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Fuzzy programming technique to solve multi-objective solid transportation problem with some non-linear membership functions

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Abstract- The Multi-objective Solid Transportation Problem (MSTP) refers to a special class of vector-minimum linear programming problems, in which constraints are all equality type and the objectives, are conflicting in nature. A generalization of multi-objective solid transportation problem, in which the supply, demand and capacity constraints are not only equality type but also of inequality type is considered. All methods either generate a set of non-dominated solution or find a compromise solution. In this paper, fuzzy programming technique is applied to solve multi-objective solid transportation problem. Special type of non-linear membership functions - Hyperbolic and Exponential are used to represent objective function into fuzzy environment. It gives an optimal compromise solution. The obtained result has been compared with the solution obtained by using a linear membership function. The method is illustrated with a numerical example.

Keywords- Solid transportation problem, Fuzzy programming, Linear and nonlinear membership functions, Multiple criteria programming.

1. Introduction

In most of the cases, it is required to solve the problem taking into account more than one decision criterion, thus giving place to the MSTP. A variety of approaches has been developed by many authors for the Linear Multi-objective Transportation Problem [1, 2, 4, 7, 8] and Bit et al [3] proposed a fuzzy programming approach to MSTP. Leberling [6] used a special- type nonlinear (hyperbolic) membership function for the vector maximum linear programming problem. He showed that solutions obtained by fuzzy linear programming with this type of non-linear membership function are always efficient. Dhingra and Moskowitz [5] defined other types of the non-linear (exponential, quadratic and logarithmic) membership functions and applied them to an optimal design problem. Verma, Biswal and Biswas [9] used the fuzzy programming technique with some non-linear (hyperbolic and exponential) membership functions to solve a multi-objective transportation problem.

2 Mathematical model

In a typical transportation problem, a homogeneous product is to be transported from each of m sources to n destinations. The sources are production facilities, warehouses, or supply point, characterized by available capacities a_i (i = 1,2,…, m). The destinations are consumption facilities, warehouses, or demand points, characterized by required levels of demand b_j (j = 1,2, …, n), let e_k (k = 1,2, …, K) be the units of this product which can be carried by k different modes of transport called conveyances, such as trucks, air freight, freight trains, ship etc. A penalty C^p_{ijk} is associated with transportation of a unit of the product from sources i to destination j by means of the k-th conveyance for the p-th criterion. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under used capacity, etc. A variable X_{ijk} represents the unknown quantity to be transported from origin O_i to destination D_j by means of the k^{th} conveyance. In the real would, however, solid transportation problems are not all-single objective type. We may have more than one objective in a solid transportation problem.

A Multi-objective solid transportation problem can be represented as:

Minimize

$$Z_p = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k} c^p_{ijk}\, x_{ijk} \text{, for } p = 1,2,\dots,P \quad (1)$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i, \quad i = 1,2,\dots,m \quad (2)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j, \quad j = 1,2,\dots,n \quad (3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k, \quad k = 1,2,\dots,K \quad (4)$$

$x_{ijk} \geq 0$ for all i, j, k (5)

Where the subscript on Z_p and superscript on C^p_{ijk} denote the P^{th} penalty criterion; a_i>o for all i,

b_j>o, for all j, e_k>o for all k, $C^p_{ijk} \geq$ o for all i, j, k, p, and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{k} e_k \text{ (Balanced condition)}$$

For P=1 problem (1-.5) reduces to a single objective solid transportation problem.

The balanced condition is treated as a necessary and sufficient condition for the existence of a feasible solution to problem (1-5). The solid transportation problem (STP) is a generalization of the classical transportation problem. The solid transportation problem may be considered as special case of linear programming problem. The necessity of considering this special type of transportation problem arises because many industrial problems are shaped in this special form. It may be noted that the necessity of the solid transportation problem arises when there are heterogeneous conveyances available for the shipment of goods. The solid transportation problem can be converted to a classical transportation problem by considering a single type of conveyance. The solid (three index/dimensional) transportation problem is of much use in public distribution systems. The LINDO (Linear Interactive and Discrete Optimization) package handles the transportation problem in an explicit equation form and thus solves the problem as a standard linear programming problem.

