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Fuzzy EOQ model for deteriorating items with two warehouses

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Abstract- In this paper an EOQ model for deteriorating items using two warehouses is developed in fuzzy sense. A rented warehouse is used to store the excess units over the fixed capacity of the own warehouse. The capacity of rented warehouse is unlimited. Deterioration rates of two warehouses are considered to be different due to change in environment. The parameters such as holding cost and deteriorating cost for two warehouses are considered as fuzzy number. Triangular and trapezoidal both types of fuzzy number are considered to represents the fuzzy parameters. Total inventory costs as well as optimum order quantity are obtained in fuzzy sense. Signed distance and Function principle methods are used to defuzzification. Solution procedure is illustrated by a numerical example.
Keywords : Fuzzy model, Crisp model, Signed distance, Function principle.

1. Introduction

In classical inventory policy the models are developed mainly for the situation of single warehouse facility, whereas in real life situations more than one warehouse would be required for holding large inventory. A two warehouse inventory system can be described as one with finite capacity say 'Own warehouse' (OW) and if the system exceeds the capacity of OW then the other warehouse with a unlimited capacity i.e., 'Rented warehouse' (RW) is used. To satisfy a demand, RW is used first and OW is used only after the inventory in RW is finished. Deterioration of an item in the storage period is a natural process. Therefore it cannot be ignored in inventory policy. It may be different in different storage places due to the difference in environment. In this model the deterioration rate of RW and OW are considered as α and β respectively. The carrying cost may also vary. Obviously the rented warehouse (RW) has a more carrying cost as compare to OW. Certain models have been developed in the area of deteriorating inventories considering the demand rate to be constant, stock-dependent, time-dependent, ramp type or selling price dependent. Time dependent demand is further classified in to two kinds such as discrete in time and continuous in time. Chang H.J. and Dye C.Y. [2], Goswami and Chudhari [4], are developed an inventory models for deteriorating items with linearly time varying demand, whereas Bahari – Kashani [1], Hariga and Benkherouf [5] used the demand rate which increases over time. In crisp inventory models, all the parameters in the total inventory cost are known and have definite values without ambiguity. But in reality, it is not so certain. Hence there is a need to consider the fuzzy inventory models. Due to the various fuzzy cases, one may consider different fuzzy inventory models as follows. Currently Jing-Shing Yao and Huey-Ming Lee[8] used trapezoidal fuzzy number to obtain fuzzy order quantity. Jing-Shing Yao and Jershan Chiang [7] discussed fuzzy inventory model without backorder. They apply Centroid and Signed distance method to defuzzify total cost. Jershan Chiang, Jing-shing Yao and Huey-Ming Lee [6] used Signed distance method for backorder fuzzy inventory model. Shan-Huo Chen and Chien-Chung Wang [9] and Shan-Huo Chen[3] used the Function principle method for defuzzification. In this paper the parameters such as holding cost and deteriorating cost for two warehouses are considered as fuzzy number. Triangular and trapezoidal both types of fuzzy number are considered to represents the fuzzy parameters. Total inventory costs as well as optimum order quantity are obtained in fuzzy sense. Signed distance and Function principle methods are used to defuzzification. Solution procedure is illustrated by a numerical example.

2. Assumptions

1) The scheduling period is constant and no lead-time.
2) Demand rate is time dependent, $D = ae^{bt}$ where a, b are non-negative constants.
3) Deteriorating rate is age specific failure rate and is different for both warehouses due to the difference in environment.
4) Replenishment rate is infinite.

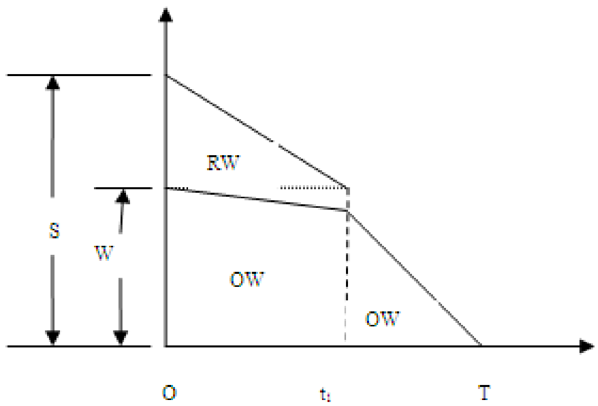
3. Notations

- T : Scheduling time of one cycle.
- OW : Own warehouse
- C_{1o} : Holding cost per unit in OW
- C_d : Deteriorating cost per unit.
- β : Deteriorating rate in OW
- W : Fixed stock level in OW
- $Qr(t)$: Inventory level at time t.
- D : Demand rate per unit time;
- RW : Rented warehouse
- C_{1r} : Holding cost per unit in RW
- α : Deteriorating rate in RW
- S : Initial stock level
- $Qo(t)$: Inventory level at time t.
- C_H : Total Holding cost per cycle.

