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AN APPROACH TO SOLVE THE FUZZY MULTY OBJECTIVE LINEAR FRACTIONAL GOAL PROGRAMMING PROBLEM

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Abstract-It is the purpose of this article to introduce a new weighted FGP technique by using only under deviation variables to fuzzy goals of fuzzy multi objective linear fractional goal programming problem (FMOLFGPP) to achieve highest degree of each of the membership goals by minimizing their under deviational variables. To assess the relative importance of the fuzzy goals properly, the weighting scheme by the decision of decision maker (DM) and Mohamed's technique (Fuzzy Sets and Systems, 89 (1997) 215-222) has been used. Illustrative numerical examples of FMOLFGPP are provided to demonstrate the feasibility of the proposed method, which clearly shows the proposed approach yields better optimal solution of FMOLFGP problem than the conventional min sum FGP approach in the sense that it gives the values of the fractional objectives closer to their aspiration level.
Keywords:-Linear programming; Goal programming; Multi objective programming; Fractional programming; Fuzzy sets; Fuzzy goal programming; Fuzzy fractional goal programming; Multiple criteria decision analysis.

Introduction

Many physical problems can be formulated as optimization problem subject to some constraints. Most real-world decision problems involve multiple criteria that are often conflicting in nature and it is sometimes necessary to conduct trade-off analysis in multiple criteria decision analysis (MCDA). As such, the estimation of the relative weights of criteria plays an important role in a MCDA process. Fractional programming gains significant stature since many of the real world problems represented as fractional function are often encountered in the situation such as return on investment, current ratio, risk-assets to capital, actual capital to required capital, foreign loans to total loans, residential mortgages to total mortgages, for finance or corporate planning, debt-to-equity ratio etc. for production planning, inventory to sales, actual cost to standard cost, output per employee, a College or an Educational Institution concerned with the optimization of student / teacher, cost / student and so forth. If the numerator and denominator in the objective function as well as the constraints are linear, then it is called a linear fractional programming problem (LFPP). Charnes and Cooper [5] and Wolf [30] derived optimal procedures for linear fractional programming problems. Fractional programming (FP) has widely been used [17]. In 1965, the concept of fuzzy sets was proposed by Zadeh [32]. Bellman and Zadeh [2] proposed that a fuzzy decision is defined as the fuzzy set of alternatives resulting from the intersection of the goals or objectives and constraints. The concept of fuzzy programming was first introduced by Tanaka et al. [28] in the framework of fuzzy decision of Bellman and Zadeh. Afterwards, fuzzy approach to linear programming (LP) with several objectives was studied by Zimmermann [11]. Too early of 1990, most of the methodologies for solving multi objective linear fractional programming problem (MOLFPP) [14] were computationally burdensome. The application of fuzzy set theory was to overcome this difficulty. Fuzzy multiple objective fractional programming (FMOFP) is an important technique for the solution of many real-world problems [22] involving the nature of vagueness, imprecision etc. Stancu Minasian and Pop's review paper [12] showed an extensive account on fuzzy fractional programming problem with a single or multiple objective functions. In 1984, Luhandjula [19] used a linguistic variable approach in order to present a procedure for solving multiple objective linear fractional programming problems (MOLFPP). In 1992, Dutta et al.[9] modified the linguistic approach of Luhandjula so as to obtain an efficient solution to problem of MOLFPP. In 1955 the roots of goal programming lie in the journal (Management Science) by Charnes, Cooper and Ferguson [4]. Earlier Goal programming has been widely implemented to different problems by the famous researchers [8, 16]. In economical and physical problems of mathematical programming generally, and in the fractional programming problems in particular, the coefficients in the problems are assumed to be exactly known. However, in practice, this assumption is seldom satisfied by great majority of real-life problems. Usually, the coefficients (some or all) are subjected to errors of measurement or they vary with market conditions. For such problems involving uncertainty, fuzzy goal programming approach is adopted. In 1980, R. Narasimhan [21] was first studied the use of fuzzy set theory in GP. In 1997, Mohammed [18] presented a new fuzzy goal programming technique which is used to achieve highest degree of each of the membership goals by minimizing their deviation variables. During the past, various researchers solved multi-objective linear goal programming problem by using fuzzy goal programming (FGP) algorithm [10, 13, 24, 26, 27, and 29]. In the fuzzy multi objective linear fractional goal programming problem, the fractional goals are transformed into the linear goals by linearization approach suggested by B.B. Pal et al [3]. In the recent past, several pioneer researchers projected some new approaches and works in the field of fuzzy multi objective linear fractional goal programming (FMOLFGP), in cooperating both the under and over deviation variables to the membership goals [1, 6, 7, 20, 23, 25, 31]. But the adopted FGP approach by introducing both the deviation variables to fuzzy goals is not correct always as its application sometimes leads to the wrong decision. So in this paper, we propose a new procedure of weighted FGP technique where only the under deviation variables are introduced to the membership goals that gets the exact a solution to overcome the problem. That the proposed technique yields a better solution of fuzzy multi objective linear fractional goal programming problem compared to the usual one in the sense that the solution provides objective values closer to their respective aspiration levels. To demonstrate the usefulness of the proposed method four examples are illustrated. Two of which are real problems. One real problem has been adopted from [7] whereas the purpose of another real problem is to analyse and introduce the concept of management, the planning of academic resource allocation with limited resources like infrastructure, human resources, equipment in the Institute namely National Institute of Technology Agartala (NITA).

Problem Formulation

The general format of the multi objective fractional programming problem (MOFPP) can be written as:

$$\text{Optimize } Z_k(x) = \frac{c_k x+\alpha_k}{d_k x+\beta_k} \quad k = 1, 2, \ldots, K$$

$$\text{Where } x \in X = \left\{ x \in R^n \mid Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b,\ x \geq 0,\ b^T \in R^m \right\}, \quad \ldots\ldots\ldots\ldots (1)$$

Where c_k^T, $d_k^T \in R^n$; α_k , β_k are constants and $d_k x + \beta_k > 0$.

Fuzzy Goal Programming Formulation

In multi objective fractional programming, if an imprecise aspiration level is introduced to each of the objectives then these fuzzy objectives are termed as fuzzy goals. Let g_k be aspiration level assigned to the k^{th} objective $Z_k(x)$. Then the fuzzy goals are

i) $Z_k(x) \gtrsim g_k$ [for maximizing $Z_k(x)$] and

ii) $Z_k(x) \lesssim g_k$ [for minimizing $Z_k(x)$].

Where '$\gtrsim$' and '$\lesssim$' represents the fuzzified version of '≥' and '≤'. These are to be understood as 'essentially greater than' and 'essentially less than' in the sense of Zimmermann [11].