Definitions:

Efficient solution

A feasible solution $\underline{x} = \{\underline{x}_{ijk}\} \in X$ is said to be an efficient solution [12] of the multi-objective solid transportation problem (1-5) if there is no other feasible solution that is in the usual sense $x = \{x_{ijk}\} \in X$ such that,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k} c^p_{ijk} x_{ijk} \le \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k} c^p_{ijk} \underline{x}_{ijk} \quad \text{for all } p$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k} c^p_{ijk} x_{ijk} < \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k} c^p_{ijk} \underline{x}_{ijk} \quad \text{for at least one } p$$

Optimal compromise solution

An optimal compromise solution [12] of the multi-objective transportation problem (1-5) is a solution $\underline{x} = \{\underline{x}_{ijk}\} \in X$ which is preferred by the decision maker to all other solutions, taking into consideration all criteria contained in the multi-objective functions. It is generally accepted that an optimal compromise solution has to be an efficient solution according to the definition of efficient solution. For a real world problem, the complete solution (set of all efficient solutions) is not always necessary. We need only a procedure, which finds an optimal compromise solution.

3 Fuzzy linear programming

The first attempt made to fuzzify a linear program is due to Zimmermann [12, 13]. Fuzzy linear programming with multiple objective functions was introduced by Zimmermann [11]. In this programming, fuzzy set theory has been applied to the linear multi criteria decision-making problems. It uses the linear fuzzy membership function. Zadeh [10] first introduced the concept of a fuzzy set. In multi criteria decision-making problems, the objective functions are represented by fussy sets and the decision set is defined as the intersection of all the fuzzy sets and constraints. The decision rule is to select the solution having the highest membership of the decision set. Zimmermann [11] presents the application of fuzzy linear programming approaches to the linear vector maximum problem. He shows that solutions obtained by fuzzy linear programming give always-efficient solutions and also an optimal compromise solution. We apply the fuzzy programming technique to solve multi-objective linear, as well as nonlinear programming problems.

4 Fuzzy programming technique to multi-objective solid Transportation problem.

The Multi-objective solid transportation problem can be considered as a vector minimum problem. Let U_p and L_p be the upper and lower bound for the p^{th} objective, where lower bound indicates aspiration level of achievement and upper bound indicates highest acceptable level of achievement for the p^{th} objective respectively.

Let $d_p = (U_p - L_p)$ = degradation allowance for the p^{th} objective.

Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. Our next step is to transform the fuzzy model into a "Crisp" model.

➢ **Algorithm**

Step 1: Solve the Multi-objective solid transportation problem as a single objective solid transportation problem using, each time, only one objective (ignore all others). Let $X_1^* = \{x^1_{ijk}\}$, $X^{2*} = \{x^2_{ijk}\}, \ldots, X^{p*} = \{x^p_{ijk}\}$,be the optimum solutions for P different single objective solid transportation problem.

Step 2: From the results of step 1, calculate the values of all the objective functions at all these P optimal points. Them a pay off matrix is formed. The diagonal of the matrix constitutes individual optimum minimum values for the P objectives. The X^{p*}`s are the individual optimal solutions and each of these are used to determine the values of other individual objectives, thus the pay off matrix is developed as:

$$\begin{array}{c} \\ Z_1 \\ Z_2 \\ \vdots \\ Z_p \end{array} \begin{array}{c} \begin{array}{cccc} X^{1*} & X^{2*} & \cdots & X^{p*} \end{array} \\ \begin{bmatrix} Z_1(x^{1*}) & Z_1(x^{2*}) & \cdots & Z_1(x^{p*}) \\ Z_2(x^{1*}) & Z_2(x^{2*}) & \cdots & Z_2(x^{p*}) \\ \vdots & \vdots & \ddots & \vdots \\ Z_p(x^{1*}) & Z_p(x^{2*}) & \cdots & Z_p(x^{p*}) \end{bmatrix} \end{array}$$

