Journal of Electronic and Electrical Engineering
Journal of Electronic and Electrical Engineering
ISSN: 0976–8106 & E-ISSN: 0976–8114, Vol. 2, Issue 2, 2011, pp-47-53
Online available at : http://www.bioinfo.in/contents.php?id=66

ACTIVE VIBRATION CONTROL OF A CANTILEVERED SMART STRUCTURE BY USING SPATIAL H∞ CONTROL TECHNIQUE

WAGHULDE K.B.[1] AND BIMLESH KUMAR[2]
[1]J.T.M. College of Engineering Faizpur, Jalgaon, MS, India - 425524
[2]J.T.M. College of Engineering Faizpur, Jalgaon, MS, India - 425524
*Corresponding Author: Email- [1]kishorwaghulde@gmail.com, [2]bimlesh2000@rediff.com

Received: November 16, 2011; Accepted: December 05, 2011

Abstract- This study presents the vibration analysis of PZT structure by using Active Vibration Control. The smart beam is taken as cantilevered beam which is made up by the aluminum and configuration with surface bonded piezoelectric (PZT) patches. Piezoelectric patches are used as actuators and sensor for vibration analysis. Initially the smart beam was analytically modelled by using the analytical method. The model only included the first two flexural vibrational modes and the model correction technique was applied to compensate the possible error due to the higher order modes. The system model was also experimentally identified and both theoretical and experimental models were used together in order to determine the modal damping ratios of the smart beam. A starting of the research paper, the study presents the design and implementation of a spatial H_∞ controller for the active vibration control of a cantilevered smart beam. The controller is applied to a Cantilevered piezoelectric-laminated beam and is validated experimentally to show the effectiveness of the proposed controller in suppressing structural vibration. The vibration is controlled by the application of H_∞ controllers. At last the paper presents the effectiveness of the technique in the modeling.
Key words- Vibration Analysis and Control, Cantilevered Smart Beam, Spatial H_∞ Controller

Introduction

Many engineering applications use structures that can be considered to be flexible. Flexible structures are distributed parameter systems. Therefore, vibration of each point is dynamically related to the vibrations of every other point over the structure. It is important to design a controller with a view to minimizing structural vibrations of the entire structure, rather than a limited number of points. This would ensure that structural vibrations of the entire structure are suppressed.

The developments in piezoelectric materials have very important role for many researchers to work in the field of smart structures. A smart structure can be defined as "A system or material which has built-in or intrinsic sensor(s), actuator(s) and control mechanism(s) whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short/ appropriate time, and reverting to its original state as soon as the stimulus is removed". Smart structures consist of highly distributed active devices which are primarily sensors and actuators either embedded or attached to an existing passive structure with integrated processor networks. Therefore our work mainly considers the application of PZT patches to smart beam-like and smart plate-like structures for the purpose of active vibration control [2].

The motivation for this work stems from the possibility of using induced strain actuation for vibration suppression, stability augmentation, and noise reduction in beam-like aerodynamic surfaces. These beams are used in such applications as helicopter and airplane wings, turbo-machine blades, missiles, space structures and civil structures. Several theories apply to the control of vibration. The best way to optimize a single mode, as being proposed in this research, is to optimize the performance metric corresponding to the mode of interest. This methodology is ideal for the design of low-order controllers. A smart structure involves distributed actuators and sensors along the structure and some type of processor that can analyze the response from the sensor and use control theory to output commands to the actuator. The actuator applies local stresses/strains to alter the behavior of the system. Therefore, a smart structure has four major components: the structure, sensor, actuator, and controller. Actuators and sensors are widely used in various applications and are generally integrated with main structures via surface bonding or embedding. When building the beam, it is taken into consideration that piezoelectric materials must be bonded to the beam in a uniform fashion along with the fact that both materials must have electrical contact on each side of the material. These facts bring about the controversy between surface bonding and embedding. Surface bonding for piezoelectric actuators is advantageous in that there is better access for fabrication, easier access for inspection, and less maintenance cost. However, since these materials are exposed, they are more vulnerable and more prone to be damaged. In this experiment, it is necessary for the piezoelectric components to be on the surface because it was the only way it could be easily manufactured [4, 5].