C_D : Total deteriorating cost per cycle. TC : Total Inventory cost

(wavy bar (~) represents the fuzzification of the parameters)

4.4 Figure



At the beginning of the period the initial stock level is ‘ S ’ from which OW is filled with fixed capacity ‘ W’, and remaining ‘S - W’ units filled in RW. Demands are met from RW and when it becomes empty the inventory in OW is used for meeting up the demand. The inventory decreases in the system (shown in the figure) is mainly by demand and partially due to the deterioration in both warehouses. As a result, RW gets emptied at time t_1 and OW is emptied at time T.

5. Crisp Model

i) Situation in RW in time interval $(0, t_1)$ is given by ,

$$\frac{d}{dt}Q_r(t)+\alpha Q_r(t)=-ae^{bt} \quad ; \quad 0\le t\le t_1 \qquad \text{---(1)}$$

Solving above differential equation using boundary condition at t = 0, Q(t) = S - W , We get ,

$$Q_r(t)=-\frac{a}{(\alpha+b)}e^{bt}+\left[(S-W)+\frac{a}{(\alpha+b)}\right]e^{-\alpha t} \; ; \; 0\le t\le t_1 \qquad \text{---(2)}$$

using boundary condition at $t = t_1$, $Q(t_1) = 0$, we get

$$t_1=\frac{1}{(\alpha+b)}\log\left\{1+\frac{(S-W)(\alpha+b)}{a}\right\} \qquad \text{---(3)}$$

ii) Situation in OW in time interval $(0, t_1)$ is given by ,

$$\frac{d}{dt}Q_o(t)+\beta Q_o(t)=0 \quad ; \quad 0\le t\le t_1$$

Solving above differential equation using boundary condition at t = 0, $Q_o(t)$ = W ,We get ,

$$Q_o(t)=We^{-\beta t} \quad ; \quad 0\le t\le t_1 \qquad \text{---(4)}$$

iii) Situation in OW in time interval (t_1, T) is given by ,

$$\frac{d}{dt}Q_o(t)+\beta Q_o(t)=-ae^{bt} \quad ; \quad t_1\le t\le T$$

Solving above differential equation using boundary condition at $t = t_1$, $Q(t_1)=We^{-\beta t_1}$, We get ,

$$Q_o(t)=-\frac{a}{(\beta+b)}e^{bt_1}+\left[W+\frac{a}{(\beta+b)}e^{(\beta+b)t_1}\right]e^{-\beta t}\; ; \; t_1\le t\le T \qquad \text{---(5)}$$

Holding Cost :

I) In RW

$$C_{H_r}=C_{1r}\int_0^{t_1}Q_r(t)dt \quad ; \quad 0\le t\le t_1 \qquad \text{---(6)}$$

II) In OW

$$CH_2=C_{1o}\left[\int_0^{t_1} Q_o(t)dt+\int_{t_1}^{T} Q_o(t)dt\right] \quad ---(7)$$

$\therefore$ Total holding cost is

$$CH=CH_1+CH_2$$

Deteriorating Cost :

I) In RW :

$$CD_1=Cd\int_0^{t_1}\alpha Q_r(t)dt \quad ; \quad 0\le t\le t_1 \quad ---(8)$$

II) In OW

$$CD_2=Cd\left[\int_0^{t_1}\beta Q_o(t)dt+\int_{t_1}^{T}\beta Q_o(t)dt\right] \quad ---(9)$$