We find the upper and lower bound for each objective from the Pay off matrix, Here

$L_p = Z_p(x^{p*})$ and $U_p = \max [Z_p(x^{1*}), Z_p(x^{2*}), \ldots, Z_p(x^{p*})]$

Step 3: From step 2, we find for each objective the worst (U_p) and the best (L_p) values corresponding set of solutions. An initial fuzzy model of the problem (1-5) can be stated as: -
Find x_{ijk}, i =1, 2, …,m; j = 1, 2, …,n; k = 1,2,…,K
So as to satisfy

$Z_p \lesssim L_p$, p = 1,2,…,P (6)

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{k} x_{ijk} = a_i, \quad i = 1,2,\ldots,m \quad (7)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j, \quad j = 1,2,\ldots,n \quad (8)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k, \quad k = 1,2,\ldots,K \quad (9)$$

$x_{ijk} \geq 0$ for all i, j, k (10)

Step 4: Define a membership function (hyperbolic u^H or exponential μ^E) for the p^{th} objective function.

Case 1: A hyperbolic membership function is defined by

$$\mu^H Z_p(x) = \begin{cases} 1, & \text{if } Z_p \leq L_p \\ \frac{1}{2}\left(\frac{e^{\{((U_p+L_p)/2)-Z_p(x)\}\alpha_p} - e^{\{((U_p+L_p)/2)-Z_p(x)\}\alpha_p}}{e^{\{((U_p+L_p)/2)-Z_p(x)\}\alpha_p} + e^{\{((U_p+L_p)/2)-Z_p(x)\}\alpha_p}}\right) + \frac{1}{2}, & \text{if } L_p \leq Z_p \leq U_p \\ 0, & \text{if } Z_p \geq U_p \end{cases} \quad (11)$$

where $\alpha_p = 6/(U_p + L_p)$

Case 2: An Exponential membership function is defined by

$\mu^E Z_p(x) =$

$$\begin{cases} 1, & \text{if } Z_p \leq L_p \\ \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}}, & \text{if } L_p \leq Z_p \leq U_p \\ 0, & \text{if } Z_p \leq U_p \end{cases} \quad (12)$$

where $\psi_p(x) = (Z_p(x) - L_p) / U_p - L_p$, p = 1,2,…,P

and s is a non –zero parameter prescribed by the decision maker.

Step 5: Find an equivalent crisp model by using a membership function (either hyperbolic or exponential) for the initial fuzzy model.

Step 6: From case 1, solve the crisp model by an appropriate mathematical programming algorithm. The solution obtained in step 6 will be the optimal compromise solution of the multi-objective solid transportation problem. If we will use the hyperbolic membership function as defined in (11) then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ (13)

subject to

$$\lambda \leq \frac{1}{2}\frac{e^{\{(U_p+L_p)/2-Z_p(x)\alpha_p\}} - e^{\{(U_p+L_p)/2-Z_p(x)\alpha_p\}}}{e^{\{(U_p+L_p)/2-Z_p(x)\alpha_p\}} + e^{\{(U_p+L_p)/2-Z_p(x)\alpha_p\}}} + \frac{1}{2},$$

p = 1,2,---,P (14)

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i, \quad i = 1,2,\ldots,m \quad (15)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j, \quad j = 1,2,\ldots,n \quad (16)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k, \quad k = 1,2,\ldots,K \quad (17)$$

$x_{ijk} \geq 0$ for all i, j, k and $\lambda \geq 0$ (18)

The above problem (13-18) can be further simplified as

Maximize X_{mn+1} (19)

subject to

$\alpha_p Z_p(x) + X_{mn+1} \leq \alpha_p(U_p + L_p)/2$, p = 1,2,-----P (20)

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i, \quad i = 1,2,\ldots,m \quad (21)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j, \quad j = 1,2,\ldots,n \quad (22)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k, \quad k = 1,2,\ldots,K \quad (23)$$

$x_{ijk} \geq 0$ for all i, j, k and $X_{mn+1} \geq 0$ (24)

