Fuzzy programming technique to solve bi-objective transportation problem

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Abstract- In a transportation problem generally a single criterion of minimizing the total transportation cost is considered but in certain practical situations two or more objectives are relevant. For example, the objectives may be minimization of total cost, consumption of certain scarce resources such as energy, total deterioration of goods during transportation etc. Clearly, this problem can be solved using any of the multiobjective linear programming techniques, but the computational efforts needed would be prohibitive in many cases. In this paper, The Bi-objective transportation problem, where only objectives are considered as fuzzy. We apply the fuzzy programming technique with hyperbolic membership function to solve a bi-objective transportation problem as vector minimum problem.
Keywords: Transportation problem, Fuzzy programming, Linear and nonlinear membership functions, Bi criteria optimization technique

Introduction

The transportation problem (TP) can be formulated as a linear programming problem, where the constraints have a special structure [1]. However, in most real world cases transportation problems can be formulated as multi-objective problems [2, 3]. In certain situations two objectives are relevant in transportation problems. For example, two linear objective may be minimization of the cost and minimization of the total deterioration. Aneja and Nair developed a criteria space approach for bicriteria.TP [1]. Leberling [5] used a special- type nonlinear (hyperbolic) membership function for the vector maximum linear programming problem. He showed that solutions obtained by fuzzy linear programming with this type of non-linear membership function are always efficient. Dhingra and Moskowitz [4] defined other types of the non-linear (exponential, quadratic and logarithmic) membership functions and applied them to an optimal design problem. Verma, Biswal and Biswas [7] used the fuzzy programming technique with some non-linear (hyperbolic and exponential) membership functions to solve a multi-objective transportation problem

Mathematical model

In a typical transportation problem, a homogeneous product is to be transported from each of m sources to n destinations. The sources are production facilities, warehouses, or supply point, characterized by available capacities a_i (i = 1,2,…, m). The destinations are consumption facilities, warehouses, or demand points, characterized by required levels of demand b_j (j = 1,2,…, n). A penalty c_{ij} and d_{ij} are associated with transportation of a unit of the product from sources i to destination j. The penalty could represent transportation cost and deterioration of a unit. A variable X_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. In the real would, however, transportation problems are not all-single objective type. We may have more than one objective in a transportation problem.

A Bi-objective transportation problem may be stated mathematically as:

$$\text{Minimize} \quad Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \quad (1)$$

$$\text{Minimize} \quad Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij} \quad (2)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\ldots,m \quad (3)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\ldots,n \quad (4)$$

$x_{ij} \geq 0$ for all i and j (5)

where c_{ij} and d_{ij} are the penalties associated with transportation of a unit from source i to destination j. The penalties may represent transportation cost, deterioration cost, delivery time, quantity of goods delivered, under used capacity, and so on.

a_i>o for all i, b_j>o, for all j, C_{ij}, $d_{ij} \geq$ o for all i, j, and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad \text{(Balanced condition)}$$

The balanced condition is treated as a necessary and sufficient condition for the existence of a feasible solution. A standard transportation problem has exactly (m + n) constraints and (m n) variables. The LINDO (Linear Interactive and Discrete Optimization) package handles the transportation problem in an explicit equation form and thus solves the problem as a standard linear programming problem.

Fuzzy programming technique to BOTP

The Bi-objective transportation problem can be considered as a vector minimum problem. Let

U_1, L_1 be the upper and lower bound for The first objective function and U_2, L_2 be the upper and lower bound for The second objective function where lower bound indicates aspiration level of achievement and upper bound indicates highest acceptable level of achievement for the objective function respectively.
Let $d_1 = (U_1 - L_1)$ and $d_2 = (U_2 - L_2)$ be degradation allowance for the Z_1 and Z_2 objective. Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. Our next step is to transform the fuzzy model into a "Crisp" model.

Algorithm

Step 1: Solve the Bi-objective transportation problem as a single objective transportation problem using, each time, only one objective (ignore all others). Let $X_1^* = \{x^1_{ij}\}$, $X^{2*} = \{x^2_{ij}\}$, be the optimum solutions for Z_1, Z_2 different single objective transportation problem.

