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A new fuzzy MADM approach used for finite selection problem

Muley A.A. and Bajaj V.H.*
*Department of Statistics, Dr. B. A. M. University, Aurangabad (M.S.)-431004, India
vhbajaj@gmail.com, aniket.muley@gmail.com

Abstract- This paper proposes a new approach to product configuration by applying the theory of Fuzzy Multiple Attribute Decision Making (FMADM), which focuses on uncertain and fuzzy requirements the customer, submits to the product supplier. The proposed method can be used in e-commerce websites, with which it is easy for customers to get his preferred product according to the utility value with respect to all attributes. The main concern of this paper, in which requirements the customer submitted to the configuration of television is vague. Further verify the validity and the feasibility of the proposed method compared with Weighted Product Method (WPM). Finally, the television is taken as an example to demonstrate the proposed methods.
Keywords- MADM, Fuzzy, Triangular fuzzy number, T.V., Uncertainty

Introduction

Real world problems are often require a decision maker (DM) to rank discrete alternatives or, at least, to select best one. The MADM theory was developed to help the DM to solve such problems. MADM has been one of the fastest growing areas during the last decades depending on the changing in the business sector; Hwang & Yoon [1], Turban [4]. We focus on MADM which is used in a finite 'selection' or choice problem. In real world problem, MADM play most important role. Now a day's television is the common in every person's life. Here, we take as an application of selection of television configuration. Generally, common people purchase 21" size for house purpose; therefore we choose the most common size. Mass customization, as a business strategy, aims at meeting diverse customer needs while maintaining near mass production efficiency, can implement both economies of scale and scope for an enterprise, and has become the goal that the companies pursue; Zhu & Jiang [7]. In order to reach the goal, companies are often forced to adopt differentiation strategy to offer customer more choices of products to meet the growing individualization of demand, by giving a more customer-centric role. The configuration approaches based on rules which are usually dependent on expert's experience to establish. The configuration is one of the most important ways to realize quickly product customization. But, in business, particularly through the internet, a customer normally develops in his mind some sort of ambiguity, given the choice of similar products. The main concern is the requirements of the customers with respect to configuration of television which are vague. The television is taken as an example to further verify the validity and the feasibility of the proposed method and compared with WPM by Millar & Starr [2].

Framework of product configuration based on uncertain customers requirements

Each attribute has a finite set of possible values, in which, the variant is defined by using attributes and attribute values. Together, all attributes and attribute values describe a complete range of the product family. Products in the same product family vary according to different attribute and its attribute value, choosing a product could be considered as a process of choosing its attributes and different attribute value. But, generally, it is difficult for a customer to express his requirements in a clear and unambiguous way, which is often due to the fact that he is not thoroughly familiar with the product which is the supplier offers. So, the requirements are often vague and fuzzy, preference weight varies with respect to different product attributes. We describe the customers' vague and uncertain requirements in the form of fuzzy number by using the method of representation of fuzzy set. It is also the design to solve the configuration problem of the uncertain environment. As we know, there are various attributes in different products, but in which some attributes, such as color, shape, and so on, are not suitable to be represented as a fuzzy number. These attributes are often clear in the customers mind and the customer could select the attribute value by seeing the virtual product model in some browser environment. In realistic configuration system, it could be achieved by selecting the corresponding attribute value that the customer prefers directly. By using the theory of fuzzy MADM, the requirements the customer decided with respect to corresponding product attribute can be regarded as an ideal product. Firstly, the uncertain attribute value the customer decided would be represented in the form of triangular fuzzy number or interval fuzzy number, which is the most common way to solve uncertain, imprecise problems. Moreover, as the attribute values of the alternate products for the customer to select are determinate, which are usually definite and known, therefore, it is impossible to measure directly the distance or similarity degree between the ideal product the customer wanted and the alternate products. The definite attribute value is converted into the form of the fuzzy number so as to compute the distance between two fuzzy numbers. When choosing a product from a number of similar alternatives, customer normally develops some sort of ambiguity. The ambiguity is mainly due to two reasons. Firstly, how to make a final product choice to purchase and secondly, on what basis the other products will be rejected. In order to answer the above questions, the customer may like to classify the products in different preference levels, preferably through some numerical strength of preference by Mohanty & Bhasker [3]. We adopt the triangular fuzzy number to represent the vague requirements provided by the customers it is shown by Fig. (1).

