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ANALYSIS OF TIME ESTIMATES ON RIGHT SKEWED DISTRIBUTION OF ACTIVITY TIMES IN PERT

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Abstract- The approximations of activity mean, project duration belong to the most important activities in managing a construction project. Several researchers made an attempt to provide better Program Evaluation Review Technique (PERT) approximations using diverse probability distribution functions, for instance beta, normal, lognormal, triangular, weibull. Several researchers approximated the time estimates with three parameters optimistic time (*a*), most likely time(*m*), pessimistic time(*b*) and a few with two parameters (*a,m*) or (*b,m*). A usual supposition in project management is that the distribution for most activities is right skewed. Mohan et al.[4] suggested lognormal PERT approximations for a right skewed project and proved that the approximations are better than Traditional PERT, normal approximation, when the distributions are highly right skewed. The prime objective of this paper is to find effective distribution in conjunction with effective PERT approximations for right skewed projects. Our PERT approximations[14] are compared with normal, lognormal approximations and also with beta approximations with three parameters. The comparison reveals that PERT [14] performs better than lognormal and also other approximations suggested.
Keywords: PERT, Estimating Mean, variance, project duration, Right skewed distributions

Introduction

One of the most important problems in project management is to obtain the distribution of the total completion time in PERT network. PERT has gained very wide recognition as an effective management tool in development programs, and there exists considerable interest in applying the principles of PERT elsewhere. PERT can be used to estimate the probability of completing either a project or individual activities by any specified time. In a PERT project network, the activity times are assumed to be a beta distribution and three parameters (optimistic time (*a*), most likely time (*m*) and pessimistic time (*b*)) are used to estimate the means and variances of the activity times. The PERT method includes precise breakdown of the projects into tasks, estimation of the duration of tasks etc.

A great deal of research has been carried out on methodologies for estimating project time distributions. The methods can be broadly grouped into four main approaches: exact analysis, analytical approximation, analytical bounding, and simulation. Yousry H. Abdelkader [6] developed the moment's method for finding project completion time when activities are weibull distributed. Cottrell [3] suggested an approximation of the normal distribution to determine expected time and variance using two time estimates. Copertari et al [7] showed that the PERT assumption of a normally distributed project completion time leads to optimistic planning. Thus the normal distribution, which is unbounded, should not be used to portray completion times. Mohan et al [4], suggested a two parameter lognormal approximation for estimating activity times in PERT and also showed that it works well when the distributions are highly right skewed. A usual supposition in project management is that the distribution for most activities is right skewed [4]. An activity in the project is said to be symmetric, right and left skewed respectively when $b-m = m-a$, $b-m > m-a$, and $b-m < m-a$. For a right skewed activity , the skewness $k = \frac{b-m}{m-a}$ >1.

The beta distribution is widely used to model probability distributions of variables or project parameters in many areas of operations research. The reasons why the beta distribution is so widely used is that it is extremely versatile, thus a variety of uncertainties can be usefully modeled by it. For example, it can accommodate a variety of skewnesses, both positive and negative, and thus, when skewness is an important factor, the beta distribution is often used [1]. Risk analysis is having a major influence on management decisions involving investments. The beta distribution is seen as a suitable model in risk analysis because it provides a wide variety of distributional shapes over a finite interval [5]. Morgan and Henrion [2] note that 'the flexibility of the beta distribution encourages its empirical use in a wide range of applications'. These applications include the simulation of systems in engineering, particularly where the random variable to be modeled is a ratio. Therefore, in a variety of cases, there is often a need to estimate the distribution. Further, the beta distribution can be estimated relatively easily from data on just the optimistic, pessimistic and most likely values, and since managers and planners find it easier to estimate these three points than other statistical parameters, the beta distribution has been applied in many practical problems.

The purpose of this paper is to show the effectiveness of PERT [14] when the activities are right skewed and suggesting a new PERT procedure for a right skewed project. In section 2, we discuss some relevant PERT formulae to estimate mean and variance. In section 3 ,we compare different approximations by calculating the activity mean , variance, and project duration of real world projects. In section 4, we draw the conclusions.