Active vibration control of a smart structure requires an accurate system model of the structure. Smart structures can be modeled by using analytical methods or system identification techniques using the experimental data. The system model of a smart structure generally involves a large number of vibrational modes. However, the performance goals are mostly related to the first few vibrational modes since their effect on structural failure is much more prominent. Hence, a reduction of the order of the model is required. On the other hand, ignoring the higher modes can affect the system behavior since directly removing the higher modes from the system model perturbs the zeros of the system. Therefore, in order to minimize the model reduction error, a correction term, including some of the removed modes, should be added to the model. Today, robust stabilizing controllers designed in respect of H_∞ control technique are widely used on active vibration control of smart structures. Controller design technique is applied, the suppression should be preferred to be achieved over the entire structure rather than at specific points, since the flexible structures are usually those of distributed parameter systems [1,2].

The aim of this paper is to present design and implementation of a spatial H_∞ controller on active vibration control of a cantilevered smart beam.

The Smart Beam Model

The cantilevered smart beam model is given in Fig. 1. In this case, a smart system consisting of a plate fixed at one side and mounted with a PZT sensor and a PZT actuator is tested for vibration control. The length, width and thickness of the plate are 0.494m, 0.051m and 0.002 m, respectively. For the PZT sensor and actuator, the length, width and thickness are taken as 0.05m, 0.04m and 0.0005m, respectively. The density and Young's modulus for the plate material is 2710 kg/m^3, and 69 Gpa respectively. For the PZT sensor and actuator, density and Young's modulus for the plate material is 7650 kg/m^3, and 64.52 Gpa. For the PZT sensor and actuator, piezoelectric charge constant is -175 x 10^{-12} m/v.

Spatial *H∞* Control

Controller design framework for structural vibration control is based on the spatial H_∞ norm concept. We use this concept to design a spatial H_∞ controller for vibration control of smart structures. In particular, to demonstrate the effectiveness in minimizing structural vibration of PZT laminated beam, we implement the controller to control the vibration. The controller is designed to minimize the spatial H_∞ norm of the closed-loop system. Minimizing the spatial H_∞ norm of the system will ensure vibration suppression over the entire structure in a spatially-averaged sense.

Consider the transfer function of a flexible structure, G(s,x) as in equation (1).

$$G(s,x) = \sum_{k=1}^{\infty} \frac{W_k(x)\varphi_k^T}{s^2 + 2\xi_{k\omega ks} + \omega_k^2} \quad (1)$$

Where $\varphi_k = [\varphi_{k1} \dots\dots\dots \varphi_{kM}]^T$ and the mode number is denoted by k. φ_{ki} is a function of the location of the ith piezoelectric actuator, the eigen function $W_k(x)$ and the properties of the structure and the piezoceramic patch. The damping ratio is denoted by ξ_k.

Furthermore, for the flexible structure with piezoelectric actuator–sensor (collocated) pairs, the multiple input–multiple output transfer function can be determined in a similar manner. The transfer function from the applied actuator-voltages V_a (s) to the induced voltages at the sensor

$V_s(S) = [V_{s1}(S), \dots\dots\dots V_{sM}(S)]^T$ is

$$G_{vs}(S) = P_{vs} \sum_{k=1}^{\infty} \frac{\varphi_k\varphi_k^T}{S^2 + 2\xi_{k\omega ks} + \omega_k^2} \quad (2)$$

Where, $P_{vs} = \gamma P > 0$ is a constant based on the properties of the structure and the piezoceramic patches.