Hence the fuzzy multi objective fractional goal programming can be stated as follows:

Find x ($\in X$)

So as to satisfy $Z_k(x) \gtrsim g_k \quad k = 1,2,\ldots,k_1$

$Z_k(x) \lesssim g_k \quad k = k_1+1,\ldots, K$

Subject to $Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$,

$x \geq 0. \quad \ldots\ldots\ldots\ldots(2)$

Construction of Membership Functions

Now the membership function μ_k for the k^{th} fuzzy goal $Z_k(x) \gtrsim g_k$ can be expressed as follows:

$$\mu_k(x) = \begin{cases} 1 & if\ Z_k(x) \geq g_k \\ \frac{Z_k(x)-l_k}{g_k-l_k} & if\ l_k \leq Z_k(x) \leq g_k \\ 0 & if\ Z_k(x) \leq l_k \end{cases} \quad \ldots\ldots\ldots\ldots(3)$$

Where l_k is the lower tolerance limit for the k^{th} fuzzy goal and $(g_k - l_k)$ is the tolerance (p_k) which is subjectively chosen. Again the membership function μ_k for the k^{th} fuzzy goal $Z_k(x) \lesssim g_k$ can be expressed as follows:

$$\mu_k(x) = \begin{cases} 1 & if\ Z_k(x) \leq g_k \\ \frac{u_k-Z_k(x)}{u_k-g_k} & if\ g_k \leq Z_k(x) \leq u_k \\ 0 & if\ Z_k(x) \geq u_k \end{cases} \quad \ldots\ldots\ldots\ldots(4)$$

Where u_k is the upper tolerance limit for the k^{th} fuzzy goal and $(u_k - g_k)$ is the tolerance (p'_k) which is subjectively chosen.

Construction of Membership Goals

In fuzzy programming approaches, the highest possible value of membership function is 1. Thus, according to the idea of Mohamed [18] the membership functions in Eq. (3) and Eq. (4) can be expressed as the following membership goals:

$$\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- - d_k^+ = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where X, d_k^- , d_k^+ (≥ 0); $d_k^- \times d_k^+ = 0$; $k = 1,2,\ldots,K$ and d_k^- and d_k^+ represent the under deviation and over deviation variable from the aspired levels.

In this paper two FGP approaches have been followed to solve the FMOFGP problems that one is suggested by Mohammad [18] and other is proposed weighted FGP approach.

The Earlier Min Sum Weighted FMOFGP Formulation

Find $x \in X$

So as to Minimize $Z = \sum w_k\ d_k^-$

and satisfy $\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1 \quad for\ Z_k(x) \gtrsim g_k$

$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- - d_k^+ = 1 \quad for\ Z_k(x) \lesssim g_k$

Subject to $Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$, ……………………………(7)

Where X ≥ 0, $d_k^-, d_k^+ \geq 0$; $d_k^- \times d_k^+ = 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; k = 1,2,…..,K; $w_k = 1 / p_k$, $w_k = 1$.

The Proposed Weighted FMOFGP Formulation

Using the concept of min sum GP, the proposed weighted fuzzy goal programming (FGP) model formulation is represented as

Find $x \in X$

So as to Minimize d^-

Subject to $w_k\,\lambda \leq \frac{Z_k(x) - l_k}{g_k - l_k}$, $for\ Z_k(x) \gtrsim g_k$

$w_k\,\lambda \leq \frac{u_k - Z_k(x)}{u_k - g_k}$, $for\ Z_k(x) \lesssim g_k$

$\lambda + d^- - d^+ = 1$

$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$, ……………………………………..(8)

Where X ≥ 0, d^-, $d^+ \geq 0$; $d^- \times d^+ = 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; k = 1,2,…..,K; λ, $\mu \in [0,1]$, $w_k = 1 / p_k$, $w_k = 1$.

It is known that in weighted FGP approach if any deviation variable is not attached to max min operator (λ), then there is no condition that $\lambda \leq 1$. In fact λ can be more than unity because $w < 1$. But the actual achieved level for each objective will never exceed unity. For this, the deviation variables are attached to the max min operator λ where $\lambda = \min(\mu_k(Z_k(x))$.

Construction of Proposed Membership Goals

In the conventional goal programming (GP), each goal is expressed in the form of constraint equation by introducing under deviation and over deviation variable denoted by d_k^-, d_k^+ respectively as follows, $G_k : f_k(x_1,x_2,..,x_n) - d_k^+ + d_k^- = b_k$, k = 1,2, ..,K. In the past Mohammad [18] studied some fuzzy goal programming models by using the concept of conventional G. P. The highest value of Zimmerman's membership function of the fuzzy objective goals is taken as unity. Accordingly the linear membership function in Eq. (3) and Eq. (4) can be expressed as membership goals in Eq. (5) and Eq. (6). Up to now, the literature [7, 13, 24, 25, 27, 30 and 31] has been only dealing with the adaptation of the min sum weighted FGP method by introducing both the deviation variables to the fuzzy goals of fuzzy multi objective linear fractional goal programming (FMOLFGP) problem to achieve highest degree of each of the membership goals by minimizing their deviational variables. In Eq. (5) or Eq. (6), if $d_k^+ > 0$ then d_k^- must be zero, since $d_k^- . d_k^+ = 0$. Thus $\mu_k(Z_k(x)) - d_k^+ = 1$ and it implies that any over deviation from the fuzzy objective goals indicates that the membership value is greater than 1, which is not possible. So d_k^+ should be zero always. On the other hand, in the construction of Zimmerman's type of membership function $\mu_k(Z_k(x))$ of the k[th] fuzzy goals like $Z_k(x) \gtrsim g_k$ given by Eq. (3), we see that, $\frac{Z_k(x) - l_k}{g_k - l_k} \leq 1$ always, when $l_k \leq Z_k(x) \leq g_k$. So the FGP approaches based on Eq. (5) and Eq. (6) are not completely correct always.Thus only under deviation variables need to be introduced in the k[th] membership goals. Also in the conventional G.P under - and / or over deviation variables are included in the achievement function for minimizing them and that depend upon the type of the objective functions to be optimized. In the proposed FGP approach, only under deviation variables are required to be minimized to achieve the aspired levels of the fuzzy goals. The proposed approach will decrease the computation burden in the solution process of the fuzzy multi objective linear fractional goal programming (FMOLFGP) problems. So the membership goals with the aspired level 1 in Eq. (5) and Eq. (6) could be written as:

$$\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (9)$$

$$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (10)$$

The First Proposed Min Sum Weighted FMOFGP Formulation

Find $x \in X$

So as to Minimize $Z = \Sigma\, w_k\, d_k^-$

and satisfy $\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- = 1 \;\; for\; Z_k(x) \gtrsim g_k$

$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- = 1 \;\; for\; Z_k(x) \lesssim g_k$

Subject to $Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix}$ b, ……………………………..(11)

Where $X \geq 0$, $d_k^- \geq 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; $k = 1,2,\ldots..,K$, $w_k = 1 / p_k$, $w_k = 1$.

