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VIBRATION ANALYSIS OF BEAMS

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Abstract- Vibration analysis of a beam is an important and peculiar subject of study in mechanical engineering. All real physical structures, when subjected to loads or displacements, behave dynamically. The additional inertia forces, from Newton's second law, are equal to the mass times the acceleration. If the loads or displacements are applied very slowly then the inertia forces can be neglected and a static load analysis can be justified. Hence, dynamic analysis is a simple extension of static analysis.

Many developments have been carried out in order to try to quantify the effects produced by dynamic loading. Examples of structures where it is particularly important to consider dynamic loading effects are the construction of tall buildings, long bridges under wind-loading conditions and buildings in earthquake zones, etc.

Dynamic structures subjected to periodic loads compose a very important part of industrial machineries. One of the major problems in these machineries is the fatigue and the cracks initiated by the fatigue. These cracks are the most important cause of accidents and failures in industrial machinery. In addition, existing of the cracks may cause vibration in the system. Thus an accurate and comprehensive investigation about vibration of cracked dynamic structures seems to be necessary. On the base of these investigations the cracks can be identified well in advance and appropriate measures can be taken to prevent more damage to the system due to the high vibration level.

Typical situations where it is necessary to consider more precisely the response produced by dynamic loading are vibrations due to equipment or machinery, impact load produced by traffic, snatch loading of cranes, impulsive load produced by blasts, earthquakes or explosions. So it is very important to study the dynamic nature of structures.

Keywords- Vibration, FSM, GQDM, FEM, CEM

1. Introduction

1.1. Overview

Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges, tall buildings, and robot arms, beams as well as the presence of cracks in the structural components can have a significant influence on the dynamic responses of the whole structure; it can lead to the catastrophic failure of the structure. To predict the Failure, vibration monitoring can be used to detect changes in the dynamic responses and/or dynamic characteristics of the structure. Knowledge of the effects of cracks on the vibration of the structure is of importance. Efficient techniques for the forward analysis of cracked beams are required. In this paper various techniques or approaches that can analyze the vibration of beams or structures with or without cracks.

A promising approach for developing a solution for structural vibration problems is provided by an advanced numerical discretisation scheme, such as; finite element method (FEM).The finite element method (FEM) is the dominant discretization technique in structural mechanics. The differential quadrature method (DQM) was first advanced by Bellman and his associates in the early 1970s aiming towards offering an efficient numerical method for solving non-linear partial differential equations. The method has since been applied successfully to various problems. in third order shear deformation theory free vibration of beams with different boundary conditions is analyzed. The boundary conditions of beams are satisfied by using Lagrange multipliers.

Fourier series will be utilized for the solution of simply supported beams with different loadings in order to arrive at a free vibration. The equation of the free vibration is $\{(\delta^2 y/ \delta t^2)/ (\delta 4y/ \delta x^4)\}$ One of the methods of solving this type of equation is the separation of the variables which assumes that the solution is the product of two functions, one defines the deflection shape and the other defines the amplitude of vibration with time. Modes of deflection with and without time along the beam were drawn for certain cases. To this end, the composite element method is then extended for free and forced vibration analysis of cracked beams. The principal advantage of the proposed method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points and the whole beam can be treated as a uniform beam. Moreover, the presented work can easily be extended to cracked beams with an arbitrary number of non-uniform segments.

1.2. Historical Prospective

Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges, tall buildings, and robot arms. The vibration of Euler–Bernoulli beams with one step change in cross-section has been well studied. Jang and Bert (1989) derived the frequency equations for combinations of classical end supports as fourth order determinants equated to zero. Bala subramanian and Subramanian (1985) investigated the performance of a four-degree-of-freedom per node element in the vibration analysis of a stepped cantilever.

De Rosa (1994) studied the vibration of a stepped beam with elastic end supports. Recently, Koplow et al. (2006) presented closed form solutions for the dynamic response of Euler–Bernoulli beams with step changes in cross section. There are also some works on the vibration of beams with more than one step change in cross-section. Bapat and Bapat (1987) proposed the transfer matrix approach for beams with n-steps but provided no numerical results. Lee and Bergman (1994) used the dynamic flexibility method to derive the frequency equation of a beam with n-step changes in cross-section. Jaworski and Dowell (2008) carried out a study for the free vibration of a cantilevered beam with multiple steps and compared the results of several theoretical methods with experiment.