Model Correction for Spatial *H∞* Norms

In practice, dynamical models of a flexible structure as described in equation (1) and (2) can be truncated to represent the system with a finite-dimensional model. The model can be truncated so to include only the modes within the frequency bandwidth of interest. However, the neglected dynamics associated with truncation of the model produces additional error in the gain and locations of the in-bandwidth zeros. This is due to the fact that the contribution of the out-of-bandwidth modes is ignored in the truncation. As a consequence, the neglected dynamics can be detrimental to the robustness of the closed-loop system. One way to improve the truncated model dynamics is to include a feed through term to correct the gain and locations of the in-bandwidth zeros. This technique is known in the aeroelasticity literature as the mode-acceleration method. Adding a feed through term to the truncated (finite dimensional) model compensates for the neglected dynamics in the model, which is important in ensuring the closed-loop stability [1,2]. Fig.-2 shows Spatial H_∞ control of flexible cantilevered beam and Fig.-3 shows Spatial H_∞ control problem.

The infinite-dimensional model of the collocated system in equation-(2) can be approximated as

$$G_{vs}^N(S) = P_{vs} \sum_{k=1}^{N} \frac{\varphi_k\varphi_k^T}{S^2 + 2\xi_{k\omega ks} + \omega_k^2} + K_{vs} \quad (3)$$

Where N is the number of modes included in the model, and K_{vs} is a $M \times M$ matrix added to compensate for the neglected dynamics.

Similarly, we describe the approximate spatial transfer function of *G(s,x)* in equation (1) by

$$G^N(s,x) = P\sum_{k=1}^{N}\frac{W_k(x)\varphi_k^T}{s^2 + 2\xi_{k\omega ks} + \omega_k^2} + K(x) \quad (4)$$

where $K(x)$ is a 1×M vector. $K(x)$ is a function of the spatial variable, x. It has to be estimated from the modal model of the system.

The term $K(x)$ is determined such that the following cost function is minimized:

$$J = \langle\langle W_c(s,x)(G(s,x) - G^N(s,x))\rangle\rangle_\infty^2 \quad (5)$$

Here, $W_c(s,x)$ is an ideal low-pass weighting function distributed spatially over X with its cutoff frequency ω_c chosen to lie within the interval $\omega_c \in (\omega_N, \omega_{N+1})$.

The cost function in equation (5) is minimized by setting

$$K(x) = \sum_{k=N+1}^{\infty} K_k^{opt} W_k(x) \quad (6)$$

Where,

$$K_k^{opt} = \frac{1}{2}\left(\frac{1}{\omega_k^2} + \frac{1}{\omega_k^2 - \omega_c^2}\right)P\varphi_k^T \quad (7)$$

Note, that in practice we can only include a finite number of modes to calculate the feedthrough term, $K(x)$. So, $K(x)$ in equation (6) is calculated from $k = N + 1$ to N_{max} with N_{max} is chosen so that the neglected dynamics in the model can be compensated sufficiently. Naturally, the larger N_{max} , the smaller the uncertainty will be. However, choosing a large enough N_{max} is quite reasonable as its effect diminishes when $N_{max} \to \infty$ since the contribution of higher frequency modes is decreasing. Furthermore, since the calculation of the feedthrough term in (6) is straightforward, there is no restriction on how many high frequency modes can be included in the calculation.

Spatial H∞ Control of a PZT Laminated Smart Cantilevered Beam

Consider the closed loop system of the smart beam shown in Fig. 2. The aim of the controller, *K*, is to reduce the effect of disturbance signal over the entire beam by the help of the PZT actuators.