2. PERT approximations

2.1 Time estimates with three parameters

The following are some activity duration estimates with three parameters a , *m* and *b* using beta distribution.

2.1.1 Time estimates of the mean and variance[9]

The creators of Traditional PERT [10-12] worked out the basic concepts of the PERT analysis, and suggested the estimates of the mean and variance values

$$\mu = \frac{1}{6}(a + 4m + b) \qquad (1)$$

$$\sigma^2 = \frac{1}{36}(b - a)^2 \qquad (2)$$

Subject to the assumption that the density distribution of the activity time is

$$f_y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}, a < y < b, \alpha, \beta > 0. \qquad (3)$$

2.1.2 Ginzburg [13] time estimates

Generalizing the assumption $p + q \cong 4$ Ginzburg proposed the estimates

$$\mu_y = \frac{2a + 9m + 2b}{13} \qquad (4)$$

$$\sigma_y^2 = \frac{(b-a)^2}{1268}\left[22 + 81\frac{m-a}{b-a} - 81\left(\frac{m-a}{b-a}\right)^2\right]. \qquad (5)$$

for the general beta distribution.

By assuming that p =1, q = 2 and m = $\frac{2a+b}{3}$, he further improved these estimates estimates when the estimated mode of the activity time is located in the tail of the distribution as follows

$$\mu_y = \frac{8a + 5b}{13} \qquad (6)$$

$$\sigma_y^2 = \frac{10(b-a)^2}{317} \cong (b-a)^2/32 \qquad (7)$$

2.1.3 Ravi shankar and Sireesha [14] time estimates

We proposed PERT approximations[14] for activity duration distribution as follows

$$\mu = \frac{5a + 17m + 5b}{27} \qquad (8)$$

$$\sigma^2 = \frac{(b-a)^2}{35} \qquad (9)$$

2.2 Time estimates with two parameters

The following are some activity duration estimates with two parameters (a , *m*) or (*b* ,*m*) .

2.2.1 Cottrell [3] time estimates

Cottrell [3] determined expected activity times with only two time estimates (*b,m*) using normal distribution to develop a simplified version of the PERT for project planning .

PERT approximations using time estimates are

$$\left.\begin{aligned} \mu &= m \\ \sigma &= \frac{b-m}{z} \end{aligned}\right\} \qquad (10)$$

where z = 3.44 .

2.2.2 Mohan et al [4] time estimates

Mohan et al [4] developed an alternative method for determining expected activity times with only two time estimates (*a,m*) or (*b,m*) using lognormal distribution.

PERT approximations using time estimates (*a,m*) are

$$\left.\begin{aligned} \sigma^* &= \frac{z}{2} - \left[\frac{z^2}{4} + \log(a/m)\right]^{1/2} \\ \mu^* &= \log[a] + z\sigma^* \end{aligned}\right\} \qquad (11)$$

PERT approximations using time estimates (*b,m*) are

$$\left.\begin{aligned} \sigma^* &= -\frac{z}{2} + \left[\frac{z^2}{4} + \log(b/m)\right]^{1/2} \\ \mu^* &= \log[b] - z\sigma^* \end{aligned}\right\} \qquad (12)$$

where σ^* *and* μ^* are the mean and standard deviation of the underlying normal distribution and $z = 3$.

2.3 Time estimates of standardized beta distribution

In order to compare the performance of the five methods to be studied, we need the exact value. Mohan et al [4] suggested that, it can be find out by transforming a standardized (0,1) beta distribution as $y = a + (b - a)x$, the mean and variance of the transformed distribution on the interval (*a,b*) can be evaluated as

$$\left.\begin{aligned} \mu &= \frac{a\beta + b\alpha}{\alpha + \beta} \\ \sigma &= \frac{(b-a)}{(\alpha+\beta)}\sqrt{\frac{\alpha\beta}{\alpha+\beta+1}} \end{aligned}\right\} \qquad (13)$$

where α, β are the shape parameters for the beta distribution.

