Journal of Information Systems and Communication
ISSN: 0976-8742 & E-ISSN: 0976-8750, Volume 3, Issue 1, 2012, pp.- 68-71.
Available online at http://www.bioinfo.in/contents.php?id=45

END-TO-END PERFORMANCE OF A DPSK MULTI-INPUT-MULTI-OUTPUT RELAYING SYSTEM IN RAYLEIGH FADING CHANNELS

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Received: January 12, 2012; Accepted: February 15, 2012

Abstract- This paper investigates the impact of multiple antennas on the end-to-end performance of an amplify-and-forward (AF) fixed relay over flat Rayleigh fading channel. The expression for the outage probability is first derived. The average bit error rate for the differential phase shift keying (DPSK) modulation is also obtained based on moment generating function (MGF) method. It is found that the end-to-end performance significantly depends on the numbers of transmit and receive antennas. In particular, calculation shows that the relay system performance improves significantly with increasing the number of input and output antennas.
Key words- Amplify-and-forward, Average bit error rate, differential phase shift keying, Fixed relay, Outage probability, Rayleigh fading channel, Moment generating function, Multiple-input-multiple-output.

Citation: Kapil Gupta, Ghosh P.K. and Anup Dey (2012) End-to-end Performance of a DPSK Multi-Input-Multi-Output Relaying System in Rayleigh Fading Channels. Journal of Information Systems and Communication. ISSN: 0976-8742 & E-ISSN: 0976-8750, Volume 3, Issue 1, pp- 68-71.

Introduction

Error performance of relay systems including maximum ratio combining (MRC) has been analyzed by several researchers [1-3]. Multi-antenna system in relaying schemes has emerged as a promising technique to combat deep fade in wireless system [4-7]. Potential benefit of multi-input-multi-output (MIMO) system over a single antenna system has attracted researchers to incorporate several antennas at relay node [8]. However, their application to wireless system often encounters various practical implementation problems [9]. In [10], the performance of selection combining (SC) based multi-antenna fixed relay for both amplify-and-forward (AF) and decode-and-forward (DF) relaying is presented. Outage probability in closed form for MIMO relay is provided in [11].

In this paper, we investigate the end-to-end performance of maximum ratio combining (MRC) and maximum ratio transmission (MRT) based multi antenna fixed relay (infrastructure-based relay) system with amplify-and-forward (AF) relaying technique. New closed form expressions for the statistics of the received SNR are developed for independent flat Rayleigh fading channel. Here, we derived the expression for moment generating function (MGF), probability density function (pdf), cumulative density function (cdf) and the output SNR. These statistical results are important to study the performance metrics of the system. Outage probability (OP) and the average bit error rates (ABERs) which are indicative of error performance are also derived in closed form.

The paper is organized as follows: Firstly, the infrastructure based relaying system and channel models are introduced. We describe here the received signal over MIMO link. The probability density function (pdf), cumulative distribution function (cdf) and moment generating function (MGF) of the received SNR are also derived. The next section provides the expressions for various performance metrics of MIMO antenna relaying system such as outage probability and average bit error rate. The results and analysis of these metrics are then discussed. Lastly, conclusion is drawn

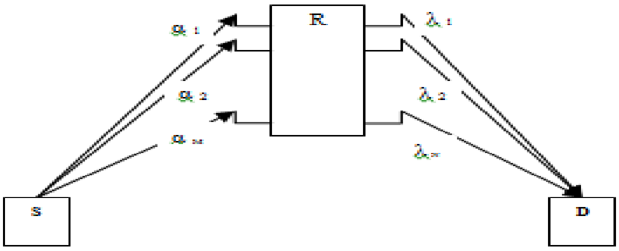

Fig. 1- A wireless communication system shows relay R equipped with M diversity antennas at the input and N diversity antennas at the output.

System and Channel Model

Fig.1 shows an infrastructure–based fixed wireless relaying system where terminal *S* is communicating with terminal *D* through the fixed relay terminal *R*. The relay *R* is equipped with *M* receiving antennas for reception of signal transmitted from source terminal S and *N* transmitting antenna to convey the signal to the receive terminal *D* after suitable amplification or decoding (as the case may be) at the relay. We assume that maximum ratio combining (MRC) is used for receiving the signal at *R* and maximum ratio transmission (MRT) is used for conveying it to the destination *D* and full channel-state information (CSI) is assumed available at the relay. In MRC, the combiner output SNR equals the sum of the SNRs in the individual branches and is given by [12].