The state space representation of the system above can be shown to be

$$(\dot{x}) = Ax(t) + B_1w(t) + B_2u(t)$$
$$y(t,r) = C_1(r)x(t) + D_1(r)w(t) + D_2(r)u(t) \quad (8)$$
$$y(t,r_L) = C_2x(t) + D_3w(t) + D_4u(t)$$

where *x* is the state vector, *w* is the disturbance input, *u* is the control input, *y*(*t*,*r*) is the performance output, $y(t,r_L)$ is the measured output at location $r_L = 0.99L_b$. The performance output represents the displacement of the smart beam along its entire body, and the measured output represents the displacement of the smart beam at a specific location *A* is the state matrix, B_1 and B_2 are the input matrices from disturbance and control actuators respectively, Π is the output matrix of error signals, C_2 is the output matrix of sensor signals, Θ_1, Θ_2, D_3 and D_4 are the correction terms from disturbance actuator to error signal, control actuator to error signal, disturbance actuator to feedback sensor and control actuator to feedback sensor respectively. The disturbance *w*(*t*) is accepted to enter to the system through the actuator channels, hence, B_1=B_2 , $D_1(r) = D_2(r)$ and $D_3 = D_4$.

The state space form of the controller can be represented as:

$$\dot{x}_k(t) = A_kx_k(t) + B_ky(t,r_L)$$
$$u(t) = C_kx_k(t) + D_ky(t,r_L) \quad (9)$$

such that the closed loop system satisfies:

$$inf_{K\in U}\, sup_{w\in L_{2(0,\infty)}} J_\infty < \gamma^2 \quad (10)$$

Where U is the set of all stabilizing controllers and γ is a constant.

The spatial cost function to be minimized as the design criterion is:

$$J_\infty = \frac{\int_0^\infty \int_R y(t,r)^T Q(r) y(t,r)drdt}{\int_0^\infty w(t)^T w(t)dt} \quad (11)$$

Where Q(r) is a spatial weighting function that designates the region over which the effect of the disturbance is to be reduced and J∞ can be considered as the ratio of the spatial energy of the system output to that of the disturbance signal. The control problem is depicted in Fig. 3.

The spatial *H*∞ control problem can be solved by the equivalent ordinary *H*∞ problem by taking:

$$\int_0^\infty \int_R y(t,r)^T Q(r) y(t,r)drdt = \int_0^\infty y(t)^T y(t)dt \quad (12)$$

Hence the following the necessary mathematical manipulations, the adapted state space representation will be:

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t)$$
$$y(t) = \begin{bmatrix}\Pi\\0\end{bmatrix}x(t) + \begin{bmatrix}\Theta_1\\0\end{bmatrix}w(t) + \begin{bmatrix}\Theta_2\\k\end{bmatrix}u(t) \quad (13)$$
$$y(t,r_L) = C_2x(t) + D_3w(t) + D_4u(t)$$

The derivation of equation (13) and the state space variables can be found as:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\omega_1^2 & 0 & -2\xi_1\omega_1 & 0\\ 0 & -\omega_2^2 & 0 & -2\xi_2\omega_2 \end{bmatrix} \quad (14)$$

$$B_1 = B_2 = \begin{bmatrix}0\\0\\P_1\\P_2\end{bmatrix} \quad (15)$$

$$C_1 = \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ 0 \\ 0 \end{bmatrix}^T \quad and \quad = \begin{bmatrix} \phi_1(r_L) \\ \phi_2(r_L) \\ 0 \\ 0 \end{bmatrix}^T \qquad (16)$$

$$D_1 = D_2 = \sum_{i=3}^{50} \phi_i(r) k_i^{opt} \; and$$

$$D_3 = D_4 = \sum_{i=3}^{50} \phi_i(r_L) k_i^{opt} \qquad (17)$$

$$\Pi = \begin{bmatrix} diag\left(L_b^{3/2}\right)_{2\times 2} & 0_{2\times 2} \\ 0_{3\times 2} & 0_{3\times 2} \end{bmatrix} \qquad (18)$$

$$\Theta_1 = \Theta_1 = \begin{bmatrix} 0_{4\times 1} \\ \left(\sum_{i=3}^{50} L_b^3 \left(k_i^{opt}\right)^2\right)^{1/2} \end{bmatrix} \qquad (19)$$