Where $X_{mn+1} = \tanh^{-1}(2\lambda-1)$

From case 2

If we use the exponential membership function as defined (12) then an equivalent crisp model for the fuzzy model can be formulated as follows:

Maximize λ (25)

subject to

$$\lambda \leq \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}}, \quad p = 1,2,\text{-----}P \quad (26)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i, \quad i = 1,2,\ldots,m \quad (27)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j, \quad j = 1,2,\ldots,n \quad (28)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k, \quad k = 1,2,\ldots,K \quad (29)$$

$x_{ijk} \geq 0$ for all i, j, k and $\lambda \geq 0$ (30)

The above problem (25-30) can be further simplified as:

Maximize X_3 (31)

Subject to

$s\{1-\psi_p(x)\} \geq X_3$ p = 1,2,-----P (32)

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i, \quad i = 1,2,\ldots,m \quad (33)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j, \quad j = 1,2,\ldots,n \quad (34)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k \;,\quad k = 1,2,\ldots,K \quad (35)$$

$x_{ijk} \geq 0$ for all i, j, k and $X_3 \geq 0$ (36)
where $X_3 = \log\{1+ \lambda(e^s -1)\}$

Case 3

However if we use a linear membership function the crisp model can be simplified as:
Maximize λ (37)
subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k} c^p{}_{ijk}\, x_{ijk} + \lambda (U_p - L_p) \leq U_p \;,\quad p = 1,2,\text{-----}P \quad (38)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i \;,\quad i = 1,2,\ldots,m \quad (39)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j \;,\quad j = 1,2,\ldots,n \quad (40)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k \;,\quad k = 1,2,\ldots,K \quad (41)$$

$x_{ijk} \geq 0$ for all i, j, k and $\lambda \geq 0$ (42)

5.5. Numerical example

To illustrate the fuzzy programming algorithm, we consider a Multi objective standard solid transportation problem having the following characteristics.

Supplies:- $a_1 = 24$, $a_2 = 8$, $a_3 = 18$, $a_4 = 10$
Demand: - $b_1 = 11$, $b_2 = 19$, $b_3 = 21$, $b_4 = 9$
Conveyance: - $e_1 = 17$, $e_2 = 31$, $e_3 = 12$
Capacities

Penalties:-

Destinations		D1			D2			D3			D4		
Conveyance / Origin		1	2	3	1	2	3	1	2	3	1	2	3
C^1	O1	15	18	17	12	22	13	10	4	12	8	11	13
	O2	17	20	19	21	21	22	21	19	18	30	10	23
	O3	14	11	12	25	34	33	20	16	15	21	23	22
	O4	22	18	13	24	35	32	18	21	14	13	23	20
C^2	O1	6	7	8	10	6	5	11	3	7	10	9	6
	O2	13	8	11	12	2	9	20	15	13	17	15	13
	O3	5	6	7	11	9	7	10	5	2	15	14	18
	O4	13	6	6	17	11	18	12	16	12	18	14	7

The penalties can be expressed in the three dimensional table this problem can be modeled as follows:
Minimize $Z_p =$

$$\sum_{3}^{4}\sum_{3}^{4}\sum_{3}^{4} c^p{}_{ijk}\, x_{iik} = a_i, \qquad p = 1,2. \quad (43)$$

subject to

$$\sum_{3}^{4}\sum_{3}^{4} x_{1ik} = 24, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{2ik} = 8,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{3ik} = 18, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{4ik} = 10,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i1k} = 11, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{i2k} = 19,$$

(44)

$$\sum_{3}^{4}\sum_{3}^{4} x_{i3k} = 21, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{i4k} = 9,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij1} = 17, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{ij2} = 31,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij3} = 12,$$

$x_{ijk} \geq 0$, i = 1,2,3,4 , j = 1, 2,3,4, k = 1,2,3 (45)

where $C^1 = (C^1{}_{ijk})$, $C^2 = (C^2{}_{ijk})$

Step 1 and **step 2**

Optimal solution which minimize the first objective Z_1 subject to constraints (44-45) are as follows:
$X_{121} = 16$, $X_{123} = 3$, $X_{132,} = 5$, $X_{242,} = 8$, $X_{312} = 11$, $X_{332} = 7$, $X_{441} = 1$, $X_{433} = 9$
With $Z_1(X_1) = 703$, $Z_2(X_1) = 537$