The Second Proposed Weighted FMOFGP Formulation

Find $x \in X$

So as to Minimize d^-

Subject to $w_k\, \lambda \leq \frac{Z_k(x) - l_k}{g_k - l_k}, for\; Z_k(x) \gtrsim g_k$

$w_k\, \lambda \leq \frac{u_k - Z_k(x)}{u_k - g_k}, for\; Z_k(x) \lesssim g_k$

$\lambda + d^- = 1$

$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix}$ b,…………………………………..(12)

Where $X \geq 0$; $d^- \geq 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; $k = 1,2,\ldots..,K$; $\lambda \in [0,1]$, $w_k = 1 / p_k$, $w_k = 1$.

Linearization Procedure

In the conventional goal programming [16], the deviational variables are included in the achievement function for minimizing them and that depend upon the type of the objective functions to be optimized. In the min sum weighted fuzzy goal programming approach, the FGP approach is used to achieve highest degree of each of the membership goals by minimizing their under deviational variables and thereby obtaining the most satisfactory solution for all decision makers. To assess the relative importance of the fuzzy goals properly, the weighting scheme by Mohamed (Fuzzy Sets and Systems, 89 (1997) 215-222) and also $w_k = 1$, $k = 1,2,\ldots K$ has been used. Now it can be easily realized that the membership goals in Eq. (5), Eq. (6) and also in Eq. (9), Eq. (10) are inherently nonlinear in nature and this may create computational difficulties in the solution process. To avoid such problems, a linearization procedure suggested by B.B. Pal et al., (Fuzzy Sets and Systems, 139 (2003) 395-405) is preferred.

Linearization of Earlier Membership Goals

In fuzzy programming approaches, the highest possible value of membership function is 1. Thus, according to the idea of linearization procedure suggested by Pal et al. (2003) [3], the k^{th} membership goal in Eq. (5) can be represented as

$$L_k\, Z_k(x) - L_k\, l_k + d_k^- - d_k^+ = 1 \; where\; L_k = \frac{1}{g_k - l_k}$$

Now we substitute $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$ in above equation,

$$L_k\,(c_k(x) + \alpha_k) + d_k^-(d_k(x) + \beta_k) - d_k^+(d_k(x) + \beta_k) = L_k'(d_k(x) + \beta_k)$$

Where $L_k' = (1 + L_k l_k)$

Or $C_k(x) + d_k^-(d_k(x) + \beta_k) - d_k^+(d_k(x) + \beta_k) = G_k$ ……………………………..(13)

Where $L_k c_k - L_k' d_k = C_k, G_k = L_k' \beta_k - L_k \alpha_k$

Similarly the linearization procedure of the k^{th} membership goal in Eq. (6) can be obtained.

Now using the method of variable change as presented by Kornbluth and Steuer [15] the goal expression in Eq. (13) can be linearised. Substituting $D_k^- = d_k^-(d_k(x) + \beta_k)$,

$D_k^+ = d_k^+(d_k(x) + \beta_k)$, the linear form of the expression in Eq. (13) is written as

$$C_k(x) + D_k^- - D_k^+ = G_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (14)$$

Where $x, D_k^-, D_k^+ \geq 0$ and $D_k^- \times D_k^+ = 0$ since $d_k^-, d_k^+ \geq 0$, $d_k(x) + \beta_k > 0$.

Now, in making decision, minimization of d_k^- means minimization of

$D_k^- = d_k^- (d_k(x) + \beta_k)$, which is also a non-linear one. It should be pointed out that when a membership goal is fully achieved $d_k^- = 0$ and when its achievement is zero then $d_k^- = 1$ are found in the solution [3]. So, involvement of $d_k^- \leq 1$ in the solution leads to impose the following constraint to the model of the problem:

$$\text{Now } \frac{D_k^-}{(d_k(x)+\beta_k)} \leq 1 \text{gives } D_k^- \leq (d_k(x) + \beta_k)$$

This implies that $-d_k(x) + D_k^- \leq \beta_k$.

Here it may be pointed out that any such constraint corresponding to d_k^+ does not arise in the min sum FGP formulation for solving FMOLFGP problems [3].

The Earlier Min Sum FGP Formulation for Solving FMOLFGP Problems

Now using min sum GP method the fuzzy goal programming model formulation of FMOLFGPP can be represented as:

$$\text{Find } x \in X$$
$$\text{so as to Minimize } Z = \Sigma\, w_k\, D_k^-$$
$$\text{and satisfy } C_k(x) + D_k^- - D_k^+ = G_k$$
$$\text{Subject to } Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b,$$
$$-d_k(x) + D_k^- \leq \beta_k$$
$$X \geq 0,\ D_k^-,\ D_k^+ \geq 0,\ k = 1,2,\ldots K \quad \ldots\ldots\ldots\ldots(15)$$

Here Z represents the achievement function. The weights w_k attached to the under deviational variables D_k^-, are to be obtained by using Mohammad's approach [18] i.e

$$w_k = \begin{Bmatrix} \frac{1}{g_k - l_k} & for\ the\ \mu_k\ in\ (3) \\ \frac{1}{u_k - g_k} & for\ the\ \mu_k in\ (4) \end{Bmatrix}$$ and also the decision maker's choice of the weights as $w_k = 1$; $k = 1,2,\ldots,K$.

Linearization of Proposed Membership Goals

Following the previous linearization procedure of the k^th membership goals in Eq. (5), the expression in Eq. (14) for the proposed k^th membership goals in Eq. (9), can be written as

$$C_k(x) + D_k^- = G_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Where $D_k^- = d_k^-(d_k(x) + \beta_k)$; x, D_k^- , ≥ 0 ; since $d_k^- \geq 0$, $d_k + d^-(x) + \beta_k > 0$.

Now, in making decision, minimization of d_k^- means minimization of

$D_k^- = d_k^- (d_k(x) + \beta_k)$, which is also a non-linear one. It should be pointed out that when a membership goal is fully achieved $d_k^- = 0$ and when its achievement is zero then $d_k^- = 1$ are found in the solution [3]. So, involvement of $d_k^- \leq 1$ in the solution leads to impose the following constraint to the model of the problem:

$$\text{Now } \frac{D_k^-}{(d_k(x)+\beta_k)} \leq 1 \text{gives } D_k^- \leq (d_k(x) + \beta_k)$$

This implies that $-d_k(x) + D_k^- \leq \beta_k$.

The Proposed Min Sum FGP Formulation for Solving FMOLFGP Problems

Now using min sum GP method the proposed fuzzy goal programming model formulation can be represented as:

$$\text{Find } x \in X$$
$$\text{so as to Minimize } Z = \Sigma\, w_k\, D_k^-$$
$$\text{and satisfy } C_k(x) + D_k^- = G_k$$
$$\text{Subject to } Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b,$$
$$-d_k(x) + D_k^- \leq \beta_k$$
$$X \geq 0,\ D_k^- \geq 0,\ k = 1,2,\ldots K \quad \ldots\ldots\ldots\ldots(17)$$

Here Z represents the achievement function. The weights w_k attached to the under deviational variables D_k^-, are to be obtained by using Mohammad's approach [18] i.e

$$w_k = \begin{Bmatrix} \frac{1}{g_k - l_k} & for\ the\ \mu_k\ in\ (3) \\ \frac{1}{u_k - g_k} & for\ the\ \mu_k in\ (4) \end{Bmatrix}$$ and also the decision maker's choice of the weights as $w_k = 1$.