One should note that, the control weight, *K*, is added to the system in order to limit the controller gain and avoid actuator saturation problem. In the absence of the control weight, the major problem of designing an *H∞* controller for the system given in equation (8) is that, such a design will result in a controller with an infinitely large gain. In order to overcome this problem, an appropriate control weight, which is determined by the designer, should be added to the system. Since the smaller *K* will result in higher vibration suppression but larger controller gain, it should be determined optimally such that not only the gain of the controller does not cause implementation difficulties but also the suppressions of the vibration levels are satisfactory [1,2]. In this study, was *K* decided to be taken as 7.87×10^{-7}. Fig. 4 shows the frequency response of the Cantilevered Beam with PZT Patches for Open and Closed Loop System. At the first two flexural resonance frequencies, the magnitude of vibration levels were found to be 27.2 dB and 23.1 dB, respectively. The results show that the designed controller is effective on the suppression of excessive vibrational levels.

Experimental Implementation

Fig. 5, shows the experimental setup for cantilevered smart beam which consists the PZT patches as the actuator and sensor. The displacement of the smart beam was measured by using the sensor and converted to a voltage output. This signal was sent to the controller unit via the connector block. The controller output was converted to the analog signal. Before applied this signal to the piezoelectric patches was amplified 30 times by high voltage power amplifier. The controller unit is hosted by a Linux machine, on which a shared disk drive is present to store the input/output data and the C programming language based executable code that is used for real-time signal processing.

Forced Vibration Control

The smart beam was analyzed in two different configurations for forced vibration control. In the first one, excitation of smart beam is taken for 160 seconds with a shaker, on which a sinusoidal chirp signal of amplitude 5V was applied. The excitation bandwidth was taken first 5 to 8 Hz and later 40 to 44 Hz to include the first two flexural resonance frequencies separately. The experimental attenuation of vibration levels were determined from the Frequency responses plots shown in Fig. 6. The resultant attenuation levels were found as 19.8 dB and 14.2 dB, respectively. In the second configuration, a constant excitation was applied for 20 seconds at the resonance frequencies again with a shaker.

Efficiency of the Controller

The efficiency of spatial controller in minimizing the overall vibration over the smart beam was compared by a pointwise controller. Point $r_L = 0.99L_b$ is used to designed and minimize the vibrationst. Due to its first two flexural modes in comparable efficiency, the implementations of the controllers showed that both controllers reduced the vibration levels of the smart beam. On the other hand, the simulated *H∞* norms of the smart beam as a function of r, shown in Fig. 7. This Figure shows that over entire beam, the spatial *H∞* controller has a slight superiority on suppressing the vibration levels.

Conclusion

This paper gives the active vibration control of a cantilevered smart beam. A spatial *H∞* controller was designed and implemented on a piezoelectric-laminated cantilever beam to control the vibration. The efficiency of the controller was demonstrated both by simulation and experimental implementations. It was observed that such a controller resulted in suppression of the transverse deflection of the entire structure by minimizing the spatial norm of the closed-loop system. The controller was obtained by solving a standard control problem for a finite-dimensional system. The effectiveness of the spatial controller on suppressing the vibrations of the smart beam over its entire body was also compared with a pointwise controller. The application of this spatial *H∞* control is not confined to a piezoelectric-laminate beam. This spatial *H∞* controller may be applied to more general vibration control and suppression problems.