$\therefore$ Total holding cost is $CD=CD_1+CD_2$

$\therefore$ Total inventory cost is $TC=CH+CD$

$$TC=(C_{1r}+Cd.\alpha)\int_0^{t_1} Q_r(t)dt+(C_{1o}+Cd.\beta)\left[\int_0^{t_1} Q_o(t)dt+\int_{t_1}^{T} Q_o(t)dt\right]$$

$$TC=(C_{1r}+Cd.\alpha)\left[-\frac{a}{b(\alpha+b)}\left(e^{bt_1}-1\right)-\left[\frac{(S-W)(\alpha+b)+a}{\alpha(\alpha+b)}\right]\left(e^{-\alpha t_1}-1\right)\right]$$
$$+(C_{1o}+Cd.\beta)\left\{\left[-\frac{W}{\beta}\left(e^{-\beta t_1}-1\right)\right]+\left[-\frac{a}{b(\beta+b)}\left(e^{-bT}-e^{-bt_1}\right)\right]\right. \quad ---(10)$$
$$\left.-\left[\frac{W(\beta+b)+a\beta e^{(\beta+b)t_1}}{\beta(\beta+b)}\right]\left(e^{-\beta T}-e^{-\beta t_1}\right)\right\}$$

The above equation can be simplified using series form of logarithmic term and ignoring second and higher terms we get, $t_1=(s-w)/a$ and eq^n (10) becomes,

$$TC=(C_{1r}+Cd.\alpha)\left[\frac{(s-w)^2}{a}\right]+(C_{1o}+Cd.\beta)\left\{\left[w\frac{(s-w)}{a}\right]+\right.$$
$$\left.\left[-\frac{a}{(\beta+b)}\left(T-\left(\frac{(S-W)}{a}\right)\right)\right]-\left[w+\beta(S-W)+\frac{a\beta}{(\beta+b)}\right]\left(\left(\frac{(s-w)}{a}\right)-T\right)\right\} \quad ---(11)$$

To obtain optimum order quantity differentiating TIC partially w.r.t. S and equate to zero. The optimum order quantity is

$$S=W+\left[\frac{a(C1o+Cd\beta)\left[\frac{(1-\beta)}{(b+\beta)}-T\beta\right]}{2\left[C1r+Cd\alpha-\beta(C1o+Cd\beta)\right]}\right] \quad ---(12)$$

6. Fuzzy Model

In the above developed crisp model, it was assumed that all the parameters were fixed or could be predicted with certainty, but in real life situations, they will fluctuate little from the actual values. Therefore the parameters of model could not be assumed constant. Usually rate of deterioration is vague in nature, thus instead of considering rate of deterioration as constant the EOQ model is developed with the assumption that deterioration rate is a fuzzy number. Similarly holding cost is also considered as a fuzzy number. In this model deterioration rates and holding costs for both warehouses are represented by triangular and trapezoidal fuzzy numbers. By using Signed distance and function principle methods fuzzy total cost and fuzzy optimum ordered quantity is obtained.

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Method I) Signed Distance Method

In this method deterioration rates and holding costs are represented by triangular fuzzy numbers.

Let the fuzzy numbers C_1r, C_1o, α and β are located in intervals as:

1) $C_1r \in [C_1r-\Delta_1, C_1r+\Delta_2]$ where $0<\Delta_1<C_1r$ and $0<\Delta_1\Delta_2$

2) $C_1o \in [C_1o-\Delta_3, C_1o+\Delta_4]$ where $0<\Delta_3<C_1o$ and $0<\Delta_3\Delta_4$

3) $\alpha \in [\alpha-\Delta_5, \alpha+\Delta_6]$ where $0<\Delta_5<\alpha$ and $0<\Delta_5\Delta_6$

4) $\beta \in [\beta-\Delta_7, \beta+\Delta_8]$ where $0<\Delta_7<\beta$ and $0<\Delta_7\Delta_8$

The signed distance of these fuzzy numbers are

$$1)\ d(\tilde{C}_1r,0)=C_1r+\frac{1}{4}(\Delta_2-\Delta_1) \qquad 2)\ d(\tilde{C}_1o,0)=C_1o+\frac{1}{4}(\Delta_4-\Delta_3)$$

$$3)\ d(\tilde{\alpha},0)=\alpha+\frac{1}{4}(\Delta_6-\Delta_5) \qquad 4)\ d(\tilde{\beta},0)=\beta+\frac{1}{4}(\Delta_8-\Delta_7)$$