3. Analysis of activity mean, variance and project duration of a right skewed project

It has already been proved that the PERT [14] is more efficient than Traditional PERT, Ginzberg [13] in finding activity mean, variance and project duration for a project network having a variety of skewnesses, both right and left. A usual supposition in project management is that the distribution for most activities is right skewed. Jan Kozłowski et al.[9] pointed out that species body size distributions are right-skewed, symmetric or left-skewed, but right-skewness strongly prevails. Mohan et al [4] developed a method for determining expected activity times with only two time estimates (*a,m*) or (*b,m*) using lognormal distribution. He pointed out that lognormal mean approximation is better than Traditional PERT [9], Premachandra[8], Cottrell[3], and works exceptionally when distributions are highly right skewed.

As the activity mean and variance plays an important role in finding project duration and project duration makes a large difference in the economic aspects of the project more than a few researchers estimated mean and variances using different distributions. To find effective method amongst them, these methods have been applied for a set of 10 right skewed project networks such as each featuring $a, m \text{ and } b$ estimates. Ship building project E is the one of the projects the network shown in fig.1 and activity times with their skewness is shown in Table I

In this paper, the comparison of the mean and variance activity times of Traditional PERT[9], PERT[14], Ginzburg [13], Cottrell[3], Mohan et al.[4] for a right skewed projects with simulated activity times has been experimented. We made an approach of comparison by calculating the standardized mean and variances of each activity using Eq. (13). The calculated mean values and their absolute percentage errors of right skewed project E are shown in Table II, Table III respectively and the calculated variances and their absolute percentage errors are shown in Table IV, Table V respectively. The graphical representation of absolute percentage errors of mean and variance are shown in Fig. 2 and Fig.3.

Fig. 1 shows an 18-activity project network in the ship building domain from Taggart[15]. The uncertainty in each activity duration could be elicited through expert judgment via a optimistic time a, most likely time m, pessimistic time b as described in Table I. Modern-day ship production is a manufacturing domain in which innovative design and build strategies require special attention to risk factors that may impact cost and delivery time. Two major risk areas are the impact of ECO's and crane unavailability. Engineering changes may come from a variety of sources- such as owner-requested changes, inadequate design specifications, interface problems for vendor-furnished equipment, etc. Cranes are used to lift large prefabricated units and their unavailability due to outages may result in substantial project delays. Generally the project delays are due to the delay of activities in the project. The activity delays can be found by finding their earliest and latest times which in turn depends up on the activity mean time.

3.1 Comparison study of activity mean

From the results of Table II it has been observed that the estimates for the mean activity time using PERT [14] are very nearer to standard mean when compared to the other approximations available till date, including the latest work being done by Mohan et al [4]. The Mohan et al [4] estimate is compared and observed in the present paper, the estimated mean values are having high percentage errors when compared to other approximations, irrespective of the skewness. In case of Cottrell [3], the normal approximations are having a very high percentage error when compared with other estimates. It has also been observed that all beta approximations are performing better than lognormal approximations. Thus, we may conclude that which ever may be the project that is either right skewed or varied skewness of activities, beta approximations execute better than normal and lognormal approximations. Among the beta approximations, PERT [14] is more efficient.

3.2 Comparison study of activity variance

The variance of a random variable is a measure of its statistical dispersion, indicating how far from the expected value its values typically are. Therefore, the right estimation for activity variance is vital. Percentage error in variance for the data in Table I is calculated. The variance is then compared with the calculated values of Mohan et al.[4]. From the results it is observed that the variance is less when compared to Mohan et al.[4]. These values in turn are compared in terms of skewness. The results depicted that the variance values of the PERT [14] are lesser than the values obtained from the method of Mohan et al[4] for $k < 5$, which includes activities Shell : Loft, I.B. Structure : Lay out, Mach Fdn. Loft,I.B. Piping Layout, Shell : Assemble, Complete #rd DK etc. When the skewness $k \geq 5$, the variance values of the PERT[14] are higher. The PERT[14] have shown lesser variance than Traditional PERT [9] and Mohan et al[4]. Even though the variance values of Ginzburg[13] are lesser than PERT[14] , from further discussion on project durations it is noticed that the error in the project duration is high.

