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UNIT COMMITMENT USING PARTICLE SWARM OPTIMIZATION

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Abstract- An important criterion in power system operation is to meet the power demand at minimum fuel cost using an optimal mix of different power plants. Moreover, in order to supply electric power to customers in a secured and economic manner, thermal unit commitment is considered to be one of the best available options. It is thus recognized that the optimal unit commitment of thermal systems results in a great saving for electric utilities. Unit Commitment is the problem of determining the schedule of generating units subject to device and operating constraints. The formulation of unit commitment has been discussed and the solution is obtained by classical dynamic programming method. An algorithm based on Particle Swarm Optimization technique, which is a population based global search and optimization technique, has been developed to solve the unit commitment problem. The effectiveness of these algorithms has been tested on systems comprising three units and four units and compared for total operating cost.
Key words- Unit commitment, dynamic Programming, PSO.

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Introduction

Unit commitment (UC) is a nonlinear mixed integer optimization problem to schedule the operation of the generating units at minimum operating cost while satisfying the demand and other equality and inequality constrains. Several solution strategies have been proposed to provide quality solutions to the UC problem and increase the potential savings of the power system operation. These include deterministic and stochastic search approaches. Deterministic approaches include the priority list method, dynamic programming, Lagrangian Relaxation and the branch and- bound methods. Although these methods are simple and fast, they suffer from numerical convergence and solution quality problems. The stochastic search algorithms such as particle swarm optimization, genetic algorithms, evolutionary programming, simulated annealing, ant colony optimization and tabu search are able to overcome the shortcomings of traditional optimization techniques. These methods can handle complex nonlinear constraints and provide high quality solutions. This formulation drastically reduces the number of decision variables and hence can overcome the shortcomings of stochastic search algorithms for UC problems. Due to simplicity and less parameter tuning, particle swarm optimization is used for solving the unit commitment problem. In this thesis we have to study the algorithm of particle Swarm optimization and formulate the algorithm for solving unit commitment using PSO. In the results we have to find the variation in the results of total operating cost of the system in the given time horizon and compare it with the results of the already existing method like dynamic programming.

Formulation of unit commitment problem

The objective of the UC problem is to minimize the total operating costs subjected to a set of system and unit constraints over the scheduling horizon. It is assumed that the production cost, PC_i for unit '*i*' at any given time interval is a quadratic function of the generator power output, p_i.

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (1)$$

Where a_i, b_i, ci are the unit cost coefficients. The generator start-up cost depends on the time the unit has been switched off prior to the start up, T_{off}. The start-up cost SC_i at any given time is assumed to be an exponential cost curve.

$$SC_i = \sigma_i + \delta_i \{1 - e^{\frac{-T_{off,i}}{\tau_i}}\} \quad (2)$$

Where σi is the hot start-up cost, δ_i the cold start-up cost and τ_i is the cooling time constant

The total operating costs, OC_T for the scheduling period T is the sum of the production costs and the start-up costs.

$$OC_T = \sum_{t=1}^{T} \sum_{i=1}^{N} PC_{i,t} U_{i,t} + SC_{i,t} (1 - U_{i,t-1}) U_{i,t} \quad (3)$$

Where $U_{i,t}$ is the binary variable to indicate the on/off state of the unit i at time t. $U_{i,t}$ =1 if unit i is committed at time t, otherwise $U_{i,t}$=0.

The overall objective is to minimize OC_T subject to a number of system and unit constraints. All the generators are assumed to be connected to the same bus supplying the total system demand. Therefore, the networks constraints are studied above are as follows briefly.

a. Power Balance Constraint

The total generated power at each hour must be equal to the Load of the correspondinghour, *D.*

$$\sum_{i=1}^{N} p_{i,t} U_{i,t} = P_{D_t} \quad (4)$$

b. Power Generation Limits

The generation of the unit is under its minimum and maximum limit

$$P_{i,\min} \le P_i \le P_{i,\max} \quad (5)$$

c. Minimum Up Time

This constraint signifies the minimum time for which a committed unit should be turned off and removed from online.