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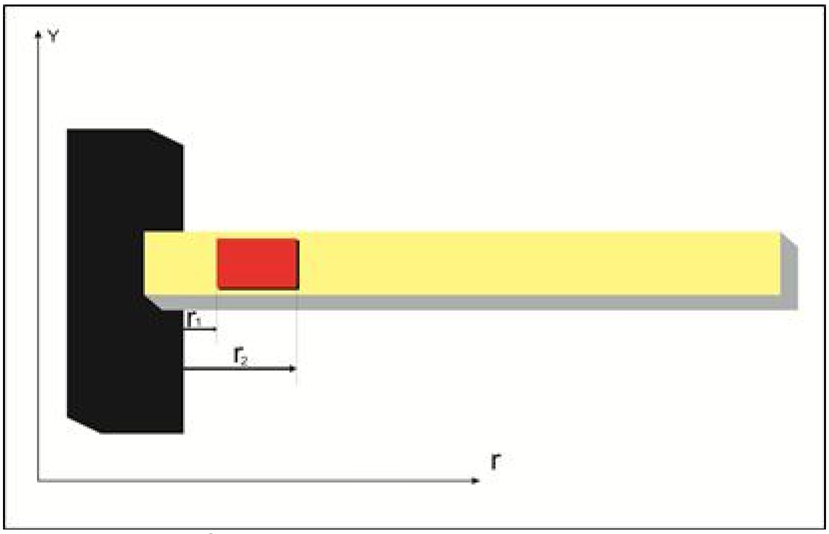

Fig. 1- Cantilevered Beam with PZT Patches.