From the above discussion it can be concluded that the PERT[14] can be applied to a wide range of projects involving more number of activities.

3.3 Finding Critical Path and project completion time for a right skewed Project

In this section, we are proposing a method to estimate mean and variance when all varied activities of a project are right skewed using beta distribution. In this new procedure, first we found time estimates by using Eq. (8) , Eq. (9) and determine the earliest occurrence times via a forward pass and the latest occurrence times via a backward pass of each activity. Then we can identify critical activity defined as an activity with the earliest occurrence time and the latest occurrence time being equal. These critical activities constitute a critical path. The total duration time of this project network can be obtained by summing these critical activity times.

The critical path for the shipbuilding project (E) is calculated using each approximation. It has been observed that the critical path (1-3-6-7-8-9-10-11-14-15) is same in each method but the project completion time varies. It has also been noticed that the project duration using PERT[14] is very nearer to the project duration obtained using standard mean and variance. This observation is shown in Table VI.

3.4 Analysis of Project completion times

In order to verify the effectiveness of the PERT[14], the experimentation has been done on randomly chosen 10 right skewed projects having different number of activities with different skewness. It has been observed that, the critical path is same for each method but the project duration of PERT[14] is very nearer to the project duration calculated using standard values of PERT. The absolute percentage error in project durations of each method is shown in Table VI. From that table values one can observe that PERT [14] is not only better than lognormal and normal approximations, but also is an improved approximation than other beta approximations.

Conclusion

The PERT method is a technique that allows us to manage the scheduling of a project. The main assumption in PERT is that the activity durations in a project can be estimated precisely and that they are statistically independent. The estimates of the mean project duration and its total costs belong to the most important activities in managing a construction project. Their importance is especially prominent in project planning and contracting. In such situations a correct estimates of project duration and costs enables managers to minimize possible losses. Estimating mean and variance is a very challenging field, in which researchers dedicate their knowledge and effort to handle interesting problems which have a direct relation with society, now and in the future.

The important conclusions of this study are summarized as follows:

- In this paper, we argue that PERT [14] is more suitable even the activities are right skewed.
- Experimental results indicate that PERT[14] could solve right skewed project more efficiently than lognormal.
- The proposed method is reliable for estimating project completion time
- PERT [14] valuably estimates activity mean irrespective of the activities' skewness..
- PERT [14] is preferred to conventional PERT.
- The comparison of the activity mean, variance and project duration of all beta distribution methods with lognormal and normal methods reveals that all beta distribution methods are far better than the lognormal and normal, especially when the activities are highly right skewed

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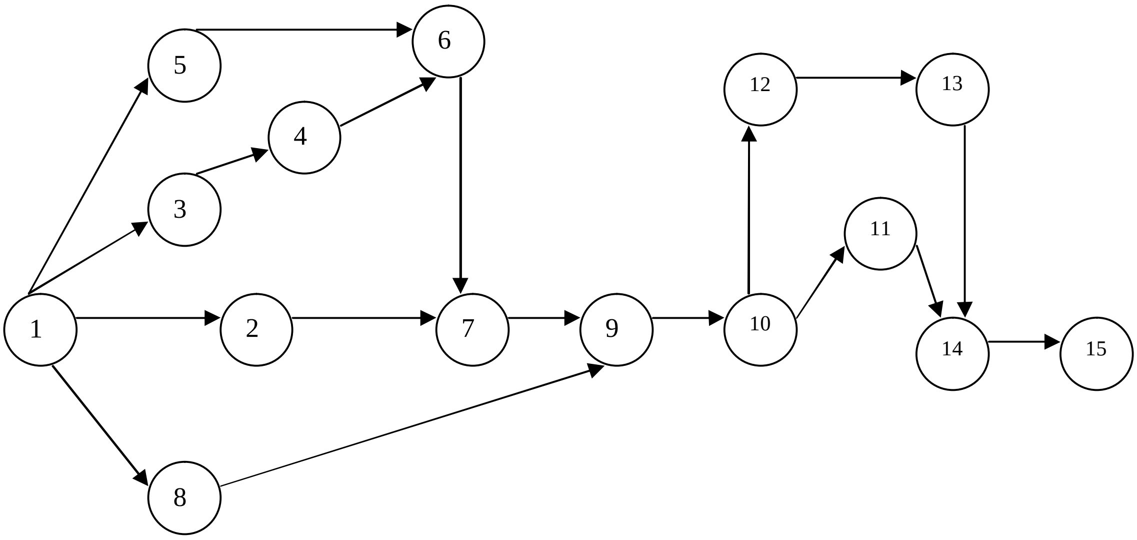