$$\gamma_{mrc} = \sum_{i=1}^{M} \gamma_i \tag{1}$$

where the instantaneous signal-to-noise ratio (SNR) of the i[th] branch is, $\gamma_i = \frac{E_s}{N_0}(\alpha_i^2), \quad i = 1,2,\ldots,M$ with α_i being the Rayleigh fading amplitude of the channel between terminals *S* and the antennas ($i = 1,2,\ldots,M$) at relay R, E_s is the energy of the transmitted signals and N_0 is the one-sided noise power spectral density per branch. We assume the channel is frequency non- selective and slowly varying such that it is constant over the transmitted symbols interval. We assume that α_i is Rayleigh distributed, and so α_i^2 is the exponentially distributed random variables. We further assume that SNR, the input signal-to-noise ratio is the same for all diversity branches (i.e., $\overline{\gamma_1} = \overline{\gamma_2} = \cdots = \overline{\gamma_M} = \overline{\gamma_s}$), where $\overline{\gamma_i}$ is the average SNR of the i-th link from S to R. The output signal-to-noise ratio of the MRC (or, the SNR between *S* and R, $\gamma_{s,r}$) has the probability density function given by [12].

$$p_{\Gamma_{s,r}}(\gamma_{s,r}) = \frac{\gamma_{s,r}^{M-1}}{\overline{\gamma_s}^{M}\Gamma(M)} e^{\frac{-\gamma_{s,r}}{\overline{\gamma_s}}} \tag{2}$$

where, Γ(.) is the Gamma function [13]. Assume that $\lambda_j, j = 1,2,\ldots\ldots,N,$ the fading amplitude of the channel between the *j*-th antenna at R and D is of Rayleigh type, and the signal-to-noise ratio is the same for all diversity branches. In the similar fashion, the pdf of $\gamma_{r,d}$, the signal-to-noise ratio between *R* and *D* can be written as

$$p_{\Gamma_{r,d}}(\gamma_{r,d}) = \frac{\gamma_{r,d}^{N-1}}{\overline{\gamma_d}^{N}\Gamma(N)} e^{\frac{-\gamma_{r,d}}{\overline{\gamma_d}}} \tag{3}$$

Assuming appropriate gain at the relay terminal R, the overall SNR Γ_{eq} at the receiving terminal *D* can be very closely upper bounded as [4],

$$\Gamma_{eq} = \frac{\Gamma_{s,r}\Gamma_{r,d}}{\Gamma_{s,r} + \Gamma_{r,d}} \tag{4}$$

where $\Gamma_{s,r}$ is the output signal-to-noise ratio of the MRC, and $\Gamma_{r,d}$ is the instantaneous SNR between *R* and *D* with MRT.

Derivation of the MGF of received signal over MIMO Link

Taking $\Gamma_{s,r} = \frac{1}{X_1}$ and $\Gamma_{r,d} = \frac{1}{X_2}$ as two independent random variables, the moment generating function MGF of $X = X_1 + X_2$ can be written as

$$M_X(s) = M_{X_1}(s).M_{X_2}(s) \tag{5}$$

$$M_X(s) = \frac{4(\sqrt{\beta_1 s})^M K_M(2\sqrt{s\beta_1})(\sqrt{s\beta_2})^N K_N(2\sqrt{s\beta_2})}{\Gamma(M)\Gamma(N)} \tag{6}$$

where $\overline{\gamma_s} = \frac{1}{\beta_1}$, $\overline{\gamma_d} = \frac{1}{\beta_2}$ and $K_V(.)$ is the modified Bessel function of second kind of order V.

By using [14, Eq. 1.11.51], we can write the MGF $M_{\Gamma_{eq}}(s)$ as

$$M_{\Gamma_{eq}}(s) = \sum_{k=0}^{\infty} \frac{(-s)^k}{\Gamma(k+1)} \frac{\beta_1^{N-k}\beta_2^{N}\Gamma(k+M+N)\Gamma(k+M)\Gamma(k+N)}{\Gamma(M)\Gamma(N)\Gamma(2k+M+N)}$$

$${}_2F_1\left(k+M+N, k+M; 2k+M+N; 1-\frac{\beta_2^2}{\beta_1^2}\right) \tag{7}$$

Here, ${}_2F_1(.,.;.;.)$ is the Gauss' hypergeometric function [13]. Here we note that Eq. (7) is in the form of infinite sum for β_1 and β_2 . However, for the special case when $\overline{\gamma_d} = \overline{\gamma_s} = \overline{\gamma}$ or, equivalently, $\beta_1 = \beta_2 = \beta = \frac{1}{\overline{\gamma}}$, $M_{\Gamma_{eq}}(s)$ can be reduced to a very compact form as follows:

$$M_{\Gamma_{eq}}(s) = {}_3F_2\left(M,N,M+N,\tfrac{1}{2}(M+N+1),\tfrac{1}{2}(M+N); -\frac{s}{4\beta}\right) \tag{8}$$

where, ${}_3F_2(.,.,.,.;.;.)$ is the generalized hypergeometric function defined in [13].