By [10] , we have $T\tilde{C}=(TC_1,TC_2,TC_3)$

where

$$TC_1=[(C_1r-\Delta_1)+Cd.(\alpha-\Delta_5)]\left[\frac{(S-W)^2}{a}\right]+[(C_1o-\Delta_3)+Cd.(\beta-\Delta_7)]$$
$$\left\{\left[W\frac{(S-W)}{a}\right]+\left[-\frac{a}{((\beta-\Delta_7)+b)}\left(T-\left(\frac{(S-W)}{a}\right)\right)\right]\right.$$
$$\left.-\left[W+(\beta-\Delta_7)(S-W)+\frac{a(\beta-\Delta_7)}{((\beta-\Delta_7)+b)}\right]\left(\left(\frac{(S-W)}{a}\right)-T\right)\right\}$$

$$TC_2=TC$$

$$TC_3=[(C_1r+\Delta_2)+Cd.(\alpha+\Delta_6)]\left[\frac{(S-W)^2}{a}\right]+[(C_1o+\Delta_4)+Cd.(\beta+\Delta_8)]$$
$$\left\{\left[W\frac{(S-W)}{a}\right]+\left[-\frac{a}{((\beta+\Delta_8)+b)}\left(T-\left(\frac{(S-W)}{a}\right)\right)\right]\right.$$
$$\left.-\left[W+(\beta+\Delta_8)(S-W)+\frac{a(\beta+\Delta_8)}{((\beta+\Delta_8)+b)}\right]\left(\left(\frac{(S-W)}{a}\right)-T\right)\right\}$$

The fuzzy total inventory cost $T\tilde{C}$ defuzzify by signed distance method as follows

$$d(T\tilde{C})=TC+\frac{1}{4}[(\Delta_2-\Delta_1)+Cd.(\Delta_6-\Delta_5)]\left[\frac{(S-W)^2}{a}\right]+\frac{1}{4}[(\Delta_4-\Delta_3)$$
$$+Cd.(\Delta_8-\Delta_7)]\left\{\left[W\frac{(S-W)}{a}\right]+\left[-\frac{a}{\left(\frac{1}{4}(\Delta_8-\Delta_7)+b\right)}\left(T-\left(\frac{(S-W)}{a}\right)\right)\right]\right.$$
$$\left.-\left[W+\frac{1}{4}(\Delta_8-\Delta_7)(S-W)+\frac{a(\Delta_8-\Delta_7)}{((\Delta_8-\Delta_7)+4b)}\right]\left(\left(\frac{(S-W)}{a}\right)-T\right)\right\}$$
$$=[TC]_d \qquad \text{---(13)}$$

For optimum order quantity S , differentiating $[TC]_d$ with respect to S and equating to zero, we get

$$S_d=W+\left[\frac{\dfrac{a}{2}\left[\begin{array}{l}[c_{1o}+cd.\beta]\left[\dfrac{\beta-1}{\beta+b}\right]+\left[\dfrac{1}{4}(\Delta_4-\Delta_3)+cd.\dfrac{1}{4}(\Delta_8-\Delta_7)\right]\left[\dfrac{\left[\frac{1}{4}(\Delta_8-\Delta_7)-1\right]}{\left[\frac{1}{4}(\Delta_8-\Delta_7)+b\right]}\right]\\ -T.\left[\left[c_{1o}.\beta+\left[\dfrac{1}{4}(\Delta_4-\Delta_3)\right]\left[\dfrac{1}{4}(\Delta_8-\Delta_7)\right]\right]+cd.\left[\beta^2+\left[\dfrac{1}{4}(\Delta_8-\Delta_7)\right]^2\right]\right]\end{array}\right]}{\begin{array}{l}\left[c_{1r}+\dfrac{1}{4}(\Delta_2-\Delta_1)+cd\left[\alpha+\dfrac{1}{4}(\Delta_6-\Delta_5)\right]\right]-\left[c_{1o}.\beta+\left[\dfrac{1}{4}(\Delta_4-\Delta_3)\right]\left[\dfrac{1}{4}(\Delta_8-\Delta_7)\right]\right]\\ -cd.\left[\beta^2+\left[\dfrac{1}{4}(\Delta_8-\Delta_7)\right]^2\right]\end{array}}\right] \quad ---(14)$$

Method II) Function Principle Method

In this method deterioration rates and holding costs are represented by trapezoidal fuzzy numbers.