Step 2: From the results of step 1, calculate the values of all the objective functions at all these X_1^*, X^{2*} optimal points. Then a pay off matrix is formed. The diagonal of the matrix constitutes individual optimum minimum values for the two objectives. The X_1^*, X^{2*} are the individual optimal solutions and each of these are used to determine the values of other individual objectives, thus the pay off matrix is developed as:

$$\begin{array}{cc} & \begin{array}{cc} x^{1*} & \quad x^{2*} \end{array} \\ \begin{array}{c} Z_1 \\ Z_2 \end{array} & \begin{bmatrix} Z_1(x^{1*}) & Z_1(x^{2*}) \\ Z_2(x^{1*}) & Z_2(x^{2*}) \end{bmatrix} \end{array}$$

We find the upper and lower bound for each objective from the pay off Matrix. Let $Z_1(x^{1*}), Z_1(x^{2*})$ be the values of the first objective Z_1 then L_1 = min[$Z_1(x^{1*}), Z_1(x^{2*})$] and U_1 = max [$Z_1(x^{1*}), Z_1(x^{2*})$]. Let $Z_2(x^{1*}), Z_2(x^{2*})$ be the values of the second objective Z_2 then L_2 = min [$Z_2(x^{1*}), Z_2(x^{2*})$] and U_2 = max [$Z_2(x^{1*}), Z_2(x^{2*})$].

Step 3: From step 2, we find for each objective the worst and the best values corresponding to the set of solutions.

An initial fuzzy model of the problem (1-4) can be stated as: -
Find x_{ij}, i =1, 2, …,m; j = 1, 2, …,n;
so as to satisfy

$Z_1 \underset{\sim}{\leq} L_1$, (6)

$Z_2 \underset{\sim}{\leq} L_2$, (7)

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i \,, \quad i = 1,2,\ldots,m \quad (8)$$

$$\sum_{i=1}^{m} X_{ij} = b_j \,, \quad j = 1,2,\ldots,n \quad (9)$$

$x_{ij} \geq 0$ for all i, j, k (10)

$\underset{\sim}{\leq}$ fuzzification symbol indicates nearly less than equal to

Step 4: Case (i)

Define a hyperbolic membership function $\mu_1^H(Z_1)$ and $\mu_2^H(Z_2)$ for the objectives (Z_1) and (Z_2) respectively, are defined as follows

$$\mu_1^H(Z_1) = \frac{1}{2}\tanh\left(\left(\frac{U_1+L_1}{2} - Z_1\right)\alpha_1\right) + \frac{1}{2} \quad (11)$$

where α_1 is a parameter. Where

$$\alpha_1 = \frac{3}{(U_1 - L_1)/2} = \frac{6}{(U_1 - L_1)}$$

$$\mu_2^H(Z_2) = \frac{1}{2}\tanh\left(\left(\frac{U_2+L_2}{2} - Z_2\right)\alpha_2\right) + \frac{1}{2} \quad (12)$$

where α_2 is a parameter. Where

$$\alpha_2 = \frac{3}{(U_2 - L_2)/2} = \frac{6}{(U_2 - L_2)}$$

The hyperbolic membership functions (11-12) has the following properties:

1. It is strictly decreasing function.
2. It is strictly concave for $Z_1 \leq (U_1 + L_1)/2$, $Z_2 \leq (U_2 + L_2)/2$
3. It is equal to 0.5 for $Z_1 = (U_1 + L_1)/2$, $Z_2 = (U_2 + L_2)/2$
4. It is strictly convex for $Z_1 \geq (U_1 + L_1)/2$, $Z_2 \geq (U_2 + L_2)/2$
5. For all $X \in R^{mn}$ holds $o < \mu_1^H(Z_1) < 1, o < \mu_2^H(Z_2) < 1$; $\mu_1^H(Z_1) = 1$, $\mu_2^H(Z_2) = 1$

is the lower asymptotic function of $\mu_1^H(Z_1),\ \mu_2^H(Z_2)$; $\mu_1^H(Z_1)$ = 0, $\mu_2^H(Z_2)$ = 0 is the upper asymptotic function of $\mu_1^H(Z_1),\ \mu_2^H(Z_2)$..