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & x > c \end{cases} \quad (1)$$

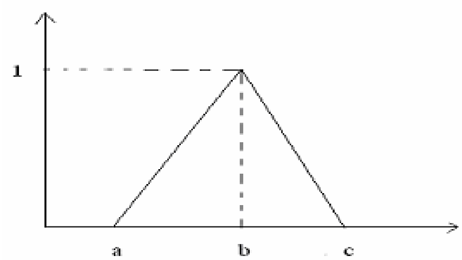

Fig. (1) Triangular fuzzy number

Fuzzy MADM methodology

As we know, when a customer chooses his preferred product from many candidate products, it is done, in fact, by comparing different attributes that could be used to describe product performance in different aspects, and ranking these products according to the customer's subjective preference. The customer requirements for products are usually uncertain and vague due to unable to understand product specifications comprehensively. On the other hand, the attribute values or specification of products offered by manufacturers are determinate and known. The model of fuzzy MADM has been introduced firstly by Yang & Chou [6]. The general MADM model can be described as follows:

- Let $X = \{X_i \mid i = 1, 2, ..., m\}$ denote a finite discrete set of $m(\ge 2)$ possible alternatives (courses of action, candidates);
- Let $A = \{A_j \mid j = 1, 2, ..., n\}$ denote a finite set of $n(\ge 2)$ attributes according to which the desirability of an alternative is to be judged,
- Let $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the vector of weights, where $\sum_{j=1}^{n} \omega_j = 1, \omega_j \ge 0, j = 1, 2..., n$, and ω_j denotes the weight of attribute A_j.
- Let $R = (r_{ij})_{m \times n}$ denote the $m \times n$ decision matrix, where $r_{ij}(\ge 0)$ is the performance rating of alternative X_i with respect to attribute A_j.

Normally, there are basically two types of attributes for a MADM problem, the first type is of 'cost' nature, and the second type is of 'benefit' nature. Since the attributes are generally incommensurate, the decision matrix needs to be normalized so as to transform the various attribute values into comparable ones. A common method of normalization is given as

$$Z_{ij} = \frac{r_{ij} - r_j^{\min}}{r_j^{\max} - r_j^{\min}}, i = 1, ..., m; j = 1, ..., n;$$

for benefit attribute (2)

and

$$Z_{ij} = \frac{r_j^{\max} - r_{ij}}{r_j^{\max} - r_j^{\min}}, i = 1, ..., m; j = 1, ..., n;$$

for cost attribute (3)

Where Z_{ij} is the normalized attribute value, $r_j^{\max}$ and $r_j^{\min}$ given by,

$$r_j^{\max} = \max(r_{1j}, r_{2j}, ..., r_{mj}) \ j = 1, ..., n; \quad (4)$$

$$r_j^{\min} = \min(r_{1j}, r_{2j}, ..., r_{mj}) \ j = 1, ..., n; \quad (5)$$

Let $Z = (Z_{ij})_{m \times n}$ be the normalized decision matrix. According to the SAW method, the overall weighted assessment value of alternative

$$d_i = \sum_{i=1}^{n} Z_{ij} \omega_j \quad i = 1, ..., m \quad (6)$$

Where d_i is a linear function of weight variables and the greater the value of d_i the better the alternative X_i. The aim of MADM is to rank alternatives or to determine the best alternative with the highest degree of desirability with respect to all relevant attributes. So, the best alternative is the one with the greatest overall weighted assessment value. The classic MADM techniques assume all r_{ij} values are crisp numbers. In the practical MADM problems, r_{ij} values can be crisp and/or fuzzy data. Fuzzy MADM methods have been developed due to the lack of precision in accessing the performance rating of alternatives with respect to an attribute, in which the representation of r_{ij} values are often linguistic terms or fuzzy numbers. Configuration approach based on fuzzy MADM is introduced in details the algorithm which includes the following steps:

Step 1: Representation of fuzzy requirements
When choosing a product from a number of similar alternatives, a customer normally develops in his mind some sort of ambiguity.