Illustrative Examples

Four examples are used to demonstrate the solution procedures of the fuzzy multi objective linear fractional goal programming problem (FMOLFGPP) by the proposed FGP approaches that two examples are real. Out of these, three are solved by first proposed min sum FGP approach and the remaining is solved by second proposed weighted FGP approach. Also the solutions are compared with the solutions obtained by earlier version of FGP approach.

Example1. The following example is considered to illustrate the proposed FGP approach

$$\text{Maximize } Z_1 = \frac{x_1-4}{-x_2+3}$$
$$\text{Maximize } Z_2 = \frac{-x_1+4}{x_2+1}$$
$$\text{Subject to } -x_1+3x_2 \le 0$$
$$x_1 \le 6$$
$$x_1, x_2 \ge 0. \quad \ldots\ldots\ldots\ldots\ldots(18)$$

First we find the aspiration level for each objective of the above example, following conventional technique [19]. In the solution process, we maximize each objective functions in the numerator and also minimize each objective functions in the denominator with respect to the crisp constraints by using linear programming technique. Therefore we solve

i) $Maximize\ N_1(x) = x_1 - 4$
Subject to $-x_1 + 3x_2 \le 0$
$x_1 \le 6$
$x_1, x_2 \ge 0.$

ii) $Maximize\ N_2(x) = -x_1 + 4$
Subject to $-x_1 + 3x_2 \le 0$
$x_1 \le 6$
$x_1, x_2 \ge 0.$

Similarly,

iii) $Minimize\ D_1(x) = -x_2 + 3$
Subject to $-x_1 + 3x_2 \le 0$
$x_1 \le 6$
$x_1, x_2 \ge 0$

iv) $Minimize\ D_2(x) = x_2 + 1$
Subject to $-x_1 + 3x_2 \le 0$
$x_1 \le 6$
$x_1, x_2 \ge 0$
……………………………………………………(19)

The results are $N_1(6, 0)=2$, $N_2(0, 0)=4$, $D_1(6,2)=1$, $D_2(6,0)=1$. Therefore $(N_1 / D_1) = (2 / 1) = 2$, $(N_2 / D_2) = (4 / 1) = 4$. So the aspiration level (g_1) for the first fractional objective $Z_1= 2$ and that (g_2) for the second fractional objective $Z_2= 4$. Here it may be mentioned that aspiration level or target value of fractional objectives could not be found out by pre-emptive or lexicographic method, as is usually done for multi objective linear goal programming problem.

Now we formulate the fuzzy multi objective fractional goal programming model:

$$Maximize\ \frac{x_1 - 4}{-x_2 + 3} \gtrsim 2$$
$$Maximize\ \frac{-x_1 + 4}{x_2 + 1} \gtrsim 4$$
$$\text{Subject to } -x_1+3x_2 \le 0$$
$$x_1 \le 6$$
$$x_1, x_2 \ge 0 \ldots\ldots\ldots\ldots\ldots(20)$$

Now we attach some tolerances p_1 and p_2 respectively to aspiration levels ($g_1 = 2$, $g_2 = 4$) of the first and second fuzzy goals, say $p_1= 3$, $p_2= 6$. Now we formulate the min sum weighted fuzzy multi objective fractional goal programming problem by introducing under deviational variables d_k^-, k = 1, 2 where aspiration level for each membership goals is 1.

Therefore the min sum weighted fuzzy multi objective fractional goal programming can be written as:

Find x_1, x_2

so as to satisfy Minimize $w_1 d_1^- + w_2 d_2^-$

$$Subject\ to\ \frac{\frac{x_1 - 4}{-x_2 + 3} - (2-3)}{3} + d_1^- = 1$$
$$\frac{\frac{-x_1 + 4}{x_2 + 1} - (4-6)}{6} + d_2^- = 1$$
$$-x_1+3x_2 \le 0$$
$$x_1 \le 6$$

Where $x_1, x_2, d_k^- \ge 0$, k = 1,2 ……………………………..(21)

Here w_k denotes the weight which is given to the under deviational variables d_k^-. Following the linearization procedure based on Eq. (16), the min sum weighted fuzzy multi objective linear fractional goal programming (FMOLFGP) problem is formulated as:

Find $x_1, x_2 (\in X)$

So as to satisfy Minimize $w_1 D_1^- + w_2 D_2^-$

$$x_1 + 2x_2 + D_1^- = 10$$
$$-x_1 - 4x_2 + D_2^- = 0$$
$$D_1^- + 3x_2 \le 9$$
$$D_2^- - 6x_2 \le 6$$
$$-x_1+3x_2 \le 0$$
$$x_1 \le 6$$

Where $x_1, x_2, D_k^- \ge 0$, k = 1,2 ………………………(22)

The optimal solutions of (22) for different choices of weights are given in the table 1.

Table 1- *Solution of the FMOLFGP problem for example 1*

Tolerance (p_k)	Weight (w_k)	Earlier FGP approach	Proposed FGP approach
p_1= 3, p_2 = 6	$w_k = 1/p_k$	$Z_1(6,0) = 2/3$, $Z_2(6,0) = -2$	$Z_1(6,2) = 2$, $Z_2(6,2) = -2/3$
	$w_k = 1$	$Z_1(1,0) = -1$, $Z_2(1,0) = 3$	$Z_1(1,0) = -1$, $Z_2(1,0) = 3$
p_1= 3, p_2 = 5	$w_k = 1/p_k$	$Z_1(6,1) = 1$, $Z_2(6,1) = -1$	$Z_1(6,1) = 1$, $Z_2(6,1) = -1$
	$w_k = 1$	$Z_1(1,0) = -1$, $Z_2(1,0) = 3$	$Z_1(1,0) = -1$, $Z_2(1,0) = 3$

Example 2

The more to illustrate the potential use of the proposed approach, the numerical example 2 is considered. Suppose under the same set of constraints of the previous example, the decision-maker (DM) desires to

$$\text{Maximize } Z_1 = \frac{x_1-4}{-x_2+3}$$
$$\text{Minimize } Z_2 = \frac{-x_1+4}{x_2+1}$$
$$\text{Subject to } -x_1+3x_2 \le 0$$
$$x_1 \le 6$$
$$x_1, x_2 \ge 0. \quad \ldots\ldots\ldots\ldots(23)$$

We know that Min Z_2 = Max $(1-Z_2)$ = Max Z_2'

Then the second fractional objective of the example 2 can be written as

$$\text{Max } Z_2' = \frac{x_1+x_2-3}{x_2+1} \quad \ldots\ldots\ldots\ldots(24)$$

In Table 2, the aspiration levels for the second objective in Eq. (23) and Eq. (24) by following the procedure based on the Eq. (19) has been shown.