Step 5: Formulate an equivalent nonlinear programming model with the help of the defined membership function (11-12) for the Bi-objective transportation problem. This is stated as follows:

Maximize λ (13)

subject to

$\lambda \le \mu_1^H(Z_1)$ (14)

$\lambda \le \mu_2^H(Z_2)$ (15)

$\sum_{j=1}^{n} X_{ij} = a_i$, i = 1,2,…,m (16)

$\sum_{i=1}^{m} X_{ij} = b_j$, j = 1,2,…,n (17)

$x_{ij} \ge 0$ for all i, j and $\lambda \ge 0$ (18)

where

$$\lambda = \underset{1}{\mathrm{Min}}\ \{\mu_1^H(Z_1)\},\ \underset{2}{\mathrm{Min}}\ \{\mu_2^H(Z_2)\}$$

This is a nonlinear programming problem with one linear objective function, two non-linear and m+n+2mn+1 linear restrictions. We shall now prove that there exists an equivalent linear programming problem.

Theorem: Define X_{mn+1} = $\tanh^{-1}(2\lambda - 1)$. The equivalent linear programming problem for the above nonlinear programming problem is as follows:

Maximize λ (19)

subject to

$\alpha_1 Z_1 + X_{mn+1} \le \alpha_1(U_1 + L_1)/2$ (20)

$\alpha_2 Z_2 + X_{mn+1} \le \alpha_2(U_2 + L_2)/2$ (21)

constraints (16),(17), (18)

Proof. For $t \in R$, we know $\tanh(t) = \dfrac{e^t - e^{-t}}{e^t + e^{-t}}$

.

Therefore, nonlinear programming problem can be formulated as:

Maximize λ (22)

subject to

$$\lambda - \frac{1}{2}\tanh\left(\left(\frac{U_1+L_1}{2} - Z_1\right)\alpha_1\right) \le \frac{1}{2} \quad (23)$$

$$\lambda - \frac{1}{2}\tanh\left(\left(\frac{U_2+L_2}{2} - Z_2\right)\alpha_2\right) \le \frac{1}{2} \quad (24)$$

and constraints (16),(17), (18)

This is equivalent to

Maximize λ (25)

subject to

$$\tanh\left(\left(\frac{U_1+L_1}{2} - Z_1\right)\alpha_1\right) \ge 2\lambda - 1 \quad (26)$$

$$\tanh\left(\left(\frac{U_2+L_2}{2} - Z_2\right)\alpha_2\right) \ge 2\lambda - 1 \quad (27)$$

and constraints (16),(17), (18)

Since tanh and $\tanh^{-1}$ are strictly increasing functions, we have equivalently

Maximize λ (28)

subject to

$$\left(\frac{U_1+L_1}{2} - Z_1\right)\alpha_1 \ge \tanh^{-1}(2\lambda - 1) \quad (29)$$

$$\left(\frac{U_2+L_2}{2} - Z_2\right)\alpha_2 \ge \tanh^{-1}(2\lambda - 1) \quad (30)$$

and constraints (16),(17), (18)

or with $X_{mn+1} = \tanh^{-1}(2\lambda - 1)$

Maximize λ (31)

subject to

$$X_{mn+1} + \alpha_1 Z_1 \le \left(\frac{U_1+L_1}{2}\right)\alpha_1 \quad (32)$$

$$X_{mn+1} + \alpha_2 Z_2 \le \left(\frac{U_2+L_2}{2}\right)\alpha_2 \quad (33)$$

and constraints (16),(17), (18)

Because of $\lambda = \dfrac{\tanh(X_{mn+1})}{2} + \dfrac{1}{2}$ and the tanh function strictly increasing, it follows equivalently:

Maximize X_{mn+1} (34)

subject to

$$X_{mn+1} + \alpha_1 Z_1 \le \left(\frac{U_1+L_1}{2}\right)\alpha_1 \quad (35)$$

$$X_{mn+1} + \alpha_2 Z_2 \le \left(\frac{U_2+L_2}{2}\right)\alpha_2 \quad (36)$$

and constraints (16),(17), (18) and $X_{mn+1} \ge 0$

This linear programming can be further simplified as:

Maximize λ (37)