Step 2: Similarity measure
In step 1, the customer's requirements have been described as the triangular fuzzy number with respect to different product attributes. In this step, we will take the requirement vector as the ideal product the customer really wants, with the purpose to measure the similarity degree with the existing product vectors, in which the specification values are known and determinate. As we know, the fuzzy numbers can not be compared with crisp ones directly unless the unfuzzy numbers have to be transformed into the form of fuzzy numbers firstly. For example, for a crisp number *b*, the form of its triangular fuzzy can be written as follows:

$$\tilde{b} = (b^L, b^M, b^U) \quad (7)$$

Where $b^L = b^M = b^U$, and Similarity measure between two triangular fuzzy numbers can be calculated with Eq. (8); Xu [5],

$$s(a,b) = \frac{a^L b^L + a^M b^M + a^U b^U}{\max((a^L)^2 + (a^M)^2 + (a^U)^2, (b^L)^2 + (b^M)^2 + (b^U)^2)} \quad (8)$$

Where the above two triangular fuzzy numbers are $a = (a^L, a^M, a^U)$ and $b = (b^L, b^M, b^U)$, respectively.

In realistic configuration system, it could be achieved by selecting an attribute value the customer prefers from the given alternate options. The similarity measure of this type of attributes is defined as follows:

$$s(a', b') = \begin{cases} 1, & a' = b' \\ 0, & a' \neq b' \end{cases} \quad (9)$$

Step 3: Construction of Decision Matrix (*DM*). Calculation result of similarity measure between alternate products and the ideal product can be concisely expressed in a matrix format which is called decision matrix in MADM problems, and in which columns indicate product attributes and rows alternate products. Thus, an element S_{ij} in the in Eq. (10) denotes the similarity degree to the ideal product of the *i*th product with respect to the j^{th} attribute.

$$DM = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \dots & \dots & S_{ij} & \dots \\ S_{m1} & S_{m2} & \dots & S_{mn} \end{bmatrix} \quad (10)$$

Step 4: Normalization: In order to eliminate the difference of dimension among different attributes, the operation of normalization is needed to transform various attribute dimensions into the non-dimensional attribute.
Here, we adopt the Eqs. (11) and (12) to complete normalization of the fuzzy number.

$$\tilde{r}_i = \left(\frac{a_i}{c_i^{\max}}, \frac{b_i}{b_i^{\max}}, \frac{c_i}{a_i^{\max}} \wedge 1\right)$$ for benefit attribute(11)

$$\tilde{r}_i = \left(\frac{c_i^{\min}}{c_i}, \frac{c_i^{\min}}{b_i}, \frac{c_i^{\min}}{a_i} \wedge 1\right)$$ for cost attribute (12)

Where

$$(g)_i^{\max} = \max_i\{(g)_i\} \text{ and } (g)_i^{\min} = \min_i\{(g)_i\}$$

Step 5: Rank of the alternate products
The element S_{ij} in the decision matrix reflects the closeness degree of the ideal product with the *i*th alternate product with respect to the j^{th} attribute. In this step, we can use the SAW method, which is widely used in MADM, to calculate the utility value with respect to all attributes, with which the ranking order of alternate products according to utility value can be obtained. And we can consider the product with the highest utility value as the closest one to that of the customer requires. The utility value of i^{th} alternate product can be calculated with Eq. (13).

$$U_i = \sum_{j=1}^{n} x_{ij}\omega_j \quad , \quad i = 1, 2, \dots, m \quad (13)$$

And the maximum of utility value can be written as Eq. (14).