Table 2

Solution

Aspiration level (g_2)	Aspiration level (g_2')
$g_2 = (N_2(6,0) / D_2(6,2)) = -2/3$	$g_2' = (N_2'(6,2) / D_2'(6,0)) = 5$

Then the fuzzy goals of the problem 2 become

$$\frac{x_1-4}{-x_2+3} \gtrsim 2$$
$$\frac{x_1+x_2-3}{x_2+1} \gtrsim 5$$

Now we attach some tolerances to the aspiration levels say p_1' = 3 and p_2' =6. Considering the linearization procedure based on Eq. (16), the resultant min sum weighted FGP problem is formulated as

Find $x_1, x_2 (\in X)$

So as to satisfy Minimize $w_1 D_1^- + w_2 D_2^-$

$$x_1 + 2x_2 + D_1^- = 10$$
$$x_1 - 4x_2 + D_2^- = 8$$
$$D_1^- + 3x_2 \le 9$$
$$D_2^- - 6x_2 \le 6$$
$$-x_1+3x_2 \le 0$$
$$x_1 \le 6$$

Where $x_1, x_2, D_k^- \ge 0$, k = 1,2 …… …………(25)

The optimal solutions of (25) for different choices of weights are given in the table 3.

Table 3- *Solution of the FMOLFGP problem for example 2*

Tolerance (p_k)	Weight (w_k)	Earlier FGP approach	Proposed FGP approach
p_1= 3, p_2 = 6	$w_k = 1/p_k$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$
	$w_k = 1$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$
p_1= 3, p_2 = 5	$w_k = 1/p_k$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$
	$w_k = 1$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$	$Z_1(6,0) = 2/3$, $Z_2'(6,0) = 3$

Example 3

This example considered by Ching-Ter Chang (2009) [7] is used to clarify the effectiveness of the proposed approach.

The fractional goal programming problem is represented as:

Max Z = (the total user satisfaction / total investment budget)

$$\text{Max } Z = \frac{(2.16\,x_1+1.095\,x_2+1.4\,x_3+1.7\,x_4+.69\,x_5+.544\,x_6+1.3\,x_7+.64\,x_8+1.7\,x_9+1.34\,x_{10}+.64\,x_{11}+2.04\,x_{12})}{(.1\,x_1+.2\,x_2+.2\,x_3+.2\,x_4+.2\,x_5+.3\,x_6+.2\,x_7+.1\,x_8+.2\,x_9+.1\,x_{10}+.2\,x_{11}+.2\,x_{12})} \quad \ldots\ldots(26)$$

Subject to $3x_1+2x_2+3x_3+3x_4+2x_5+x_6+2x_7+3x_8+4x_9+3x_{10}+2x_{11}+x_{12} \le 15$ ……………(27)

(Manpower constraint)

$.1x_1+.2x_2+.2x_3+.2x_4+.2x_5+.3x_6+.2x_7+.1x_8+.2x_9+.1x_{10}+.2x_{11}+.2x_{12} \le 1.6$ ……………..(28)

(Capital constraint)

$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}+x_{11}+x_{12} \ge 6$ (At least six E-Learning Systems) ……(29)

Where $x_k \ge 0$; k = 1,2,…,12.

Following the procedures based on the Eq. (19) the aspiration level (g) for the above objective in Eq. (26) is given by $g = \frac{N(x_1, x_{12})}{D(x_1, x_{12})} = \frac{N(2.8,\ 6.6)}{D(4.5,\ 1.5)} = 26$.

Then the fuzzy goal of the problem become

$$\frac{(2.16\,x_1+1.095\,x_2+\cdots+.64\,x_{11}+2.04\,x_{12})}{(.1\,x_1+.2\,x_2+\cdots+.2\,x_{11}+.2\,x_{12})} \gtrsim 26$$

Assume that the tolerance (p) of the fuzzy objective goal is 9. The membership function of the problem is obtained as follows:

$$\mu = \frac{\frac{2.16\,x_1+1.095\,x_2+\cdots+2.04\,x_{12}}{(.1\,x_1+.2\,x_2+\cdots+.2\,x_3)} - 17}{9} \quad \ldots\ldots\ldots(30)$$

Therefore the min sum weighted FGP model of the above problem is given by

Minimize $\Sigma w\, d^-$

$$\text{Subject to } \frac{\frac{2.16\,x_1+1.095\,x_2+\cdots+2.04\,x_{12}}{(.1\,x_1+.2\,x_2+\cdots+.2\,x_{12})} - 17}{9} + d^- = 1 \quad \ldots\ldots\ldots(31)$$

Equation (27)-(29)

Where $x_k \ge 0$, k =1,2,…,12; $d^- \ge 0$; w = 1 / p and w = 1.

Following the linearization strategy based on Eq. (16), Eq. (31) can be converted as:

$$(2.16\,x_1 + 1.095\,x_2 + \ldots + 2.04\,x_{12}) + D^- = 26\,(.1\,x_1 + .2\,x_2 + \ldots + .2\,x_{12}) \quad \ldots\ldots(32)$$

Where $D^- = 9\,d^-\,(.1\,x_1 + .2\,x_2 + \ldots + .2\,x_{12})$.

Now $d^- \le 1 \Rightarrow D^- \le 9\,(.1\,x_1 + .2\,x_2 + \ldots + .2\,x_{12})$; $x_k \ge 0$, k=1,2,…,12; $D^- \ge 0$

The min sum weighted FGP model formulation of FLFGPP is given by

Minimize $\Sigma w\, D^-$

Subject to $(2.16\,x_1 + 1.095\,x_2 + \ldots + 2.04\,x_{12}) + D^- = 26\,(.1\,x_1 + .2\,x_2 + \ldots + .2\,x_{12})$

$3x_1+2x_2+3x_3+3x_4+2x_5+x_6+2x_7+3x_8+4x_9+3x_{10}+2x_{11}+x_{12} \le 15$

$.1x_1+.2x_2+.2x_3+.2x_4+.2x_5+.3x_6+.2x_7+.1x_8+.2x_9+.1x_{10}+.2x_{11}+.2x_{12} \le 1.6$

$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}+x_{11}+x_{12} \ge 6$

$D^- \le 9\,(.1\,x_1 + .2\,x_2 + \ldots + .2\,x_{12})$ ……………………………(33)

Where $x_k \ge 0$, k=1,2,…,12; $D^- \ge 0$; w = 1 / p and w = 1.

The solution is $x_1 = 4.5$, $x_{12} = 1.5$, Z = 17.04.