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \frac{X_{mn+1}}{\alpha_1} \le \left(\frac{U_1+L_1}{2}\right) \quad (38)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij} + \frac{X_{mn+1}}{\alpha_2} \le \left(\frac{U_2+L_2}{2}\right) \quad (39)$$

and constraints (16),(17), (18) and $X_{mn+1} \ge 0$

Case (ii)
However, if we use a linear membership function $\mu_1(x)$ and $\mu_2(x)$, for the objectives (Z_1) and (Z_2). respectively, are defined as follows:

$$\mu_1(z_1(x)) = \begin{cases} 1, & \text{if } Z_1(x) \le L_1 \\ \frac{U_1 - Z_1(x)}{L_1 - U_1}, & \text{if } L_1 < Z_1(x) < U_1 \\ 0, & \text{if } Z_1(x) \ge U_1 \end{cases} \quad (40)$$

$$\mu_2(z_2(x)) = \begin{cases} 1, & \text{if } Z_2(x) \le L_2 \\ \frac{U_2 - Z_2(x)}{L_2 - U_2}, & \text{if } L_2 < Z_2(x) < U_2 \\ 0, & \text{if } Z_2(x) \ge U_2 \end{cases} \quad (41)$$

where $L_1 \neq U_1$, if $L_1 = U_1$, then $\mu(Z_1(x)) = 1$ for any value of Z_1 and $L_2 \neq U_2$. if $L_2 = U_2$, then $\mu(Z_2(x)) = 1$ for any value of Z_2
.
Following the fuzzy decision of Bellman and Zadeh [9] together with the linear membership function (40-41), a fuzzy optimization model of Two-objective Transportation Problem can be written as follows:

P1: Max $\text{Min}\, \mu_1(z_1(x))$ (42)

$\text{Min}\, \mu_2(z_2(x))$ (43)

subject to

$\sum_{j=1}^{n} X_{ij} = a_i$, i = 1,2,…,m (44)

$\sum_{i=1}^{m} X_{ij} = b_j$, j = 1,2,…,n (45)

$x_{ij} \geq 0$ for all i and j (46)

By introducing an auxiliary variable λ, problem P1 can be transformed into the following equivalent conventional linear programming problem [10].

P2: Max λ (47)

subject to

$\lambda \le \mu_1(z_1(x))$ (48)

$\lambda \le \mu_2(z_2(x))$ (49)

$\sum_{j=1}^{n} X_{ij} = a_i$, i = 1,2,…,m (50)

$\sum_{i=1}^{m} X_{ij} = b_j$, j = 1,2,…,n (51)

$0 \le \lambda \le 1,$ (52)

$x_{ij} \geq 0$ for all i and j (53)

In problem constraint (P2) can be reduced to the following form:

$$\lambda\,(U_1 - L_1) \le (U_1 - Z_1(x))$$
$$\lambda\,(U_1 - L_1) + Z_1(x) \le U_1$$
$$\frac{\lambda\,(U_1 - L_1)}{U_1} + \frac{Z_1(x)}{U_1} \le 1 \quad (54)$$

In problem constraint (P2) can be reduced to the following form

$$\lambda\,(U_2 - L_2) \le (U_2 - Z_2(x))$$
$$\lambda\,(U_2 - L_2) + Z_2(x) \le U_2$$
$$\frac{\lambda\,(U_2 - L_2)}{U_2} + \frac{Z_2(x)}{U_2} \le 1 \quad (55)$$

To determine the degree of closeness of the fuzzy approach result to the ideal solution, let us define the following family of distance functions [8]

$$L_p(\lambda, k) = \left[\sum_{k=1}^{k} \lambda_k^p (1 - d_k)^p \right]^{\frac{1}{p}} \quad (56)$$

where d_k represents the degree of closeness of the compromise solution vector X* to the ideal solution vector with respect to the k-th objective. $\lambda = (\lambda_1, \lambda_2, ..., \lambda_k)$ is vector of objective attention level. The power p represents a distance parameter $1 \le p \le \infty$.

Definition: Ideal solution: The solution to the Two-objective Transportation Problem is a point X^{1*}, X^{2*} in the outcome space such that $Z_2(x^{1*}), Z_2(x^{2*})$ is an optimal objective function value of the sub problems:

$$\text{Minimize} \quad Z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{Minimize} \quad Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij}$$

subject to the given set of constraints.