$$T_{i,t}^{on} \ge MUT_i \quad (6)$$

d. Minimum Down Time

This constraint signifies the minimum time for which a de-committed unit should be turned on and brought on-line**.**

$$T_{i,t}^{off} \ge MDT_i \quad (7)$$

e. Spinning Reserve Constraints

Spinning reserve is the term used to describe the total amount of generation available from all the units synchronized on the system minus the present load plus losses being incurred. Spinning reserve must be carried so that the loss of one or more units does not cause too far a drop in system frequency

$$\sum_{i=1}^{N} p^{\max}{}_{i,t} U_{i,t} \ge P_{D_t} + R_t \quad (8)$$

Unit commitment using dynamic programming

Dynamic programming acts as an important optimization technique with broad application areas. It decomposes a problem into a series of smaller problems, solves them, and develops an optimal solution to the original problem step-by-step. The optimal solution is developed from the sub problem recursively. In its fundamental form, the dynamic programming algorithm for unit commitment problem examines every possible state in every interval. Some of these states are found to be infeasible and hence they are rejected instantly. But even, for an average size utility, a large number of feasible states will exist and the requirement of execution time will stretch the capability of even the largest computers. Hence many proposed techniques use only some part of simplification and approximation to the fundamental dynamic programming algorithm. Dynamic programming has many advantages over the enumeration scheme. The chief advantage of this technique is the reduction in the dimensionality of the problem. Suppose we have found units in a system and any combination of them could serve the single load. A maximum of 2N-1 combinations are available for testing. The imposition of priority list,

arranged in order of the full load average cost rate would result in a theoretically correct dispatch and commitment only if

- No load costs are zero.
- Unit input-output characteristics are linear between zero output and full load.
- There are no other restrictions.
- Start-up costs have a fixed amount

In dynamic programming algorithm:

- A state consists of an array of units with only specified units operating at a time and rest off-lin
- The start-up cost of a unit is independent of the time it has been off-line (i.e., it is a fixed amount).
- There are no costs for shutting down a unit.
- There is a strict priority order, and in each interval a specified minimum amount of capacity must be operating.

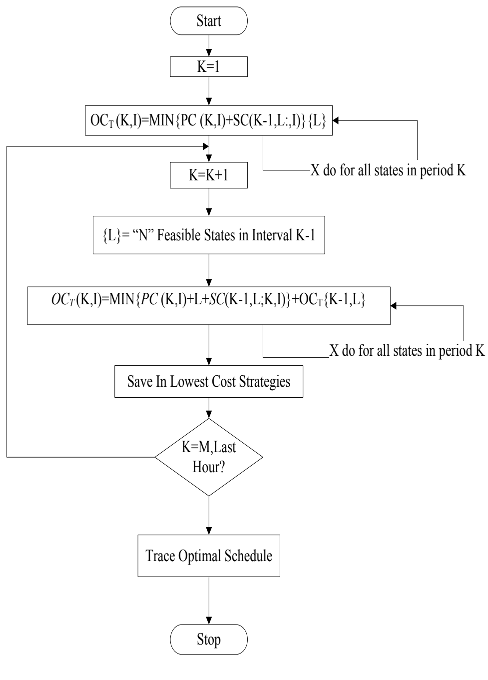

Fig. 1- Unit commitment by dynamic programming

A feasible state is one in which the committed units can be supply the required load and that meets the amount of capacity at each period The dynamic programming algorithm can be run backward in time starting from the final hour to be studied, back to the initial hour. Conversely, we have set the algorithm to run forward in time from the initial hour to the final hour. DP approach has distinct advantages in solving generator unit commitment. For example, if the start-up cost of a unit is a function of time it has been off-line (i.e., its temperature), then a dynamic programming approach is more suiTable since the previous history of the unit can be computed at each stage. There are other practical reasons for going for D.P. The initial conditions are easily specified and the computations can go forward in time as long as required. The flowchart for the Dynamic programming approach to Unit commitment problem is given below in “Fig. (1)”.

Particle swarm optimization

Particle swarm optimization is a stochastic, population-based search and optimization algorithm for problem solving. It is a kind of swarm intelligence that is based on social-psychological principles and provides insights into social behaviour, as well as contributing to engineering applications. The particle swarm optimization algorithm was first described in 1995 by *James Kennedy and Russell C. Eberhart*. The techniques have evolved greatly since then, and the original version of the algorithm is barely used at present. Social influence and social learning enable a person to maintain cognitive consistency. People solve problems by talking with other people about them, and as they interacts their beliefs, attitudes, and behaviour changes, the changes could typically be depicted as the individuals moving toward one another in a socio-cognitive space.