Fig. 1-Project Network

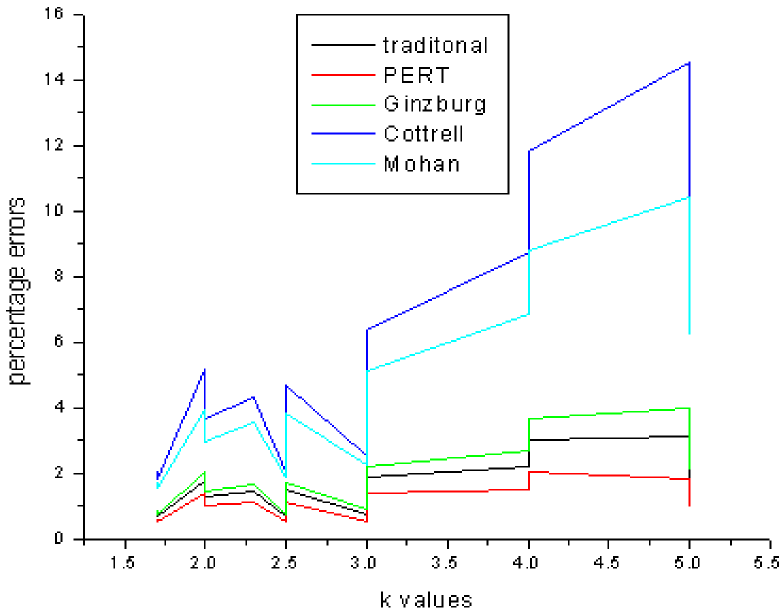

Fig. 2-The graphic representation of percentage errors in mean

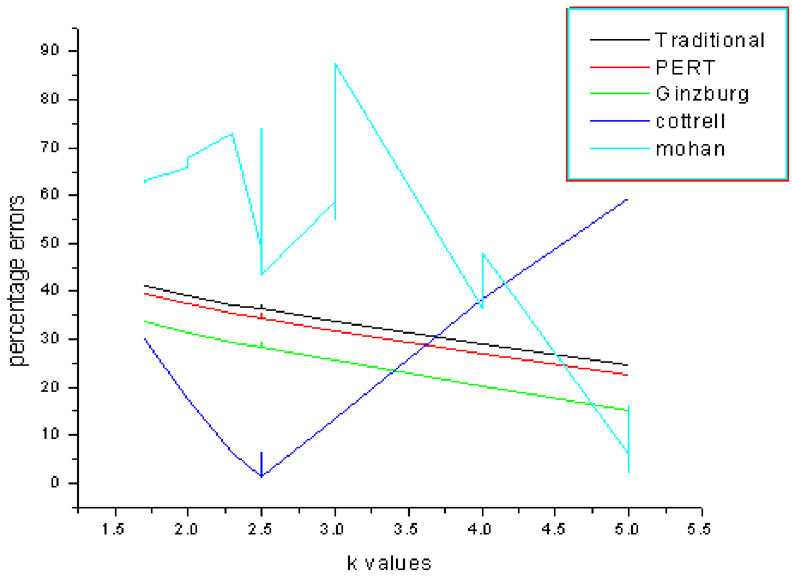

Fig.3-The graphic representation of percentage errors in variance

Table I-Activity times and their skewness

Activity	**Activity Name**	***a***	***m***	***b***	$k = \frac{b-m}{m-a}$
1-2	Shell : Loft	22	25	30	1.7
1-3	I.B. Structure : Lay out	23	26	31	1.7
1-8	Mach Fdn. Loft	25	28	33	1.7
1-5	I.B. Piping Layout	19	22	29	2.3
2-7	Shell : Assemble	35	37	43	3
3-4	I.B. Structure : Fab.	16	18	24	3
8-9	Mach Fdn. Fabricate	33	35	40	2.5
5-6	I.B. Piping Fab.	4	5	10	5
4-6	I.B. Structure : Assemb.	11	14	20	2
6-7	I.B. Structure : Install	6	7	12	5
7-9	Erect I.B.	27	30	37	2.5
9-10	Erect Foundation	6	7	11	4
10-11	Engine : Install	6	7	12	5
10-12	Complete #rd DK	4	5	9	4
11-14	Engine : Finish	17	20	26	2
12-13	Boiler: Install	6	7	10	3
13-14	Boiler : Test	9	10	15	5
14-15	FINAL Test	13	15	20	2.5

Table II- A comparison of the estimated mean activity times

Activity times			**Traditional PERT [9]**	**PERT [14]**	**Ginzberg [13]**	**Cottrell [3]**	**Mohan et al [4]**	**Standardized mean**
a	***m***	***b***						
22	25	30	25.33	25.37	25.31	25	25.09	25.52
23	26	31	26.33	26.37	26.31	26	26.09	26.52
25	28	33	28.33	28.37	28.31	28	28.08	28.52