Assuming $\sum_{k=1}^{k} \lambda_k = 1,$ (k =1, 2) we can write $L_p(\lambda, K)$ with p = 1,2,and ∞ as follows:

$$L_1(\lambda, K) = \left[1 - \sum_{k=1}^{k} \lambda_k d_k \right] \quad (57)$$

$$L_2(\lambda, K) = \left[\sum_{k=1}^{k} \lambda_k^2 (1 - d_k)^2 \right]^{\frac{1}{2}} \quad (58)$$

$$L_\infty(\lambda, K) = \text{Max}_k \{\lambda_k (1 - d_k)\} \quad (59)$$

where in the minimum problem d_k takes the form:

$$d_k = \frac{\text{The ideal value of } Z_k(x)}{\text{The compromise value of } Z_k(x)} \quad (60)$$

Thus, we can state that the approach which gives a compromise solution close to the ideal solution, is better than the other if

Min $L_p(\lambda, K)$ (61)

is achieved for its solution with respect to some p.

Example 1.

Let us consider two-objective transportation problems with following characteristics:

Supplies: - $a_1 = 8$, $a_2 = 19$, $a_3 = 17$.

Demand: - $b_1 = 11$, $b_2 = 3$, $b_3 = 14$, $b_4 = 16$.

Penalties:

$$C^1 = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix},$$

$$C^2 = \begin{bmatrix} 4 & 4 & 3 & 4 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}$$

This problem can be modeled as follows:

$$\text{Minimize} \quad Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34} \quad (62)$$

$$\text{Minimize} \quad Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + 1x_{34} \quad (63)$$

subject to

$$\sum_{j=1}^{4} x_{1j} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17.$$

$$\sum_{i=1}^{3} x_{i1} = 11, \quad \sum_{i=1}^{3} x_{i2} = 3, \quad \sum_{i=1}^{3} x_{i3} = 14, \quad \sum_{i=1}^{3} x_{i4} = 16.$$

(64)

$x_{ij} \geq 0$, i = 1,2,3, j = 1, 2,3,4. (65)

where $C^1 = (C^1_{ij})$, $C^2 = (C^2_{ij})$.

Step 1 and step 2:

Optimal solution, which minimizes the first objective Z_1 subject to constraints (64-65) are as follows:

$X_{11} = 5$, $X_{12} = 3$, $X_{21} = 6$, $X_{24} = 13$, $X_{33} = 14$, $X_{34} = 3$.

With $Z_1(X_1) = 143$, $Z_2(X_1) = 208$,

Optimal solutions, which minimizes the second objective Z_2 subject to constraints (64-65) are as follows:

$X_{13} = 8$, $X_{21} = 11$, $X_{22} = 2$, $X_{23} = 6$, $X_{32} = 1$, $X_{33} = 16$.

With $Z_1(X_1) = 167$, $Z_2(X_1) = 265$.

Step 3: Pay-off matrix is

$$\begin{array}{c|cc} & X_1^* & X_2^* \\ \hline Z_1 & 143 & 208 \\ Z_2 & 265 & 167 \end{array}$$

From the pay-off matrix, we find the upper and lower bound of each objective as follows:

$U_1 = 208$, $L_1 = 143$,

$U_2 = 265$, $L_2 = 167$,

Find { x_{ij}, i = 1,2,3; j = 1,2,3,4.} so as to satisfy

$Z_1 \leq 143, Z_2 \leq 167$ and constraints (64-65)

Step 4:

If we use hyperbolic membership function, with

$$\alpha_1 = \frac{6}{(U_1 - L_1)} = \frac{6}{65},$$

$$\alpha_2 = \frac{6}{(U_2 - L_2)} = \frac{6}{98}$$

$$\frac{U_1 + L_1}{2} = 175.5, \quad \frac{U_2 + L_2}{2} = 216,$$

we get the membership functions $\mu_1^H(Z_1)$ and $\mu_2^H(Z_2)$ for the objectives (Z_1) and (Z_2) respectively, are defined as follows:

$$\mu_1^H(Z_1) = \begin{cases} 1, & \text{if } Z_1(x) \leq 143 \\ \frac{1}{2}\tanh\left((175.5 - Z_1(x))\frac{6}{65}\right) + \frac{1}{2}, & \text{if } 143 \leq Z_1(x) \leq 208 \\ 0, & \text{if } Z_1(x) \geq 208 \end{cases}$$

$$\mu_2^H(Z_2) = \begin{cases} 1, & \text{if } Z_2(x) \leq 167 \\ \frac{1}{2}\tanh\left((216 - Z_2(x))\frac{6}{98}\right) + \frac{1}{2}, & \text{if } 167 \leq Z_2(x) \leq 265 \\ 0, & \text{if } Z_2(x) \geq 265 \end{cases}$$