The particle swarm simulates a kind of social optimization. A problem is given, and some way to evaluate a proposed solution to it exists in the form of a fitness function. A communication structure or social network is also defined, assigning neighbours for each individual to interact with a population of individuals defined as random guesses as the problem solutions is initialized. These individuals are candidate solutions and are also known as the particles, hence the name particle swarm. An iterative process to improve these candidate solutions is set in motion. The particles iteratively evaluate the fitness of the candidate solutions and remember the location where they had their best success. The individual's best solution is called the particle best or the local best. Each particle makes this information available to their neighbours. They are also able to see where their neighbours have had success. Movements through the search space are guided by these successes, with the population usually converging, by the end of a trial, on a problem solution better than that of non-swarm approach using the same methods.The particle swarm optimization (PSO) algorithm is a population-based search algorithm inspired by the social behaviour of birds within a flock. The initial intent of the particle swarm concept was to graphically simulate the graceful and unpredicTable choreography of a bird flock, the aim of discovering patterns that govern the ability of birds to fly synchronously, and to suddenly change direction with a regrouping in an optimal formation. From this initial objective, the concept evolved into a simple and efficient optimization algorithm. In PSO, individuals, referred to as particles, are "flown" through hyper dimensional search space. Changes to the position of particles within the search space are based on the social-psychological tendency of individuals to emulate the success of other individuals. The changes to a particle within the swarm are therefore influenced by the experience, or knowledge, of its neighbours. The search behaviour of a particle is thus affected by that of other particles within the swarm therefore PSO is the kind of symbiotic cooperative algorithm. The consequence of modelling this social behaviour is that the search process is such that particles stochastically return toward previously successful regions in the search space. The operation of the PSO is based on the neighbourhood principle as social network structure.

Particle Swarm Optimization Algorithm

- Initialize the swarm, *p*(*t*), of particles such that the position $x_i(t)$ of each particle . $p(t)$ is random within the hyperspace, with t = 0.
- Evaluate the fitness function for each particle and find out the pbest.
- For each individual particle, compare the particle’s fitness value with its *pbest*. If the current value is better than the *pbest* value, then set this value as the and the current particle’s position, x_i, as p_i.
- Identify the particle that has the best fitness value. The value of its fitness function is identified as *gbest* and its position as p_g.
- Update the velocities and positions of all the particles.

$$v_i(t)=v_i(t-l)+C_1(xpbest_i-x_i(t))+C_2(xgbest-x_i(t)) \quad (9)$$

Where C_1 and C_2 are random variables. The second term above is referred to as the cognitive component, while the last term is the social component.

$$x_i\,(t) = x_i(t - 1) + v_i(t) \quad (10)$$

The flow chart is given as under. “Fig. (2)”

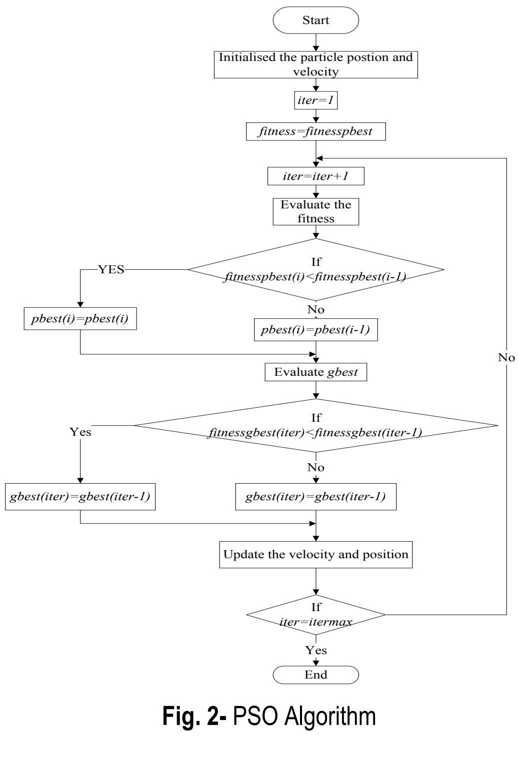

Fig. 2- PSO Algorithm

According to the discussion in above sections, the following procedure can be used for implementing the PSO algorithm.