Optimal solutions which minimize the second object Z_2 subject to constraints (44-45) are as follows:
$X_{121} = 2$, $X_{132,} = 18$, $X_{122} = 4$, $X_{222} = 8$
$X_{311,} = 11$, $X_{321} = 4$, $X_{333} = 3$, $X_{431} = 1$, $X_{433} = 9$
With $Z_1(X_2) = 866$, $Z_2(X_2) = 293$

Step 3

From the pay-off matrix, we find
$U_1 = 866$, $L_1 = 703$
$U_2 = 537$, $L_2 = 293$
If we use the hyperbolic membership function $u^H z_1(x)$, $u^H z_2(x)$ for the objective Z_1 and Z_2 respectively are defined as follows:
Maximize X_{mn+1} (46)
subject to
$X_{mn+1} \leq \alpha_1\{(U_1 + L_1)/2 - Z_1(x)\}$, (47)
$X_{mn+1} \leq \alpha_2\{(U_2 + L_2)/2 - Z_2(x)\}$, (48)

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i \;,\quad i = 1,2,\ldots,m \quad (49)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j \ , \quad j = 1,2,\ldots,n \ (50)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k \ , \quad k = 1,2,\ldots,K \ (51)$$

$x_{ijk} \geq 0$ for all i, j, k and $X_{mn+1} \geq 0$ (52)

where $X_{mn+1} = \tanh^{-1}(2\lambda-1)$

OR

Maximize X_{mn+1}

subject to

$6[Z_1(x)] + 163X_{mn+1} \leq 4707$

$3[Z_2x)] + 122X_{mn+1} \leq 1245$

$$\sum_{3}^{4}\sum_{3}^{4} x_{1ik} = 24, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{2ik} = 8,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{3ik} = 18, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{4ik} = 10,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i1k} = 11, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{i2k} = 19,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i3k} = 21, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{i4k} = 9,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij1} = 17, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{ij2} = 31,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij3} = 12,$$

$x_{ijp} \geq 0$, i = 1,2,3,4 , j = 1, 2,3,4, k = 1,2,3

and $X_{mn+1} \geq 0$

OR

Max x_{10}

subject to

$90x_{111}+108x_{112}+102x_{113}+72x_{121}+132x_{122}+78x_{123}+60x_{131}+24x_{132}+72x_{133}+8x_{141}+6648x_{142}+784x_{143}+102x_{211}+120x_{212}+114x_{213}+126x_{221}+126x_{222}+132x_{223}+126x_{231}+114x_{232}+108x_{233}+180x_{241}+60x_{242}+138x_{243}+84x_{311}+66x_{312}+72x_{313}+150x_{321}+204x_{322}+198x_{323}+120x_{331}+96x_{332}+90x_{333}+126x_{341}+138x_{342}+132x_{343}+132x_{411}+108x_{412}+78x_{413}+144x_{421}+210x_{422}+192x_{423}+108x_{431}+126x_{432}+84x_{433}+78x_{441}+138x_{442}+120x_{443}+163x_{10} \leq 4707$

$18x_{111}+21x_{112}+24x_{113}+30x_{121}+18x_{122}+15x_{123}+33x_{131}+9x_{132}+21x_{133}+30x_{141}+27x_{142}+18x_{143}+39x_{211}+24x_{212}+33x_{213}+36x_{221}+6x_{222}+18x_{223}+60x_{231}+45x_{232}+39x_{233}+51x_{241}+45x_{242}+39x_{243}+15x_{311}+18x_{312}+21x_{313}+33x_{321}+27x_{322}+21x_{323}+30x_{331}+15x_{332}+6x_{333}+45x_{341}+42x_{342}+54x_{343}+39x_{411}+18x_{412}+18x_{413}+51x_{421}+33x_{422}+54x_{423}+36x_{431}+48x_{432}+36x_{433}+54x_{441}+42x_{442}+21x_{443}+122x_{10} \leq 1245$