19	22	29	22.66	22.74	22.62	22	22.18	22.99
35	37	43	37.66	37.74	37.62	37	37.09	37.95
16	18	24	18.66	18.74	18.62	18	18.16	18.95
33	35	40	35.5	35.56	35.46	35	35.07	35.73
4	5	10	5.66	5.740	5.62	5	5.24	5.85
11	14	20	14.5	14.56	14.46	14	14.18	14.76
6	7	12	7.66	7.74	7.62	7	7.20	7.85
27	30	37	30.67	30.74	30.62	30	30.14	30.99
6	7	11	7.5	7.56	7.46	7	7.15	7.67
6	7	12	7.67	7.74	7.62	7	7.20	7.85
4	5	9	5.5	5.56	5.46	5	5.17	5.67
17	20	26	20.5	20.56	20.46	20	20.14	20.76
6	7	10	7.33	7.37	7.31	7	7.09	7.47
9	10	15	10.67	10.74	10.62	10	10.17	10.85
13	15	20	15.5	15.56	15.46	15	15.13	15.73

Table III- Percentage error in mean

Activity times			**Traditional PERT [9]**	**PERT [14]**	**Ginzberg [13]**	**Cottrell [3]**	**Mohan et al [4]**
a	*m*	*b*					
22	25	30	0.74	0.59	0.84	2.04	1.7
23	26	31	0.71	0.57	0.81	1.97	1.64
25	28	33	0.66	0.53	0.75	1.83	1.54
19	22	29	1.41	1.09	1.63	4.31	3.54
35	37	43	0.74	0.54	0.87	2.5	2.26
16	18	24	1.48	1.09	1.75	5	4.17
33	35	40	0.66	0.5	0.76	2.05	1.87
4	5	10	3.1	1.83	3.98	14.5	10.4
11	14	20	1.77	1.4	2.03	5.16	3.91
6	7	12	2.31	1.37	2.96	10.8	8.19
27	30	37	1.04	0.81	1.21	3.2	2.74
6	7	11	2.21	1.49	2.71	8.73	6.83
6	7	12	2.31	1.37	2.96	10.8	8.19
4	5	9	2.99	2.01	3.67	11.8	8.76
17	20	26	1.26	0.99	1.45	3.67	2.97
6	7	10	1.88	1.38	2.22	6.34	5.1
9	10	15	1.67	0.99	2.14	7.82	6.26
13	15	20	1.49	1.14	1.73	4.67	3.84