- Initialize the swarm by assigning a random position in the problem search space to each particle.
- Evaluate the fitness function for each particle and find out the pbest.
- For each individual particle, compare the particle's fitness value with its *pbest*. If the current value is better than the *pbest* value, then set this value as the and the current particle's position, x_i, as p_i.
- Identify the particle that has the best fitness value. The value of its fitness function is identified as *gbest* and its position as p_g.
- Update the velocities and positions of all the particles using equation (9) and (10).
- Repeat steps b-e until a stopping criterion is met (e.g., maximum number of iterations or a sufficiently good fitness value). The flow chart is given above in Fig 2.

Unit Commitment Using Particle Swarm Optimization

The Particle swarm optimization (PSO) has been briefed earlier.PSO is a population based searching algorithm. This approach simulates the simplified social system such as fish schooling and birds flocking. PSO is initialized by a population of potential solutions called particles. Each particle flies in the search space with a certain velocity. The particle's flight is influenced by cognitive and social information attained during its exploration. It has very few tuneable parameters and the evolutionary process is very simple. It is capable of providing quality solutions to many complex power system problems. One such problem is the unit commitment of thermal units in the power system. PSO is used to minimize the total operating cost by committing those optimal combinations of the units which satisfy the constraints and gives the minimum cost corresponding to that combination.

Our main aim is to minimise the operating cost, so we are using the ALM method for handling equality and in equality constraints. In this problem the up and down time of the units are not taken into consideration. the algorithm for UC is detailed as follows

Algorithm

The following steps are used by the PSO technique to solve the unit commitment problem

- Initialize a population of particles p_i and other variables. Each particle is usually generated randomly with in allowable range.

$$P_{i,\min} \le P_i \le P_{i,\max} \quad (11)$$

Here p_i represented as i^{th} unit in the power system.

- Initialize the parameters such as the size of population, initial and final inertia weight, random velocity of particle, acceleration constant, the max generation, Lagrange's multiplier (λ_i), etc.
- Calculate the fitness of each individual in the population using the fitness function or cost function.

$$OC_T = \sum_{t=1}^{T}\sum_{i=1}^{N} PC_{i,t}U_{i,t} + SC_{i,t}(1-U_{i,t-1})U_{i,t} \quad (12)$$

Where $PC_{i,t}$ is represented as

$$F_i(P_i) = a_i + b_iP_i + c_iP_i^2 \quad (13)$$

With equality constraint as

$$\sum_{i=1}^{n} P_i = P_D \quad (14)$$

Where P_i is the i^{th} generators and P_D is the load or demand. And inequality constraints as

$$P_{i,\min} \le P_i \le P_{i,\max} \quad (15)$$

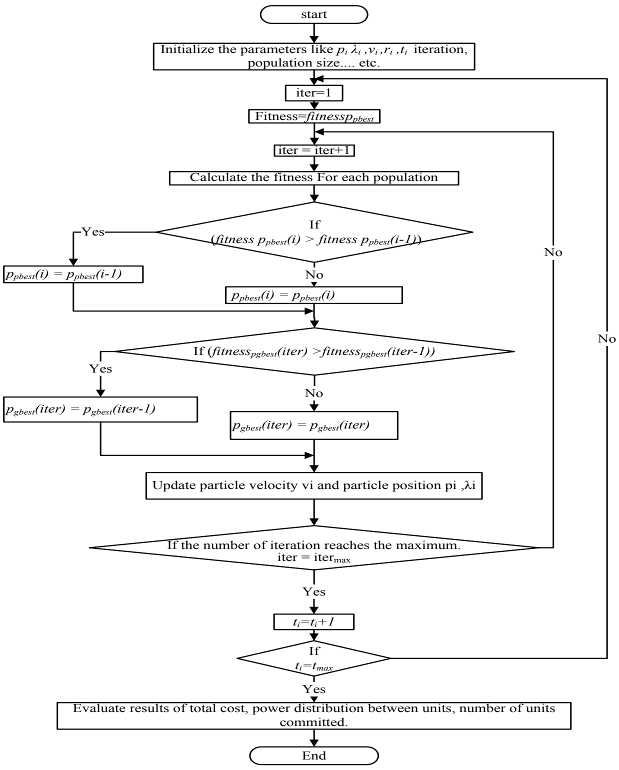

Fig. 3- Flow chart for solving unit commitment using PSO

- Compare each individual's fitness value with its *pbest*. The best fitness value among *pbest* is denoted as *gbest*.
- Modify the individual's velocity v_{id} of each individual p_i as