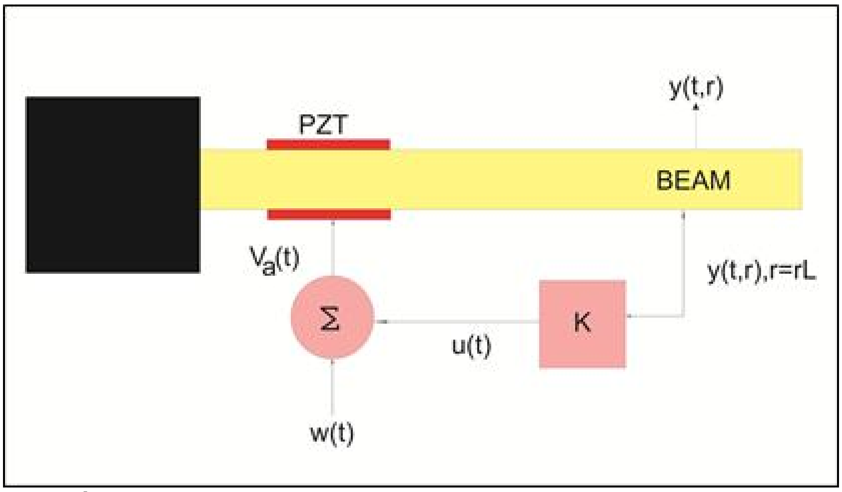

Fig. 2- Closed Loop System of the Cantilevered Beam with PZT Patches

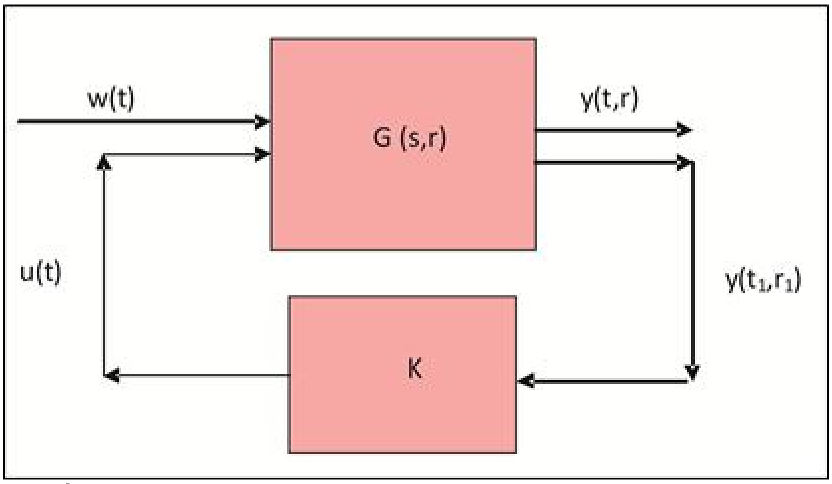

Fig. 3- Spatial H_∞ Control Problem of Cantilevered Beam with PZT Patches

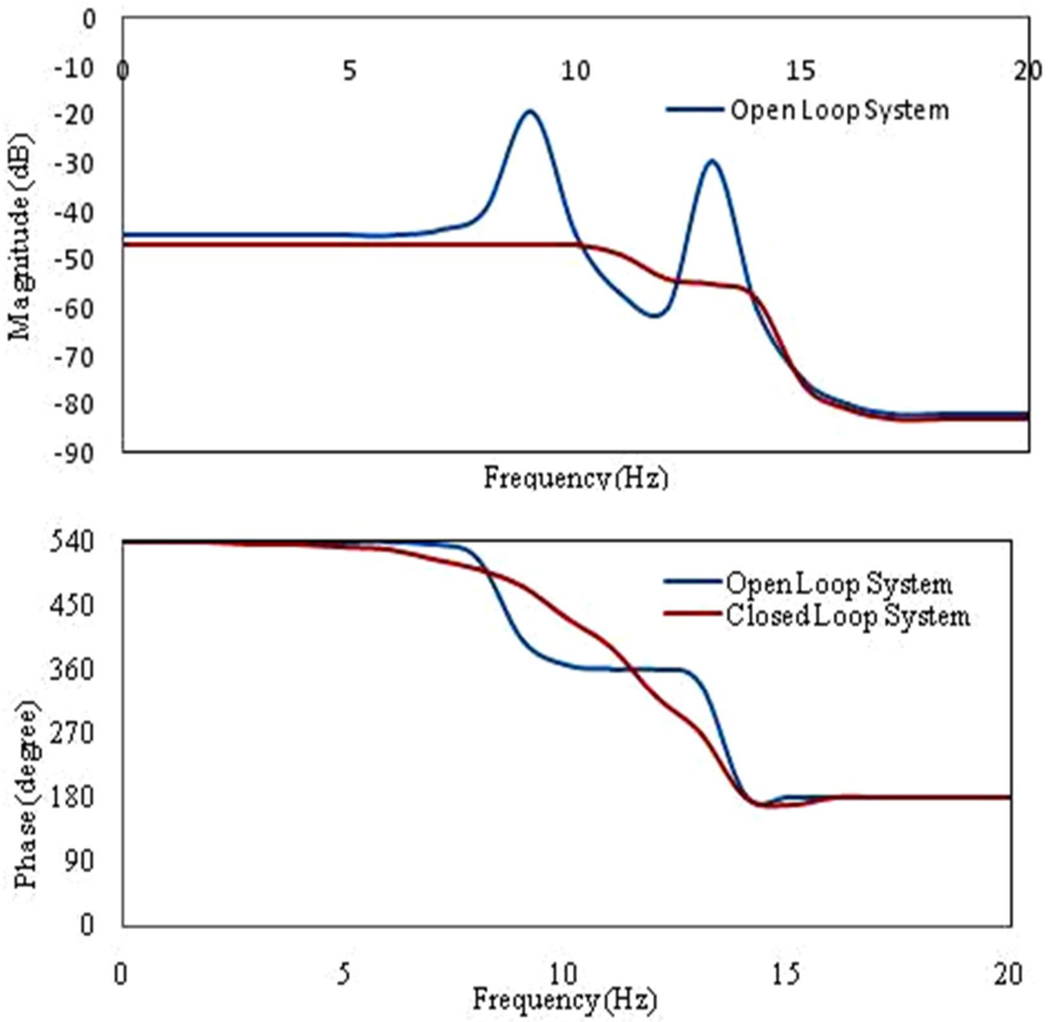

Fig. 4- Frequency Responses of the Cantilevered Beam with PZT Patches for Open and Closed Loop System