Let the fuzzy numbers C_1r, C_1o, α and β are represented as:

1) $\tilde{C}_1r=(C_1r_1,C_1r_2,C_1r_3,C_1r_4)$ 2) $\tilde{C}_1o=(C_1o_1,C_1o_2,C_1o_3,C_1o_4)$

3) $\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ 3) $\tilde{\beta}=(\beta_1,\beta_2,\beta_3,\beta_4)$

By using Function Principle method, the membership function of $T\tilde{C}$ can be defined as

$T\tilde{C}=(TC_1,TC_2,TC_3,TC_4)$, where

$$TC_i=(C_{1ri}+Cd.\alpha_i)\left[\frac{(S-W)^2}{a}\right]+(C_{1oi}+Cd.\beta_i)\left\{\left[W\frac{(S-W)}{a}\right]+\left[-\frac{a}{(\beta_i+b)}\left(T-\left(\frac{(S-W)}{a}\right)\right)\right]-\left[W+\beta_i(S-W)+\frac{a\beta_i}{(\beta_i+b)}\right]\left(\left(\frac{(S-W)}{a}\right)-T\right)\right\}$$

$i=1,2,3,4.$

let by applying the median rule to find minimization of $T\tilde{C}$. The median TC_m of $T\tilde{C}$ can be derived from

$$[(TC_m-TC1)+(TC_m-TC2)]/2=[(TC4-TC_m)+(TC3-TC_m)]/2$$

$$TC_m=(TC1,+TC2,+TC3,+TC4)/4$$

By using median rule TC_m can be obtained as

$$TC_m=\frac{1}{4}\Sigma(C1ri+Cd.\alpha i)\left[\frac{(S-W)^2}{a}\right]+\left[\frac{1}{4}\Sigma(C1oi+Cd.\beta i)\right]\left\{\left[W\frac{(S-W)}{a}\right]+\left[-\frac{a}{\left(\frac{1}{4}\Sigma\beta i+b\right)}\left(T-\left(\frac{(S-W)}{a}\right)\right)\right]-\left[W+\frac{1}{4}\Sigma\beta i(S-W)+\frac{a\frac{1}{4}\Sigma\beta i}{\left(\frac{1}{4}\Sigma\beta i+b\right)}\right]\left(\left(\frac{(S-W)}{a}\right)-T\right)\right\}$$

$i=1,2,3,4.$

---(15)

and optimum order quantity is

$$S=W+\left[\frac{a\left(\frac{1}{4}\sum C1o+Cd\frac{1}{4}\sum\beta\right)\left[\frac{\left(1-\frac{1}{4}\sum\beta\right)}{\left(b+\frac{1}{4}\sum\beta\right)}-T\frac{1}{4}\sum\beta\right]}{2\left[\frac{1}{4}\sum C1r+Cd\frac{1}{4}\sum\alpha-\frac{1}{4}\sum\beta\left(\frac{1}{4}\sum C1o+Cd\frac{1}{4}\sum\beta\right)\right]}\right] \qquad ---(16)$$

7. Numerical Example

1) Crisp model

$C_{1o}=4$, $C_{1r}=4.5$, $C_d=1.1$, $T=3$, $W= 50$

$a=5$, $b=0.5$, $\alpha=2.0$, $\beta=1.5$

Then we obtain $S=83.8204$ $t_1=1.1541$ $TC=293.3733$

2) Fuzzy model

I) Signed Distance Method

Let $\Delta_1=1.5$, $\Delta_2=1$, $\Delta_3=1.5$, $\Delta_4=1$, $\Delta_5=.15$, $\Delta_6=.1$, $\Delta_7=.15$, $\Delta_8=.1$

Then $t_1=1.1247$, $S=80.9707$, $TC_d=275.2286$

II) Function Principle Method

Let $c1r= (4,4.5,5,5.5)$, $c1o= (3.5,4,4.5,5)$, $\alpha = (1.8,2,2.2,2.4)$, $\beta = (1.3,1.5,1.7,1.9)$

Then $t_1=1.0364$, $S=76.5364$, $TC_d=274.7250$

8. Conclusion

The optimum results of fuzzy model defuzzified by Signed distance method and Function principal method are based on more realistic situations. For simplicity only one case is to be taken for decision parameters in both methods but they are changes in different manner. Decision maker can select the suitable values for these parameters to obtain optimum results.

Future Scope

The proposed methods can be used for another inventory models. One can use the linear and / or different non-linear membership functions to develop the proposed model in fuzzy sense.

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