$$v_i^{(t)} = v_i^{(t-1)} + C_1 \times \text{rand}() \times \left(p_{pbesti} - p_i^{(t)}\right) + C_2 \times \text{rand}() \times \left(p_{gbest} - p_i^{(t)}\right) \quad (16)$$

- Modify the individual's position p_i as

$$p_i^{(t)} = p_i^{(t-1)} + p_i^{(t)} \quad (17)$$

where i is the i^{th} unit and t is the hour

- If the evaluation value of each individual is better than the previous p*pbest*, the current value is set to be *ppbest.* If the best *ppbest* is better than *pgbest* the value is set to be *pgbest.*
- Modify the λ and α for each equality and Inequality constraint

For Inequality Constraint

$$\alpha = max\left(inequality\ constraint, -\lambda\left(iter-1\right)/\left(2\times r\right)\right) \quad (18)$$

$$\lambda(iter) = \lambda(iter-1) \ + (2 \times r \times \alpha) \quad (19)$$

- For equality Constraint

$$\alpha = max\left(inequality\ constraint, -\lambda(iter-1)/(2 \times r)\right) \quad (20)$$

- Minimize the fitness function using PSO method for the number of units running at that time.
- If the number of iteration reaches the maximum then go to step k. Otherwise go to step c.
- The individual that generates the latest is the optimal generation power of each unit with the minimum total generation cost.

The flow chart of the above mention steps is developed as under in "Fig. (3)".

Result and discussion

The performance has been studied for three generator and four generator test data. The results for the respective systems are discussed as in Table 1, 2and 3 respectively

Test System

Three units are to be committed to serve 15-h load pattern. Data on the units and load pattern are contained in the given Table (1).The details of fuel cost components, initial conditions and load pattern are given

Table 2- Initial conditions.

Units	Initial condition	Start up cost hot	Start-up cost cold	Cold start-Time (h)
1	-5	150	350	4
2	8	170	400	5
3	8	500	1100	5

Table 3- Units characteristics, load pattern and initial status of the unit.

Hour	1	2	3	4	5	6	7	8
Load	1200	1150	1100	1050	1000	950	900	850
Hour	9	10	11	12	13	14	15	
Load	800	750	700	650	600	550	500	

Table 4-Result of 3-units, unit commitment problem using Dynamic Programming

S.NO	Load	Unit	Distribution of load among the units			Total Oper.cost 10^4(R)
1	1200	1 1 1	600	400	200	1.384896
2	1150	1 1 1	600	400	150	2.515248
3	1100	1 1 1	600	400	100	3.593869
4	1050	1 1 1	600	400	50	4.622505
5	1000	1 1 0	600	400	0	5.591561
6	950	1 1 0	550	400	0	6.540129
7	900	1 1 0	500	400	0	7.452047
8	850	1 1 0	450	400	0	8.340131
9	800	1 1 0	400	400	0	9.159183
10	750	1 1 0	350	400	0	9.943715
11	700	1 1 0	350	400	0	10.6581.6
12	650	1 1 0	250	400	0	11.298124
13	600	1 1 0	200	400	0	11.899672
14	550	1 1 0	150	400	0	12.463296
15	500	1 1 0	150	350	0	12.883241
		Total Operating Cost				12.883241

Dynamic Programming Results

The results obtained for the test system1 using dynamic programming are summarized in Table(4).

PSO Results

The results obtained from PSO are detailed in Table (5) for tree generator system. Correspondingly, the variation of fitness and Xgbest are shown in "Fig.(4)" and "Fig. (5)" respectively. The total operating cost is calculated,

the unit combination selected in each hour and the distribution of load among each unit. From "Fig.(4)", it is concluded that at first there is variation in the operating cost (fitnessgbest) and after some iteration the operating cost is set to its optimal point. i.e. The operating cost is minimized. Same is the case with "Fig. (5)", there are three units i.e. Unit1, Unit2, Unit3. As these are denoted by Xgbest1, Xgbest2, Xgbest3, the behaviour of these three units are also varying at first and then these are set to their optimal point.