Fig. 5- Cantilevered Beam with PZT Patches and FFT Analyser

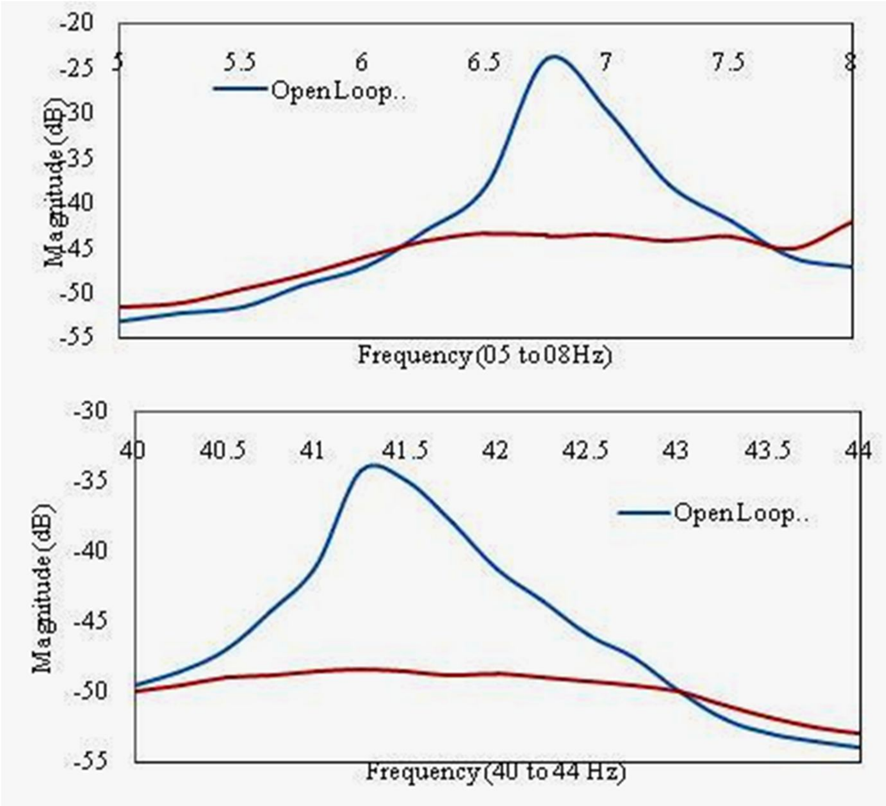

Fig. 6- Frequency Responses of the Cantilevered Beam with PZT Patches for Open and Closed Loop Systems (05 to08 Hz and 40 to 44 Hz)

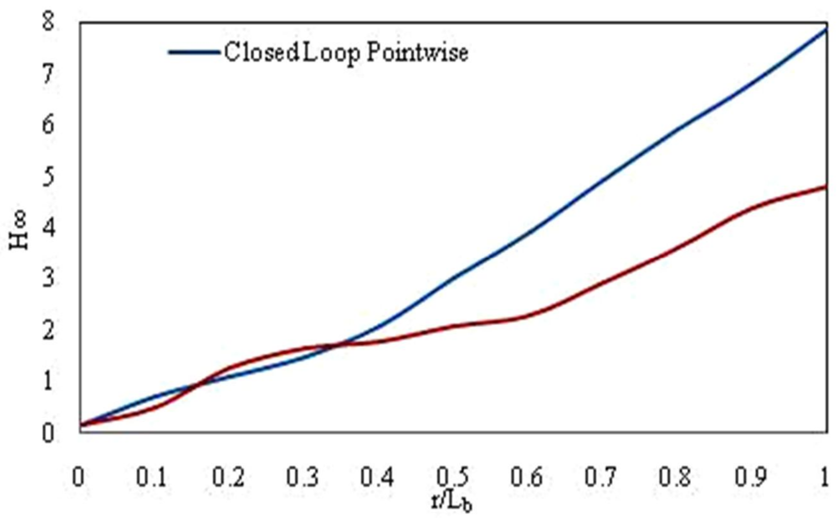

Fig. 7- Simulated H∞ Norm Plot of Closed Loop Systems under the Effect of Controllers