Step 5:

We get an equivalent crisp model, which can be formulated as:

Maximize X_{mn+1}

subject to

$$6\,[Z_1] + 65X_{mn+1} \leq 1053$$

$$6\,[Z_2] + 98X_{mn+1} \leq 1296$$

constraints (64-65) and $X_{mn+1} \geq 0$

The problem was solved by the Linear Interactive and Discrete Optimization (LINDO) Software The optimal solution is presented as follows:

$X_{mn+1} = 1.351464$, $X_{11} = 3.785216$, $X_{12} = 3.0$, $X_{13} = 1.214784$,

$X_{21} = 7.214784$, $X_{23} = 11.785216$, $X_{33} = 1.0$, $X_{34} = 16.0$, and $\lambda = 0.937$

Transportation cost Z_1 = 160. 8591,

Deterioration of goods Z_2 = 193. 926.

Ringuest and Rinks [6] have obtained 186 and 174 as the interactive approach values of objectives Z_1 and Z_2 respectively

Table 1-

Objective function	Ideal solution	Fuzzy approach results	Interactive approach results
$Z_1(x)$	143	160. 8591,	186
$Z_2(x)$	167	193. 926.	174

The example is solved by the given interactive approach in [6]. The procedure begins with constructing a linear compromise solution and a search is conducted among all non-dominated solutions corresponding to extreme points adjacent to the most preferred extreme point. This search is continued until a satisfactory solution is obtained. Solution of the above example by using this procedure is summarized in table 1.

Using fuzzy programming (with hyperbolic membership function) approach the result is as follows:

Assuming $\sum_{k=1}^{k} \lambda_k = 1$, we can write $L_k(\lambda, K)$ with p = 1,2,and ∞ as follows:

$\lambda_1 = \lambda_2 = 1/2$, i. e. the objectives are equally important

$$L_1(\lambda, K) = \left[1 - \sum_{k=1}^{k} \lambda_k d_k\right]$$

= 0.5{(1-0.8889)+(1-0.8612)}

= 0.125

$$L_2(\lambda, K) = \left[(o.5)^2\{1\text{-}0.8889)^2 + (1\text{-}0.0.8612)^2\}\right]^{\frac{1}{2}}$$

= 0.08889

$$L_\infty(\lambda, K) = \text{Max}_k\{\lambda_k(1\text{-}d_k)\}$$

$$L_\infty(\lambda, K) = (0.5)\{(1 - 0.8612)\}$$

= 0.0694

Interactive approach the results

Assuming $\sum_{k=1}^{k} \lambda_k = 1$, we can write $L_k(\lambda, K)$ with p = 1,2,and ∞ as follows:

$\lambda_1 = \lambda_2 = 1/2$, i. e. the objectives are equally important

$L_1(\lambda, P)$ = 0.5{(1-0.7688) + (1-0.9598)}

= 0.1357

$$L_2(\lambda, K) = \left[(o.5)^2\{1\text{-}0.7688)^2 + (1\text{-}0.9598)^2\}\right]^{\frac{1}{2}}$$

= 0.1173

$$L_\infty(\lambda, K) = (0.5)\{(1 - 0.7688)\}$$

= 0.1156

Table 2- Numerical values of L_1, L_1 and L_∞

Objective function	Ideal solution	Fuzzy approach results	Interactive approach results
$Z_1(x)$	143	160. 8591,	186
$Z_2(x)$	167	193. 926.	174
d_1	--	0.8889	0.7688
d_2	--	0.8612	0.9598
L_1	--	0.125	0.1357
L_2	--	0.08889	0.1173
L_∞	--	0.0694	0.1156

The family of the distance functions for solutions of the given fuzzy approach and the interactive procedure [6] are summarized in table 2. In above example it is observed that the fuzzy approach gives compromise solution better than the interactive compromise solution with respect to L_1. L_2 and L_∞.