Table IV- A comparison of the estimated variance activity times

Activity times								
a	m	b	Traditional PERT [9]	PERT [14]	Ginzberg [13]	Cottrell [3]	Mohan et al [4]	standardized variance
22	25	30	1.78	1.83	2	2.11	1.13	3.02
23	26	31	1.78	1.83	2	2.11	1.12	3.02
25	28	33	1.78	1.83	2	2.11	1.11	3.02
19	22	29	2.78	2.86	3.13	4.14	1.2	4.43
35	37	43	1.78	1.83	2	3.04	1.1	2.68
16	18	24	1.78	1.83	2	3.04	1.2	2.68
33	35	40	1.36	1.4	1.53	2.11	1.09	2.14
4	5	10	1	1.03	1.13	2.11	1.54	1.33
11	14	20	2.25	2.31	2.53	3.04	1.26	3.7
6	7	12	1	1.03	1.13	2.11	1.41	1.33
27	30	37	2.78	2.86	3.13	4.14	1.15	4.43
6	7	11	0.69	0.71	0.78	1.35	1.33	0.98
6	7	12	1	1.03	1.13	2.11	1.41	1.33
4	5	9	0.69	0.71	0.78	1.35	1.45	0.98
17	20	26	2.25	2.31	2.53	3.04	1.19	3.7
6	7	10	0.44	0.46	0.5	0.76	1.26	0.67
9	10	15	1	1.03	1.13	2.11	1.3	1.33
13	15	20	1.36	1.4	1.53	2.11	1.2	2.14

Table V- Percentage error in variance

Activity times							
a	m	b	Traditional PERT [9]	PERT[14]	Ginzberg[13]	Cottrell[3]	Mohan et al [4]
22	25	30	41.09	39.4	33.72	29.99	62.67
23	26	31	41.09	39.4	33.72	29.99	62.82
25	28	33	41.09	39.4	33.72	29.99	63.1
19	22	29	37.23	35.43	29.38	6.426	72.98
35	37	43	33.72	31.83	25.44	13.42	58.86
16	18	24	33.72	31.83	25.44	13.42	55.1
33	35	40	36.32	34.5	28.36	1.158	48.92
4	5	10	24.62	22.46	15.19	59.25	16
11	14	20	39.11	37.37	31.5	17.68	65.97
6	7	12	24.62	22.46	15.19	59.25	5.91
27	30	37	37.23	35.43	29.38	6.426	74.09
6	7	11	28.98	26.95	20.1	38.28	36.3
6	7	12	24.62	22.46	15.19	59.25	5.91
4	5	9	28.98	26.95	20.1	38.28	47.9
17	20	26	39.11	37.37	31.5	17.68	67.92
6	7	10	33.72	31.83	25.44	13.42	87.5
9	10	15	24.62	22.46	15.19	59.25	2.32
13	15	20	36.32	34.5	28.36	1.158	43.65

Table VI- Absolute percentage errors in project durations

Projects	Number of activities	Traditional PERT [9]	PERT[14]	Ginzberg[13]	Cottrell[3]	Mohan et al [4]
A	10	1.141	0.941	1.53	4.53	3.721
B	6	1.252	0.862	1.484	4.454	3.651
C	12	1.242	0.891	1.521	4.362	3.252
D	5	1.343	0.991	1.591	4.442	3.625
E	18	1.380	0.995	1.622	4.686	3.770
F	8	1.031	0.832	1.425	4.111	3.451
G	18	1.304	0.921	1.541	4.611	3.573
H	10	1.223	0.822	1.472	4.342	3.392
I	9	1.181	0.983	1.565	4.181	3.712
J	8	1.372	0.921	1.545	4.852	3.601