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A FIXED POINT THEOREM IN QUASI SEMI 2- METRIC SPACES

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Abstract -The aim of the present paper is to establish a fixed point theorem in Quasi Semi 2- Metric Spaces introducing Φ' - contraction.

Keywords: Fixed point, 2- metric spaces, Convergent Sequence, Quasi semi 2- metric space, Contraction mapping, Φ' - contraction, Compatibility, Commuting mappings.

Introduction

Since 1963,when S.Gahler[1,2] initially introduced the concepts of 2- metric spaces there was a spat of papers[4,8,15] dealing with this generalized space. G.Jungck [7] introduced the idea of compatibility. B.E. Rhoades, G,Jungck, S.L. Singh [7,14,10] worked on fixed point theorems for weakly commuting and compatible maps and established several fixed point theorems. Further , the work in Quasi semi 2- metric spaces was extended by M. Telsi, K. Tas [5] , S. Romaguera [6], T.L. Hicks [12,13], Naidu [11], Takahashi [9].

A 2- metric space is a space X in which for each triple of points x,y,z there exists a real function d(x,y,z) such that

(i) to each pair of distinct points x, y in X, there exists a point $z \in X$ such that $d(x, y, z) \neq 0$,
(ii) d(x, y, z) = 0, when at least two of x, y, z are equal,
(iii) d(x, y, z) = d(y, z, x) = d(z, x, y),
(iv) $d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(w, y, z)$, for all $w \in X$.

It is known that a 2- metric is a non –negative real- valued function . Naidu and Prasad made the following observations:

(i) A convergent sequence in a 2- metric space need not be Cauchy.

(ii) In a 2- metric space (X,d) every convergent sequence is Cauchy if d is continuous on X.

Definition:

[1] *Quasi Semi 2- Metric Space* –
A nonempty set X, together with a nonnegative function d: $X^3 \rightarrow$ R is called a quasi semi 2- metric space such that

(i) to each pair of distinct points x, y in X, there exists a point $z \in X$ such that
$d(x, y, z) \neq 0$.
(ii) d(x, y, z) = 0, when at least two of x, y, z are equal.

[2] *Φ' - contraction in Quasi Semi 2- Metric Spaces*:
The set Φ' of all real valued functions $\phi' : R_+^3 \rightarrow R_+$ satisfies the following properties:

(a) $\phi'(1,1,1) = h < 1$, where $h \in R_+$,
(b) Let u, v $\in R_+$ be such that if either $u \leq \phi'(u, v, v)$

or $u \leq \phi'(v, u, v)$ or $u \leq \phi'(v, v, u)$,
Then $u \leq kv$, for some $k \in [h, 1)$.

Analogous to the p' - contraction by M. Telsi and K.Tas [5], we introduce ϕ' - contraction mappings in quasi semi 2- metric spaces.

Definition:

A self mapping T on a quasi semi 2- metric space (X,d) is called a *Φ'-contraction,* if there exists a map $\phi' \in \Phi'$ such that

$$d(Tx, Ty, a) \leq \phi'(d(x, y, a), d(x, Tx, a), d(y, Ty, a)) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Theorem:

Let (X, d) be a quasi semi 2- metric space and T a Φ' - contraction. If there exists a point $x_0 \in$ X such that for all a $\in$ X

$d(x_0, Tx_0, a) \le \min\{d(x, y) + (y, Tx, a) : x, y \in X\}$
then T has a unique fixed point.

Proof:
Suppose $x_0 \ne Tx_0$. We put $x = x_0$, $y = Tx_0$ in (1).
Therefore,
$d(Tx_0, T^2x_0, a) \le \Phi'(d(x_0, Tx_0, a), d(x_0, Tx_0, a), d(Tx_0, T^2x_0, a))$
So by (b) we obtain

$d(Tx_0, T^2x_0, a) \le k.\, d(x_0, Tx_0, a)$ for some $k \in [h,1)$
since $k < 1$,
we have $d(Tx_0, T^2x_0, a) \le d(x_0, Tx_0, a)$
But given that
$d(x_0, Tx_0, a) \le \min\{d(x, y) + d(y, Tx, a): x, y \in X\}$
is a contradiction.
Hence, $Tx_0 = x_0$.
This proves that x_0 is the fixed point of T.
Uniqueness:
Let y_0 be another fixed point of T i.e. $y_0 = Ty_0$
Now, $d(x_0, y_0, a) = d(Tx_0, Ty_0, a)$

$$\le \phi'(d(x_0,y_0,a), d(x_0, Tx_0, a), d(y_0, Ty_0, a))$$
$$\le \phi'(d(x_0,y_0,a), d(x_0, x_0, a), d(y_0, y_0, a))$$
$$\le \phi'(d(x_0,y_0,a), 0, 0).$$

Therefore, by (Definition 2, b), we obtain
$d(x_0, y_0, a) \le 0$, or
$d(x_0, y_0, a) = 0$
$\Rightarrow x_0 = y_0$

Hence proved that fixed point is unique. This completes the proof of the Theorem.

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