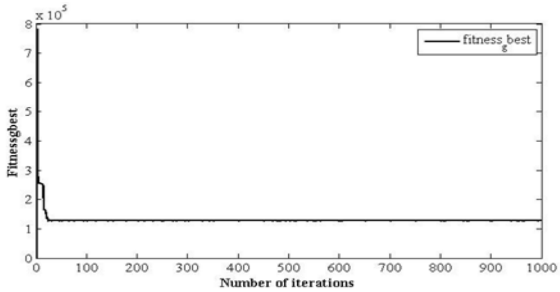

Fig. 4- Variation of fitness global best (Total Operating Cost)

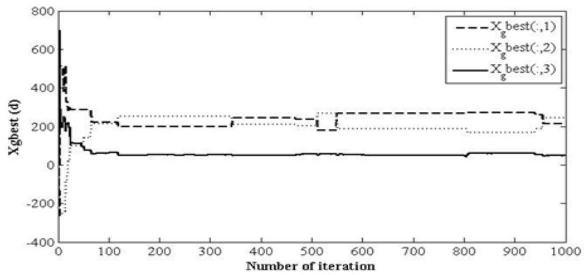

Fig. 5- Variation of Xgbest (Generating Units)

Test System 2

Four units are to be committed to serve an 8-h load pattern. The details of unit characteristics, fuel cost components, initial conditions and load pattern are given in Table(6),(7),(8),(9) respectively

Table 7- Fuel cost components

ai(R/h)	bi(R/MWh)	ci(r/MW²h)
684.74	16.83	.0021
585.62	16.95	0.0042
213	20.74	.0018
252	23.60	.0034

Table 8- Start up and start down costs and Initial conditions

Unit	Initial condition	Start up cost Hot (R)	Start up cost Cold (R)	Cold start Time(h)
1	-5	150	350	4
2	8	170	400	5
3	8	500	1100	5
4	-6	0	0.02	0

Table 9- load pattern

Hour(h)	1	2	3	4	5	6	7	8
Load(MW)	450	530	600	540	400	280	290	500

Table -10- Result of Dynamic Programming

S. No	Load (MW)	Unitcombination selected	Distribution of load among the units(MW)	Total production cost× 10⁴(R)
1	450	0 1 1 0	0 150 300 0	1.070836
2	530	0 1 1 0	0 230 300 0	2.135672
3	600	0 1 1 1	50 250 300 0	3.380708
4	540	0 1 1 0	0 240 300 0	4.463546
5	400	0 1 1 0	0 100 300 0	5.294382
6	280	0 0 1 0	0 0 280 0	5.851736
7	290	0 0 1 0	0 0 290 0	6.426550
8	500	0 1 1 0	0 200 300 0	7.477386
		Total Operating Cost		7.477386

Table11- Results of unit commitment using PSO

S. No	Load (MW)	Unit-combination selected	Distribution of load among the units(MW)	Total production cost× 10⁴ (R)
1	450	0 1 1 0	0 150.614 298.9599 0	1.06453544
2	530	0 1 1 0	0 230.154 299.8036 0	2.12745187
3	600	0 1 1 1	0 253.054 306. 35 40.0609	3.37214399
4	540	0 1 1 0	0 239.7138 300.635 0	4.45397219
5	400	0 1 1 0	0 125.0072 274.8632 0	5.27815693
6	280	0 0 1 0	0 0 279.9979 0	5.8343343
7	290	0 0 1 0	0 0 290.0055 0	6.40853994
8	500	0 1 1 0	0 199.4972 299.8515 0	7.4551818
		Total Operating Cost		7.4551818

Dynamic Programming Results

The results obtained for the test system2 using dynamic programming are summarized above in Table (10)

PSO Results

Results are coming according to given data for the four generator unit commitment problem. Here the total operating cost is calculated, the unit combination selected in each hour and the distribution of load among each unit. It is seen from the Table(11) that the total operating cost in this case is minimum as compared to the results obtained as seen in the Table(10) in case of dynamic programming Now, in "Fig.(6)" As at first the there is variation in the operating cost of the four units, but after few iteration the operating cost is minimized as it is set to its optimal point. In "Fig.(7)" Units (Xbgest) also shows the random behaviour at first then they also reach their optimal point.