$x_{111}+x_{112}+x_{113}+x_{121}+x_{122}+x_{123}+x_{131}+x_{132}+x_{133}+x_{141}+x_{142}+x_{143} = 24$

$x_{211}+x_{212}+x_{213}+x_{221}+x_{222}+x_{223}+x_{231}+x_{232}+x_{233}+x_{241}+x_{242}+x_{243} = 8$

$x_{311}+x_{312}+x_{313}+x_{321}+x_{322}+x_{323}+x_{331}+x_{332}+x_{333}+x_{341}+x_{342}+x_{343} = 18$

$x_{411}+x_{412}+x_{413}+x_{421}+x_{422}+x_{423}+x_{431}+x_{432}+x_{433}+x_{441}+x_{442}+x_{443} = 10$

$x_{111}+x_{112}+x_{113}+x_{211}+x_{212}+x_{213}+x_{311}+x_{312}+x_{313}+x_{411}+x_{412}+x_{413} = 11$

$x_{121}+x_{122}+x_{123}+x_{221}+x_{222}+x_{223}+x_{321}+x_{322}+x_{323}+x_{421}+x_{422}+x_{423} = 19$

$x_{131}+x_{132}+x_{133}+x_{231}+x_{232}+x_{233}+x_{331}+x_{332}+x_{333}+x_{431}+x_{432}+x_{433} = 21$

$x_{141}+x_{142}+x_{143}+x_{241}+x_{242}+x_{243}+x_{341}+x_{342}+x_{343}+x_{441}+x_{442}+x_{443}=9$

$x_{111}+x_{121}+x_{131}+x_{141}+x_{211}+x_{221}+x_{231}+x_{241}+x_{311}+x_{321}+x_{331}+x_{341}+x_{411}+x_{421}+x_{431}+x_{441} = 17$

$x_{112}+x_{122}+x_{132}+x_{142}+x_{212}+x_{222}+x_{232}+x_{242}+x_{312}+x_{322}+x_{332}+x_{342}+x_{412}+ x_{422}+x_{432}+x_{442} = 31$

$x_{113}+x_{123}+x_{133}+x_{143}+x_{213}+x_{223}+x_{233}+x_{243}+x_{313}+x_{323}+x_{333}+x_{343}+x_{413}+x_{423}+x_{433}+x_{443} = 12$

$x_{ijk} \geq 0$, i = 1,2,3,4 , j = 1, 2,3,4, k = 1,2,3

and $X_{10} \geq 0$

The problem is solved by Linear Interactive and Discrete optimization (LINDO) software. The optimal solution is presented as follows:

$X_{121} = 10.142877$, $X_{132,} = 13.857123$, $X_{222} = 8$, $X_{311} = 1.714246$, X312 =9.142877, $X_{333} = 7.142877$, $X_{413} = 0.142877$, $X_{421} = 0.857123$, $X_{441} = 4.285754$, $X_{443} = 4.714246$,

rest all x_{ijk} are zero.

$X_{10} = 1.296245$

$\tanh^{-1}(2\lambda-1) = 1.296245$ and $\lambda = 0.93$

Therefore $Z_1 = 749.28418$ and $Z_2 = 362.28514$

If we use the exponential membership function with the parameter s = 1

An equivalent crisp model can be formulated as:

Maximize X_3 (53)

subject to

$s\,[Z_1(x)] + x_3\,(U_1 - L_1) \leq s\,[U_1]$ (54)

$s\,[Z_2(x)] + x_3\,(U_2 - L_2) \leq s\,[U_2]$ (55)

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i \ , \quad i = 1,2,\ldots,m \ (56)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j \ , \quad j = 1,2,\ldots,n \ (57)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k \ , \quad k = 1,2,\ldots,K \ (58)$$

$x_{ijk} \geq 0$ for all i, j, k and $X_3 \geq 0$ (59)

where, $\psi_p^{(x)} = (Z_k(x) - L_p) / U_p - L_p$ and $X_3 = \log(1+(e^s-1)\lambda$