Next we assume that g = 12 and p = 9. Then the solution is $x_{10} = 4.472$, $x_{11} = .0545$, $x_{12} = 1.472$, Z = 11.99. Therefore the setting of aspiration level (g) of above fuzzy fractional objective goal will be 10 with tolerance (p) equal to 9 in the sense that solution provides objective value closer to their respective aspiration levels. Then the solutions of linearised fuzzy fractional goal programming problem in Eq. (33) using proposed FGP approach under different weighting scheme are listed in Table 4.

Table 4- *Solution*

Aspiration level	Tolerance	Weight (w)	solution
10	9	w = 1 /p	$Z(x_{11}=.1714,\ x_{12}=5.828,\ x_1=..,\ x_{10}=0) = 10.07$
		w = 1	$Z(x_{11}=.1714,\ x_{12}=5.828,\ x_1=..,\ x_{10}=0) = 10.07$

Where the value of membership function = 1 i.e. $\mu(Z_k(x)) = 1$.

Comparison with Earlier FGP Method

A comparison between the solutions of FMOLFGP problems in examples 1, 2 and 3 by using FGP procedure based in Eq. (15) and in Eq. (17) are listed in Table 5.

Table 5- *Comparison*

Type	Earlier linearised FGP method	Proposed linearised FGP method
Ex. 1	$Z_1(6, 0) = 2/3$, $Z_2(6, 0) = -2$	$Z_1(6,2) = 2$, $Z_2(6,2) = -2/3$
Ex. 2	$Z_1(6, 0) = 2/3$, $Z_{2'}(6, 0) = 3$	$Z_1(6,0) = 2/3$, $Z_{2'}(6, 0) = 3$
Ex. 3	$Z(x_{12} = 6,\ x_k = 0) = 10.20$, k = 1, 2,…., 11; g = 10.	$Z(x_{11}=.1714,\ x_{12}= 5.828,\ x_k=0) = 10.07$, k = 1, 2,….,10; g = 10.

Example 4: Another real example is considered to make clear the usefulness of the proposed approach. In this example, the academic resource allocation with limited resources like infrastructure, human resources, equipment in National Institute of Technology Agartala (NITA) is finding out by proposed weighted FGP technique with the concept of min sum GP technique. The National Institute of Technology, Agartala (NITA) was established in 1st April, 2006 with Deemed to be University status and centre of excellence, in the north eastern state Tripura of India. The Institute NITA offers the various UG, PG (M. Tech, M.Sc.), PhD, MCA courses. In order to expound the model, the cases of various departments viz. Civil Engineering (CE), Mechanical Engineering (ME), Electrical Engineering (EE), Computer Science & Engineering (CSE), Electronics & Communication Engineering (ECE), Electronics & Instrumentation Engineering (EIE), Chemical Engineering(CHEM. ENGG.), Production Engineering (PE), Mathematics, Physics, Chemistry, Humanities has been considered. At present, the total student strength of the institute NITA = 1741. According to UGC (University Grant Commission) rule (RF: Student = 1:12), the number of regular faculties comes out to be 1741 ÷ 12 = 145. But the existing number of regular faculties = 85 with the break up Director = 1, Professor = 4, Associate Professor = 20, Assistant Professor = 60. Apart from these 35 numbers of contractual faculties has been engaged. In the Institute some departments like civil (CE), electrical (EE), electronics and communication (ECE), computer science (CSC) take common classes of the first year students in one semester only where the number of students in the first year = 546 (according to 2010-2011 session). The goal of this paper is that the number of regular faculties, contractual faculties of the institute NITA is optimized according to the actual need of the institute by giving special attention in respect of the idle semester. Therefore, the proposed approach may serve as an effective decision support tool for the distribution and lateral academic resource allocation of the institute NITA as compared the usual approach. In this paper actual data are used to find an optimal solution for the possible implementation.

The parameters and decision variables are as follows:

Parameters:- i) FS_{ijt} = Number of regular faculties (RF) in the department i (i=1,2,..I), rank j (j=1,2,…J) at the time period t; ii) CF_{it} = Number of contractual faculties (CF) in the department i (i=1,2,.,.I), at the time period t; iii) SC_{it} = Total number of students or total weakly class load in the department i at the time period t; v) S_{ijt} = Annual (average) salary for a RF in the department i, rank j at the time period t; vi) M_{it} = Annual remuneration for a contractual faculties (CF) in the department i at the time period t; viii) $fs_{ij,t+1}$ = Number of new regular faculties (NRF) to be employed at the department i, rank j at the next time period t+1; ix) Q_{it} = CF and RF ratio in the department i at the time period t; x) T_{it} = (Student and RF ratio) / (weekly teaching – learning process hour per teacher and RF ratio) in the department i at the time period t; xii) B_t = Total pay roll budget at the time period.

Decision variables:- i) F_{ijt} = Number of RF required in the department i, rank j for smooth functioning of the departments at the time period t; ii) C_{it} = Number of CF required in the department i at the time period t. It is noted that according to the rule of university grant commission (UGC) of India, the regular faculty (RF) - student ratio is 1: 12 and RF - average weekly teaching learning process hour per teacher ratio is 1: 14.

Description of Fuzzy Goals and Crisp Goals

The fuzzy goals and the crisp goals are as follows:

Fuzzy Goals:- i)RF goal: $F_{ijt} \gtrsim FS_{ijt}$; i =1,2,…,I; j = 1,2,…,J;

ii) CF / RF ratio goal:

$$C_{it} - Q_{it}\sum_{j=1}^{J} F_{ijt} \gtrsim CF_{it}\,; i = 1,2,\dots,I\,;$$

iii)Budget Goal:

$$\left[\sum_{j=1}^{J} S_{ijt} F_{ijt} + \sum_{i=1}^{I} M_{it} C_{it}\right] \lesssim B_t\,;$$

Crisp goal:- (RF / Student) ratio or (RF / weekly Class Load) ratio goal:

$$SC_{it} - T_{it}\sum_{j=1}^{J} F_{ijt} + d_k^- = 0\,; i = 1,2,..,I;\ k = 1,2,\dots,K$$

………………………………………………………(34)

Faculty Flow: In order to balance the level of goal variables in each department i at the next planning period t +1 in different rank j, the flow is considered, which represents that total number of RF at time period t + 1 will be equal to the sum of the number of RF who are promoted from rank j-1 to rank j at the time period t + 1, the number of those who remain at the same rank from period t to t + 1 and number of new RF who are newly recruited at time period t + 1.

For data collection related to research parameters, we study the official documents in the academic year 2010-2011 which has been collected from the Institute's finance section, establishment section and examination section. Here the existing number of regular faculties and contractual faculties is considered as the lower tolerances (LT) for the membership goals of RF and CF. The aspiration levels or target levels of both regular and contractual faculties are evaluated by experts of the Institute. The lower tolerances (LT) and aspiration levels (AL) of RF, CF are presented in the Table 6. The pay-roll budget for the financial year 2010-2011, INR[1] 1320.50 Lakh is considered as the aspiration level for the fuzzy budget goal. The upper tolerance (UT) limit is determined by increasing 15 % of the current year budget, which is conventionally taken into account with a view to increase of salary / Dearness Allowance etc. The estimated amount is INR 1518.58 Lakh which is the estimated pay-roll budget allocation for the next financial year 2011-2012. The data for annual average salaries of RF and remuneration for CF were obtained from the Institute's finance section which is described in the Table 7. The number of students, the number of class load and RF-student ratios, RF - number of class's ratios of the concerned departments are shown in the Table 8.