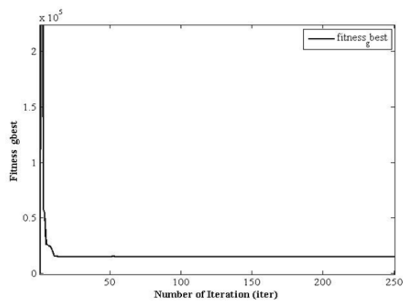

Fig. 6- Variation of Fitness global best (Total Operating Cost)

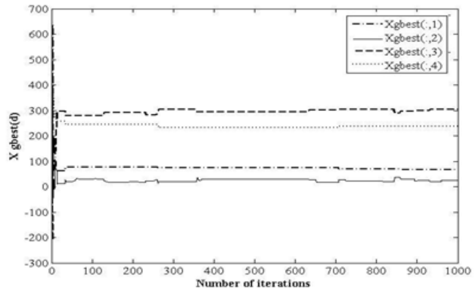

Fig. 7- Variation of X global best (Generation Units

Conclusions

It is recognized that the optimal unit commitment of thermal systems results in a great saving for electric utilities. Unit Commitment is the problem of determining the schedule of generating units subject to device and operating constraints. The formulation of unit commitment has been discussed and the solution is obtained by classical dynamic programming method. An algorithm based on Particle Swarm Optimization technique, which is a population based global search and optimization technique, has been developed to solve the unit commitment problem. The effectiveness of these algorithms has been tested on systems comprising three units and four units and compared for total operating cost. It is found that the result obtained from the unit commitment using particle swarm optimization are minimum than the results obtained from classical Dynamic programming.

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Table 1- Fuel cost component

Units	Max (MW)	Min (MW)	No-Load Cost (R/h)	Full load Ave. Cost (R/mWh)	Minimum Uptime (h)	Minimum DownTime (h)	Fuel cost component		
							ai (R/h)	bi (R/MWh)	Ci (R/MW²h)
1	600	150	213.00	9.79	4	2	561	7.92	0.001562
2	400	100	585.62	9.48	5	3	310	7.85	0.00194
3	200	50	684.74	11.188	5	1	93.6	9.564	0.005784

Table 5- Result of 3-units, unit commitment problem using PS0

S.NO	Load	Unitcombination selected	Distribution of load among the units			Total Operatingcost× 10^4 (R)
1	1200	1 1 1	603.9896	399.599	198.136	1.37046339
2	1150	1 1 1	601.5616	400.5913	150.6513	2.50086212
3	1100	1 1 1	591.0499	398.3093	110.6391	3.5765278
4	1050	1 1 1	589.3339	393.5762	67.5547	4.59971254
5	1000	1 1 0	598.918	400.0272	0	5.56370474
6	950	1 1 0	549.9872	400.0272	0	6.47869488
7	900	1 1 0	505.8089	394.8238	0	7.34589761
8	850	1 1 0	456.5077	391.9949	0	8.16592328
9	800	1 1 0	416.9848	383.22	0	8.93955607
10	750	1 1 0	408.8343	340.5459	0	9.66701498
11	700	1 1 0	373.684	325.8019	0	10.34873111
12	650	1 1 0	350.6772	299.3466	0	10.98512888
13	600	1 0 0	600.0123	0	0	11.57266111
14	550	1 0 0	550.0049	0	0	12.1116111
15	500	1 0 0	500.0048	0	0	12.60276111
		Total Operating Cost				12.60276111

Table 6- Unit characteristics

Units	Max(MW)	Min(MW)	Incremental cost(R/MWh)	NoLoadCost (R/h)	Full load Ave.Cost (R/Mh)	Min.Uptime(h)	Min.DownTime (h)
1	80	25	20.88	213.00	23.54	4	2
2	250	60	18.00	585.62	20.34	5	3
3	300	75	17.46	684.74	19.74	5	1
4	60	20	23.80	252.00	28.00	1	1