Derivation of pdf and cdf of the received SNR

Using [15, Eq.(07.27.26.0004.01)], we express the MGF as given in (8) as

$$M_{\Gamma_{eq}}(s) = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,3}^{1,3}\left[\frac{s}{4\beta}\,\middle|\,{}^{1-M,1-N,1-M-N}_{0,1-\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}\right] \tag{9}$$

where *G[.]* is the Meijer-G function [13]. The inverse Laplace transform of $M_{\Gamma_{eq}}(s)$ gives the pdf of Γ_{eq} as follows:

$$P_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{2,3}^{3,0}\left[4\beta\,\gamma_{eq}\,\middle|\,{}^{1-M,1-N,1-M-N}_{0,1-\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}\right] \tag{10}$$

The cumulative distribution function (cdf) $F_{\Gamma_{eq}}(\gamma_{eq})$ of Γ_{eq}, is given by $F_{\Gamma_{eq}}(\gamma_{eq}) = \int_0^{\gamma_{eq}} p_\Gamma(\gamma)\,d\gamma.$ The cdf can be written as

$$F_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,4}^{3,1}\left[4\beta\gamma_{eq}\,\middle|\,{}^{1,\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}_{M,N,M+N,0}\right] \tag{11}$$

As a check, if one puts $M = N$ in Eq. (11) and uses the property of Meijer-G-function as well as gamma function, the equation reduces to the same form of equation (36) of [16].

Outage Probability

The outage probability of an *amplify-and-forward* relaying system is defined as the probability that the instantaneous SNR γ falls bellow some prescribed threshold (γ_{th}). Mathematically stated, given by

$$P_{op} = \Pr[0 \le \gamma \le \gamma_{th}] = F_\Gamma(\gamma_{th}) \tag{12}$$

Using (11) and (12), the outage probability for amplify-and - forward systems can be shown to be given by

$$P_{op} = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,4}^{3,1}\left(4\beta\,\gamma_{th}\,\middle|\,{}^{1,\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}_{M,N,M+N,0}\right) \tag{13}$$

Please note that the lower bound of the outage probability for the fixed relay MIMO-antenna system can be achieved for fixed *M* by letting $N \to \infty.$ If *N* goes to infinity, the SNR of the $R \to D$ link can be much larger compared to that of the $S \to R$ link and the overall received SNR is dictated only by the $S \to R$ link. The lower bound of outage probability for *amplify-and-forward* relaying systems can be written as

$$P_{op,lb} = 1 - \left(\frac{\Gamma(M,\beta\gamma_{th})}{\Gamma(M)}\right) \tag{14}$$

Average Bit Error Rate (ABER)

The average bit error rate (ABER) for different digital modulation schemes on MIMO relay link can be derived by adopting the MGF-based approach [17]. For *amplify-and-forward* relaying, the ABER $P_b(E)$ for DPSK is given by

$$P_b(E) = \frac{1}{2} M_{\Gamma_{eq}}(E) \tag{15}$$

By equation (9) we get $P_b(E)$ as

$$P_b(E) = \frac{1}{2} M_{\Gamma_{eq}}(E)$$

$$= \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,3}^{1,3}\left[\frac{s}{4\beta}\,\middle|\,{}^{1-M,1-N,1-M-N}_{0,1-\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}\right] \tag{16}$$

The lower bound of ABER of DPSK for *amplify-and-forward* relaying system can be obtained for fixed M by letting N→ ∞ and is given by

$$P_{b,lb}(E) = \frac{1}{2}\left(\frac{\beta}{1+\beta}\right)^M \tag{17}$$

Result and Analysis

Fig. (2) shows the variation of ABER with SNR in dB of the MIMO antenna relaying system for different antennas *(M,N)* using amplify-and-forward (AF) protocol. In this graph, we have considered fixed number *N* of the transmit antenna at the relay output. The MIMO antenna system offers significant gains over the reference single-input-single-output antenna system *(M=1, N=1)* at the relay. As a comparison, for the ABER of 10^{-1}, the SNR requirement for the conventional single antenna relaying is 10 dB *(M=1, N=1)* whereas, the same for *(M=2, N=1)* , *(M=3, N=1)*, *(M=4, N=1)* , *(M=5, N=1)* are obtained as 7.6 dB , 7.0 dB, 6.5 dB and 6 dB a gain of 2.4 dB, 3.0 dB, 3.5dB and 4 dB respectively. Thus the gain improvement is significant in MIMO-antenna relay. However, the gain of the system increases only marginally for values of (M>3,N=1) over (M=3,N=1). If M is increased still further, the gain values attain saturation, the lower bound of performance value. The simulation results for N=1, M=1 are indicated in Fig (2) as asterisk marks. The results are very close to that computed and are not resolvable to our chosen scales.