OR

Max X_3

subject to

$Z_1(x)+163\ X_3 \leq 866$

$Z_2(x)+244\ X_3 \leq 537$

$$\sum_{3}^{4}\sum_{3}^{4} x_{1ik} = 24, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{2ik} = 8,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{3ik} = 18, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{4ik} = 10,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i1k} = 11, \qquad \sum_{3}^{4}\sum_{3}^{4} x_{i2k} = 19,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i3k} = 21, \quad \sum_{3}^{4}\sum_{3}^{4} x_{i4k} = 9,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij1} = 17, \quad \sum_{3}^{4}\sum_{3}^{4} x_{ij2} = 31,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij3} = 12,$$

$x_{ijp} \geq 0$, i = 1,2,3,4 , j = 1, 2,3,4, k = 1,2,3 and $X_3 \geq 0$

The optimal solution of the problem is presented as:-

$X_{121} = 10.143$, $X_{132} = 13.857$, $X_{222} = 8$, $X_{311} = 1.7114$ $X_{312} = 9.140$, $X_{333} = 7.143$, $X_{421} = 0.857$, $X_{413} = 0.143$, $X_{441} = 4.286$, $X_{443} = 4.714$

rest all x_{ijk} are zero.

and $X_3 = 0.716$ therefore $\lambda = 0.608$

$Z_1 = 749.2853$ and $Z_2 = 362.2860$

From case 3

However if we use a linear membership function the crisp model can be simplified as:

Maximize λ (60)

Subject to

$Z_1(x) + \lambda (U_1 - L_1) \leq U_1$ (61)

$Z_2(x) + \lambda (U_2 - L_2) \leq U_2$ (62)

$$\sum_{j=1}^{n}\sum_{k=1}^{k} X_{ijk} = a_i \ , \quad i = 1,2,\ldots,m \quad (63)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} X_{ijk} = b_j \ , \quad j = 1,2,\ldots,n \quad (64)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ijk} = e_k \ , \quad k = 1,2,\ldots,K \quad (65)$$

$x_{ijk} \geq 0$ for all i, j, k and $\lambda \geq 0$ (66)

OR

Maximize λ

Subject to

$Z_1(x) + 163\lambda \leq 866$

$Z_2(x) + 244\lambda \leq 573$

$$\sum_{3}^{4}\sum_{3}^{4} x_{1ik} = 24, \quad \sum_{3}^{4}\sum_{3}^{4} x_{2ik} = 8,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{3ik} = 18, \quad \sum_{3}^{4}\sum_{3}^{4} x_{4ik} = 10,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i1k} = 11, \quad \sum_{3}^{4}\sum_{3}^{4} x_{i2k} = 19,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{i3k} = 21, \quad \sum_{3}^{4}\sum_{3}^{4} x_{i4k} = 9,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij1} = 17, \quad \sum_{3}^{4}\sum_{3}^{4} x_{ij2} = 31,$$

$$\sum_{3}^{4}\sum_{3}^{4} x_{ij3} = 12,$$

$x_{ijk} \geq 0$, i = 1,2,3,4 , j = 1, 2,3,4, k = 1,2,3 and $\lambda \geq 0$

The optimal solution of the problem is presented as: -

$X_{121} = 10.143$, $X_{132} = 13.857$, $X_{222} = 8$, $X_{311} = 1.7114$, $X_{312} = 9.140$, $X_{333} = 7.143$, $X_{421} = 0.857$, $X_{413} = 0.143$ $X_{441} = 4.286$, $X_{443} = 4.714$

rest all x_{ijk} are zero. and $\lambda = 0.716$

$Z_1 = 749.2853$, $Z_2 = 362.2860$

6. Conclusion

In this paper, two special types of non-linear membership functions have been used to solve the multi-objective solid transportation problem. If we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function. However, if we use the exponential type membership function, with different values of s (parameter) then the crisp model becomes linear and the optimal compromise solution does not change significantly, if we compare with the solution obtained by the linear membership function.

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