]	-418.9829*n
$f_5(x) = (\sum_{i=0}^{n-1}(i+1)x_i^4) + rand[0,1]$ [-1.28,1.28]	0
$f_6(x) 20 + e - 20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)$ [-32,32]	0

Experiments and Results

The efficiency of the developed PSO is tested against benchmark functions given in Table 1. The functions of Table 1 are considered here for 30 variables. To avoid attributing the optimization results to the choice particular initial population and to conduct fair comparisons, each test is performed 100 times. Starting from various randomly selected points in the hyper rectangular search domain. The four PSOs (SPSO[2], LXPSO[1], CPSO[3], and QPSO[4]) are implemented in MATLAB R2008a. A run during which the algorithm finds a solution satisfying $f_{min} - f_{opt} \leq 0.001$, when f_{min} the best solution found when an algorithm terminates and f_{opt} is known as a global minimum of the problem, is called a successful run. For each method and problem following are recorded:

a. Success Rate (SR) $= \frac{(\#\ of\ success\ of\ runs)}{total\ runs}$ x 100

b. Average number of function evaluations of successful runs (AFE).

c. Average Error (AE) = Average error of $f_{min} - f_{opt}$ over 100 runs.

d. Standard Deviation (SD) = Standard deviations of error

$f_{min} - f_{opt}$.

Success performance (SP) = $\frac{AFE}{\# \, of \, successful \, runs}$ x $(\# \, of \, total \, runs)$ (Liang et al., 2006)

Based on this evaluation and performance criteria we have compared the results by the designed PSO.

Result

Table 2- Success Rate

	SPSO	LXPSO	CPSO	QPSO	Quantum PSO
f_1	95	99	79	99	99
f_2	92	100	100	100	100
f_3	68	58	45	56	80
f_4	94	80	85	86	100
f_5	93	99	100	100	100
f_6	98	100	100	100	100

Table 3- Success Performance

	SPSO	LXPSO	CPSO	QPSO	Quantum PSO
f_1	277	283	193	212	175
f_2	406	429	400	402	511
f_3	19354	19210	21678	17979	18921
f_4	900	819	902	710	590
f_5	617	587	507	605	552
f_6	932	992	925	1095	812

Conclusions

The algorithm has been tested on six benchmark functions and also compared with the four existing algorithms LXPSO, CPSO, QPSO and SPSO. In order to study the performance of proposed PSO we have obtained the results which are shown in Table 2 and Table 3. All the results are generated by programs of concern PSO coded on MATLAB. For the validation of comparison for each run we have generated the same pattern of random numbers as an initial population. The parameters used in the evaluation process are taken from the concern paper of each version of PSO.

Conclusion

An an Harmonic potential well based particle swarm optimization method has been proposed. Velocity update equation is defined in a new way. Based on proposed algorithm, different PSO variants, such as global version of the PSO as well as the local versions of the PSO can be extended to this version. Since it is proposed to solve complex multimodal problems and from the statistical validity of the algorithm, it can be use to solve real life high complex problems. The most important the parameter selection for this has been done in a smarter manner that guaranties the desired accuracy of the algorithm. We have implemented experiments to compare the propose algorithm with existing versions of PSO. Experimental results have shown that the proposed algorithm is promising.

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