$$U_{\max} = \max_i \sum_{j=1}^{n} x_{ij}\omega_j \quad i = 1, 2, \dots, m \quad (14)$$

Here, we compared with the WPM by Millar & Starr [2] and check the feasibility of the customer's requirement

$$U_i = \prod_{j=1}^{n} x_{ij}^{\omega_j} \quad i = 1, 2, \dots, m \quad (15)$$

Case study
In this section, we take the television as an example to illustrate the method mentioned above. Table 1 shows the television that could

be used to configure for the different customers with respect to different attributes, in which the corresponding attributes are described as follows:

Table 1 Configuration of Television

Sr. No.	Speakers	Watt	Channels	Price
P1	6	1800	200	10300
P2	2	110	100	9790
P3	5	500	200	11990
P4	4	1200	200	12400
P5	2	200	200	9400
P6	2	400	100	11490
P7	2	250	200	9300
P8	4	500	200	9900

Suppose that the ideal product the customer wants according to the above attributes and the corresponding preference weight are shown in Table 2.

Table 2 The ideal product and attribute weight

Attributes	Ideal	Lower	Upper	Weight
Speakers	5	2	8	0.25
Watt	1000	200	2400	0.2
Channels	150	100	250	0.25
Price	10,000	9,000	12,000	0.30

The vector of the ideal product can be represented as the following form of the triangular fuzzy number.

$$\tilde{C} = [(2,5,8),(200,1000,2400),(100,150,250),(9'000,10'000,12'000)]$$

The corresponding vector of the attribute weight can be written as the follows: $\omega = (0.25, 0.20, 0.25, 0.30)$

The decision matrix, which shows the similarity degree with respect to each attribute between the ideal television that the customer desired and the candidate ones, is shown in Table 3 by using Eqs. (8)-(12).

Table 3 Decision Matrix

Sr. No.	U1	U2	U3	U4
P1	0.8333	0.6666	0.8333	0.9824
P2	0.3225	0.0582	0.5263	0.738
P3	0.8064	0.2647	0.8333	0.8618
P4	0.6451	0.6329	0.8333	0.8333
P5	0.3225	0.1058	0.8333	0.8966
P6	0.3225	0.2117	0.5263	0.897
P7	0.3225	0.1323	0.8333	0.887
P8	0.6451	0.2647	0.8333	0.9443

The utility value of all candidate products with respect to all attributes can be calculated by Eq. (13) and the final calculated results are given below:

Table 4 Utility value of each product configuration by SAW

P1	P2	P3	P4	P5	P6	P7	P8
0.8447	0.5039	0.7214	0.7462	0.5791	0.5243	0.5815	0.7058

The Table 4 presents the final utility value, with which the customer can rank the candidate products according to his preference to different attributes, and the order that shows the closeness degree to the customer requirements can be written as follows:

$$P_1 > P_4 > P_3 > P_8 > P_7 > P_5 > P_6 > P_2$$

Here, we compare the above method with the WP method and check the feasibility of the customer requirement calculated by Eq. (15), we get

Table 5 Utility value of each product configuration by WPM

P1	P2	P3	P4	P5	P6	P7	P8
0.8373	0.356	0.6637	0.7398	0.4446	0.4558	0.4635	0.6452

$$P_1 > P_4 > P_3 > P_8 > P_7 > P_6 > P_5 > P_2$$

Due to the uncertainty of the customers' requirements and the fact that different algorithms may yield different results, therefore, in the realistic configuration system, several products that have the higher similarity degree to that of the customer requires can be presented for customer to choose so as to satisfy the customer requirements to the greatest degree.

Conclusion

This paper proposes an approach to realize product level configuration according to the fuzzy and uncertain customer requirements by using the theory of the fuzzy MADM. The television is taken as an example to demonstrate feasibility of the proposed method for solving uncertain customer requirements. When results of SAW and WPM are compared, we get the same preferences to our problem and the optimal solution for selection of television is P_1.

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