[1] Indian Rupee

Table 6

Rank	CE		ME		EE		CSC	
	LT	AL	LT	AL	LT	AL	LT	AL
Professor	2	4	1	4	0	3	0	2
Associate Professor	5	7	4	7	4	6	1	5
Assistant Professor	11	14	9	15	8	11	10	14
Contractual Faculty	2	3	5	8	4	5	7	8

Rank	**ECE**		**EIE**		**CHEM.ENGG.**		**PE**	
	LT	AL	LT	AL	LT	AL	LT	AL
Professor	0	3	0	0	0	0	0	1
Associate Professor	0	6	0	1	0	1	1	1
Assistant Professor	7	12	1	2	1	2	5	5
Contractual Faculty	3	5	0	0	2	2	2	3

Rank	**MATH**		**PHYSICS**		**CHEMISTRY**		**HUMANITIES**	
	LT	AL	LT	AL	LT	AL	LT	AL
Professor	0	1	0	1	0	1	1	1
Associate Professor	2	3	1	2	1	2	1	1
Assistant Professor	3	6	2	4	2	4	1	2
Contractual Faculty	3	4	3	5	2	3	2	3

Table 7:

Rank	Salary / Remuneration
Director + Professor	INR 1192072.8
Associate Professor	INR 1066576.80
Assistant Professor	INR 579756.40
Contractual Faculty	INR 331885.71

Table 8:

Here ratios of the respective departments are shown.

Department	No. of student (ST)	CF: RF	RF: ST
CE	289	1: 24	1: 12
ME	258	2: 21	1: 12
EE	195	3:16	1: 12
CSE	190	10: 20	1: 12
ECE	190	4: 20	1: 12

Department	No. of Classes	CF: RF	RF: weekly teaching learning process hour per teacher
EIE	24	0:2	1:14
CHEM. ENGG.	33	0:2	1:14
PE	90	1:6	1:14
Mathematics	132	1:14	1:14
Physics	90	1:7	1:14
Chemistry	90	1:7	1:14
Humanities	48	1:4	1:14

Proposed Weighted Fuzzy Goal Programming (FGP) Model Formulation for the Academic Resource Allocation of the Institute NITA

Now following the Eq. (34) and using min sum GP concept, the proposed weighted fuzzy goal programming (FGP) model formulation for the academic resource allocation of the Institute NITA under proposed approach is represented as

Minimize $d_1^- + d_2^- + d_3^- + d_4^- + d_5^- + d_6 + d_7^- + d_8^- + d_9^- + d_{10}^- + d_{11}^- + d_{12}^- + d_{13}^-$

$$\text{Subject to } w_1\lambda \le \frac{f_{11}-2}{2};\ w_2\lambda \le \frac{f_{12}-5}{2};\ w_3\lambda \le \frac{f_{13}-11}{3}$$

$$w_4\lambda \le \frac{f_{21}-1}{3};\ w_5\ \lambda \le \frac{f_{22}-4}{3};\ w_6\ \lambda \le \frac{f_{23}-9}{6}$$

$$w_7\ \lambda \le \frac{f_{31}-0}{3};\ w_8\ \lambda \le \frac{f_{32}-4}{2};\ w_9\ \lambda \le \frac{f_{33}-8}{3}$$

$$w_{10}\lambda \le \frac{f_{41}-0}{2};\ w_{11}\lambda \le \frac{f_{42}-1}{4};\ w_{12}\ \lambda \le \frac{f_{43}-10}{4}$$

$w_{13}\ \lambda \le \frac{f_{51}-0}{3}$; $w_{14}\ \lambda \le \frac{f_{52}-0}{6}$; $w_{15}\ \lambda \le \frac{f_{53}-7}{5}$

$f_{61} = 0$; $w_{16}\ \lambda \le \frac{f_{62}-0}{1}$; $w_{17}\ \lambda \le \frac{f_{63}-1}{1}$

$f_{71} = 0$; $w_{18}\lambda \le \frac{f_{72}-0}{1}$; $w_{19}\ \lambda \le \frac{f_{73}-1}{1}$

$w_{20}\ \lambda \le \frac{f_{81}-0}{1}$; $f_{82} = 1$; $f_{83} = 5$

$w_{21}\ \lambda \le \frac{f_{91}-0}{1}$; $w_{22}\ \lambda \le \frac{f_{92}-2}{1}$; $w_{23}\ \lambda \le \frac{f_{93}-3}{3}$

$w_{24}\lambda \le \frac{f_{101}-0}{1}$; $w_{25}\lambda \le \frac{f_{102}-1}{1}$; $w_{26}\lambda \le \frac{f_{103}-2}{2}$

$w_{27}\ \lambda \le \frac{f_{111}-0}{1}$; $w_{28}\ \lambda \le \frac{f_{112}-1}{1}$; $w_{29}\ \lambda \le \frac{f_{113}-2}{2}$

$f_{121} = 1$; $f_{122} = 1$; $w_{30}\ \lambda \le \frac{f_{123}-1}{1}$

$w_{31}\ \lambda \le \frac{(24\,c_1-(f_{11}+f_{12}+f_{13}))-2}{1}$; $w_{32}\ \lambda \le \frac{(21\,c_2-2(f_{21}+f_{22}+f_{23}))-5}{3}$

$w_{33}\ \lambda \le \frac{(16\,c_3-3(f_{31}+f_{32}+f_{33}))-4}{1}$; $w_{34}\ \lambda \le \frac{(20\,c_4-10(f_{41}+f_{42}+f_{43}))-7}{1}$

$w_{35}\ \lambda \le \frac{(20\,c_5-4(f_{51}+f_{52}+f_{53}))-3}{2}$; $c_6 = 0$; $c_7 = 0$; $w_{36}\ \lambda \le \frac{(6\,c_8-(f_{81}+f_{82}+f_{83}))-2}{1}$

$w_{37}\ \lambda \le \frac{(14\,c_9-(f_{91}+f_{92}+f_{93}))-3}{1}$; $w_{38}\ \lambda \le \frac{(7\,c_{10}-(f_{101}+f_{102}+f_{103}))-3}{2}$

$w_{39}\ \lambda \le \frac{(7\,c_{11}-(f_{111}+f_{112}+f_{113}))-2}{1}$; $w_{40}\ \lambda \le \frac{(4\,c_{12}-(f_{121}+f_{122}+f_{123}))-2}{1}$

$(12 / 289)\ (f_{11} + f_{12}+ f_{13}) + d_1^- = 1$; $(12 / 258)\ (f_{21} + f_{22}+ f_{23}) + d_2^- = 1$;