If we use the linear membership function as defined in (40-41), an equivalent crisp model can be presented as follows:

We get the membership function $\mu_1(x)$ and $\mu_2(x)$, for the objectives (Z_1) and (Z_2). respectively, are defined as follows:

$$\mu_1(x) = \begin{cases} 0, & \text{if } Z_1(x) \le 143 \\ \dfrac{208 - Z_1(x)}{208\text{-}143}, & \text{if } 143 < Z_1(x) < 208 \\ 1, & \text{if } Z_1(x) \ge 208 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0, & \text{if } Z_2(x) \le 167 \\ \dfrac{265 - Z_2(x)}{265\text{-}167}, & \text{if } 167 < Z_2(x) < 265 \\ 1, & \text{if } Z_2(x) \ge 265 \end{cases}$$

Now (54) is written as follows:

$$\frac{\lambda(208\text{-}143)}{208} + \frac{Z_1(x)}{208} \le 1$$

(55) is written as follows :

$$\frac{\lambda\ (265\text{-}167)}{265} + \frac{Z_2(x)}{265} \le 1$$

The equivalent linear programming problem of this example is given below

Maximize λ

Subject to

$0.0048x_{11} + 0.0096x_{12} + 0.0337x_{13} + 0.0337x_{14} + 0.0048x_{21} + 0.0433x_{22} + 0.01442x_{23} + 0.0192x_{24}$

$+0.0385x_3 + 0.0433x_{32} + 0.0192x_{33} + 0.02881x_{34}$
$+0.3125\lambda \le 1$

$0.0151 + 0.0151\, x_{12} + 0.01132\, x_{13} + 0.01509\, x_{14} + 0.01887\, x_{21} +$
$0.03019x_{22} + 0.03396x_{23} + 0.03774x24$
$+0.02264x_{31} + 0.007547x_{32} + 0.01887x_{33} +$
$0.00377x_{34} + 0.3698\lambda \le 1$

$x_{11} + x_{12} + x_{13} + x_{14} = 8$
$x_{21} + x_{22} + x_{23} + x_{24} = 19$
$x_{31} + x_{32} + x_{33} + x_{34} = 17$
$X_{11} + X_{21} + X_{31} = 11$
$x_{12} + x_{22} + x_{32} = 3$
$x_{13} + x_{23} + x_{33} = 14$
$x_{14} + x_{24} + x_{34} = 16$
$x_{ij} \ge 0$, $i = 1,2,3,$, $j = 1, 2,3,4.$ and $\lambda \ge 0$

The problem was solved by the Linear Interactive and Discrete Optimization (LINDO) Software The optimal solution is presented as follows:
$X_{11} = 3.765676$, $X_{12} = 3.0$, $X_{13} = 1.234324$, $X_{21} = 7.234324$, $X_{23} = 11.765676$, $X_{33} = 1.0$, $X_{34} = 16.0$, and $\lambda = 0.726343$
Transportation cost $Z_1 = 160.9368$,
Deterioration of goods $Z_2 = 193.8273$.

The family of distance functions for solutions of the given fuzzy linear membership approach and the interactive procedure [6] are summarized in table 3.

Table 3

Objective function	Ideal solution	Fuzzy approach results	Interactive approach results
$Z_1(x)$	143	160. 9368,	186
$Z_2(x)$	167	193. 8373.	174
d_1	--	0.8885	0.7688
d_2	--	0.8615	0.9598
L_1	--	0.125	0.1357
L_2	--	0.08890	0.1173
L_∞	--	0.06925	0.1156

The family of the distance functions for solutions of the given fuzzy approach and the interactive procedure [6] are summarized in table 3. In above example it is observed that the fuzzy approach gives compromise solution better than the interactive compromise solution with respect to L_1. L_2 and L_∞

Conclusion

In the present paper, fuzzy linear and non- linear programming technique has been used to find an optimal compromise solution for Two-objective Transportation Problem. If we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function. Further, we conclude that for a transportation problem if the demand parameters are gamma random variables, then the deterministic problem becomes non-linear. To solve this type of problem, these non-linear membership functions can be used Apart from the transportation problems for the multiobjective non-linear programming problems, non-linear membership functions are useful. The family of the distance functions for solutions of the given Fuzzy programming (with Hyperbolic and Linear membership function) approach and the interactive procedure [6] are summarized in table 2 or 3. In above example it is observed that the fuzzy approach gives compromise solution better than the interactive compromise solution with respect to L_1. L_2 and L_∞. As a result, adaptation of the fuzzy approach leads to better solution than the interactive algorithm.

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