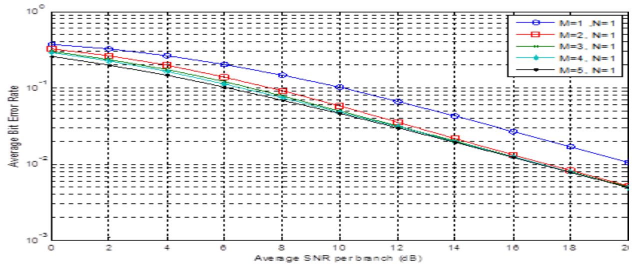

Fig. 2- Plot of ABER versus SNR (dB) for amplify-and-forward relaying with one transmitting antenna N = 1 and varying receiving antennas M = 1, 2, 3, 4 & 5.

We have also studied the performance of *amplify-and-forward relay* for the same values of *M* and *N* as shown in Fig. (3). Numerical results show that gain advantage is attained for (M=2,N=2) and (M=3, N=3) system over (M=1, N=1) system. Notably, at a given SNR, the bit error rate is lower as the number of receive and transmit antennas at the relay terminals increases. This is highly expected because of increased diversity for higher *M* and *N* values. The gaps in error performance in the high SNR regimes also increase for large values of *M* and N. The labels of the curves are the same as in Fig. (2). Fig. (4) shows the plot of ABER versus SNR for fixed number *M* of the receive antenna at the relay input and with varying number N of transmit antennas. Significant gain advantage is observed for (M=1, N=2) over (M=1, N=1) system.

As *N* increases still further, the gain advantage increases but marginally. This observation can be explained as in Fig. (2).

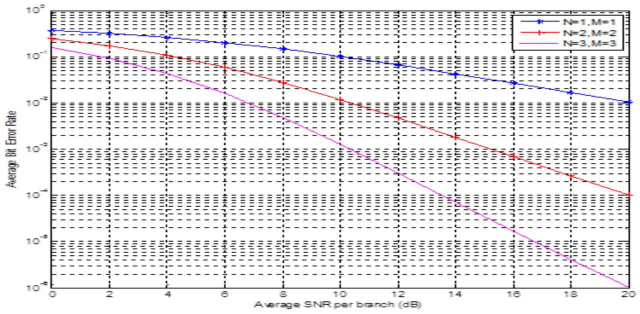

Fig. 3- Plot of ABER versus SNR (dB) for amplify-and-forward relaying with same number of receiving and transmitting antennas M = N = 1, 2 & 3.

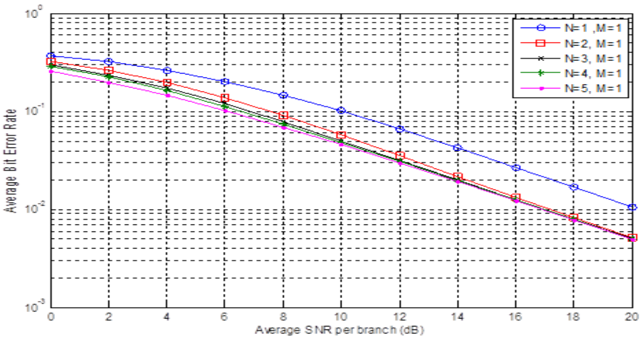

Fig. 4- Plot of ABER versus SNR (dB) for amplify-and-forward relaying with one receiving antenna M = 1 and varying transmitting antennas N = 1, 2, 3,4 & 5.

Our expressions for ABER shows that the interchange of and does not change the ABER values which is expected because of symmetrical structure.

Conclusion

In this paper, we derived the closed form expressions for the pdf, cdf, and MGF of the received SNR to study the end-to-end performance of maximum ratio combining (MRC) and maximum ratio transmission (MRT) based multi antenna fixed relay (infrastructure-based relay) system with amplify-and-forward (AF) relaying technique. We applied these expressions to study the average bit error rates for various combinations of number of receive and transmit antennas at the relay terminals. Significant gain advantage is observed for and combinations over the single combination. Best gain performance is achieved from multiple antennas relaying system when the number of transmit or receive antennas at the relay is 2.

Acknowledgement

The first two authors would like to thank Faculty of Engineering and Technology of Mody Institute of Technology and Science, Lakshmangarh for providing necessary facilities to carry out this research work.

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