$(12 / 195)\ (f_{31} + f_{32}+ f_{33}) + d_3^- = 1$; $(12 / 190)\ (f_{41} + f_{42}+ f_{43}) + d_4^- = 1$;

$(12 / 190)\ (f_{51} + f_{52}+ f_{53}) + d_5^- = 1$; $(14 / 24)\ (f_{61} + f_{62}+ f_{63}) + d_6^- = 1$;

$(14 / 33)\ (f_{71} + f_{72}+ f_{73}) + d_7^- = 1$; $(14 / 90)\ (f_{81} + f_{82}+ f_{83}) + d_8^- = 1$;

$(14 / 132)\ (f_{91} + f_{92}+ f_{93}) + d_9^- = 1$; $(14 / 90)\ (f_{101} + f_{102}+ f_{103}) + d_{10}^- = 1$;

$(14 / 90)\ (f_{111} + f_{112}+ f_{113}) + d_{11}^- = 1$; $(14 / 48)\ (f_{121} + f_{122}+ f_{123}) + d_{12}^- = 1$;

$w_{41}\ \lambda \le (7.67 - .06\ (f_{11} + f_{21}+ f_{31}+\ldots\ldots+ f_{101} + f_{121}+ f_{131}) - .05\ (f_{12} + f_{22}+ f_{32}+\ldots..+ f_{102} + f_{112}+ f_{122}) - .03\ (f_{13} + f_{23}+ f_{33}+\ldots.+ f_{103} + f_{113}+ f_{123}) - .01\ (c_1 + c_2+ c_3+\ldots.+ c_{11} + c_{12}+ c_{13})$;

$\lambda + d_{13}^- = 1$……………………………………………………….(35)

Where, $d_k^- \ge 0$; $k = 1,2,,\ldots,13$; f_{ij} , $c_i \ge 0$; $i = 1,2,\ldots,12$; $j = 1,2,3$; $\lambda\in[0,1]$; the weights are taken as $w_k = 1 / p_k$ and $w_k = 1$.

The optimal solution of the Eq. (35) is given in the following table 9.

Table 9- *Resulting solution:*

Departments	$w_k = 1/ p_k$				$w_k = 1$			
	(Prof.)	(Asso.Prof.)	(Assi.Prof.)	(C.F)	(Prof.)	(Asso.Prof.)	(Assi.Prof.)	(C.F)
CE	2	5	16	1	3	6	15	1
ME	8	4	9	2	5	5	11	2
EE	0	7	8	3	1	6	9	3
CSC	4	1	10	3	2	2	11	3
ECE	8	0	7	3	5	2	9	3
EIE	0	0	1	0	0	0	2	0
CHEM.ENG.	0	1	1	0	0	1	2	0
PE	0	1	5	1	0	1	5	1
MATH	3	2	3	1	3	2	4	1
PHYSICS	2	1	2	1	2	1	3	1
CHEMISTRY	2	1	2	1	2	1	3	1
HUMANITIES	1	1	1	1	1	1	1	1

Where $\lambda = .35$

Solution of the Problem 4 Using the Earlier FGP Approach

Similarly, the solution of the proposed weighted fuzzy goal programming (FGP) model formulation for the academic resource allocation of the Institute NITA under earlier FGP approach is given in the table 10, where weights are taken as $w_k = 1 / p_k$; p_k is subjectively chosen and $w_k = 1$.

Table 10- *Resulting solution*

Departments	$w_k = 1/p_k$				$w_k = 1$			
	(Prof.)	(Asso.Prof.)	(Assi.Prof.)	(C.F)	(Prof.)	(Asso.Prof.)	(Assi.Prof.)	(C.F)
CE	3	8	13	1	5	8	14	1
ME	3	7	11	2	5	7	15	2
EE	1	5	10	3	3	6	11	4
CSC	2	2	12	2	2	5	14	4
ECE	2	4	10	3	3	6	12	4
EIE	0	1	2	0	0	1	2	0
CHEM.ENG.	0	1	2	0	0	1	2	0
PE	1	1	5	2	1	1	5	2
MATH	2	3	4	1	2	3	6	1
PHYSICS	2	2	3	2	2	2	4	2
CHEMISTRY	2	3	3	2	2	2	4	2
HUMANITIES	1	1	2	2	1	1	2	2

Where $\lambda = 1$

Comparison

Comparing the solution of the problem 4 obtained from the table 9 and table 10 it has been shown that solution obtained from the table 9 is suitable when $w_k = 1$. These solutions are optimal solution for the academic resource allocation of the institute NITA.

It should be noted that some departments of the institute NITA like CE, EE, CSC and ECE take the classes of the first year students in one semester only whereas the mechanical (ME) department take the classes of the first year students in both the semesters. The data of the class loads of various departments has been collected from academic section of the institute NITA. The fuzzy goal programming (FGP) approach to academic resource allocation problem of the institute NITA, presented in this paper provides a new look into the way of allocation of resources within a technical institute in an imprecise decision making environment which optimizes the idle semester for faculties. At present if the required number of regular faculty (RF) could not be employed due to the administrative reasons then the management provides a number of contractual faculties (CF) in place of regular faculties.

But in this paper it has been shown that if the regular faculties (RF) are not engaged due to limitations of the budget and also to avoid the idle semester of the institute, the new post of the contractual faculties obtained from the optimal solution has been created. These posts are always permanent contractual but never abolish. This optimization in respect of faculty has been achieved here by giving special attention in respect of the idle semester / time. Under the present condition of available infrastructure facility in the institute in respect of accommodating the student in class rooms, the results found is realistic. When the sufficient infrastructure will be available, the faculties requirement much higher.

There are other limitations and intangible factors that the Management may consider implementing the solution, such as strategic movements during the implementation period.

Conclusion

In this paper it has been observed that the min sum weighted FGP technique with only under deviation variables d_k^- (proposed technique) gives the better solution for fuzzy multi objective linear fractional goal programming problem (FMOLFGPP) than the earlier adopted FGP technique, in the sense that the solution provides objective values closer to their respective aspiration levels. The proposed approach can be easily applied to decision / management problems, which involve the achievement of fuzzy goals, some of which are met and some are sufficiently close to the target levels. Through solving the practical / numerical problem, decision makers will obtain more assurance in the usefulness of the proposed model for their fuzzy multiple objective linear fractional goal programming (FMOLFGP) problems. The capable results encourage the need for further research on the FMOLFGP. In the problem 4, this paper addresses a new approach of weighted fuzzy goal programming (FGP) methodology with the concept of min sum GP (where only under deviation variables are introduced to the goal constraints) for determining possible implementable solutions to teaching staff (regular and contractual) selection problems in National Institute of Technology, Agartala (NITA). Also the proposed approach decreases the computation burden in the solution process of the FMOLFGP problems. In this paper, the software LINGO (version 11) is used to solve the FMOLFGP problems.

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