A new method is presented to analyze the free and forced vibrations of beams with either a single step change or multiple step changes using the composite element method (CEM) (Zeng, 1998; Lu & Law, 2009).

1.3. Introductions to vibration

Vibrations are time dependent displacements of a particle or a system of particles with respect to an equilibrium position. If these displacements are repetitive and their repetitions are executed at equal interval of time with respect to equilibrium position the resulting motion is said to be periodic. One of the most important parameters associated with engineering vibration is the natural frequency.. Each structure has its own natural frequency for a series of different modes which control its dynamic behavior. Whenever the natural frequency of a mode of vibration of a structure coincides with the frequency of the external dynamic loading, this leads to excessive deflections and potential catastrophic failures. This is the phenomenon of resonance. An example of a structural failure under dynamic loading was the well-known Tacoma Narrows Bridge during wind induced vibration.

In practical application the vibration analysis assumes great importance. For example, vehicle-induced vibration of bridges and other structures that can be simulated as beams and the effect of various parameters, such as suspension design, vehicle weight and velocity, damping, matching between bridge and vehicle natural frequencies, deck roughness etc., on the dynamic behavior of such structures have been extensively investigated by a great number of researchers . The whole matter will undoubtedly remain a major topic for future scientific research, due to the fact that continuing developments in design technology and application of new materials with improved quality enable the construction of lighter and more slender structures, vulnerable to dynamic and especially moving loads. Every structure which is having some mass and elasticity is said to vibrate. When the amplitude of these vibrations exceeds the permissible limit, failure of the structure occurs. To avoid such a condition one must be aware of the operating frequencies of the materials under various conditions like simply supported, fixed or when in cantilever conditions.

1.3.1 Classification of vibration

Vibration can be classified in several ways. Some of the important classifications are as follows:

Free and forced vibration: If a system, after an internal disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of the simple pendulum is an example of free vibration. If a system is subjected to an external force (often, a repeatating type of force), the resulting vibration is known as forced vibration. The oscillation that arises in machineries such as diesel engines is an example of forced vibration. If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs, and the system undergoes dangerously large oscillations. Failuers of such structures as buildings, bridges, turbines and airplane have been associated with the occurrence of resonance.

Undamped and damped vibration: If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as undamped vibration. If any energy lost in this way, however, it is called damped vibration. In many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes. However, consideration of damping becomes extremely important in analyzing vibratory system near resonance.

Linear and nonlinear vibration: If all the basic components of vibratory system—the spring, the mass and the damper—behave linearly, the resulting vibration is known as linear vibration. If however, any of the basic components behave non-linearly, the vibration is called nonlinear vibration.

1.4 Crack

A crack in a structural member introduces local flexibility that would affect vibration response of the structure. This property may be used to detect existence of a crack together its location and depth in the structural member. The presence of a crack in a structural member alters the local compliance that would affect the vibration response under external loads.

1.4.1 Classification of Crack

Based on geometries, cracks can be broadly classified as follows:

(1) Transverse crack: These are cracks perpendicular to beam axis. These are the most common and most serious as they reduce the cross section as by weakens the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity or crack tip.

(2) Longitudinal cracks: These are cracks parallel to beam axis. They are not that common but they pose danger when the tensile load is applied at right angles to the crack direction i.e. perpendicular to beam axis.

(3) Open cracks: These cracks always remain open. They are more correctly called “notches”. Open cracks are easy to do in laboratory environment and hence most experimental work is focused on this type of crack

(4) Breathing crack: These are cracks those open when the affected part of material is subjected to tensile stress and close when the stress is reversed. The component is most influenced when under tension . The breathing of crack results in non-linearity in the vibration behavior of the beam. Most theoretical research efforts are concentrated on “transverse breathing” cracks due to their direct practical relevance.

(5) Slant cracks: These are cracks at an angle to the beam axis , but are not very common . There effect on lateral vibration is less than that of transverse cracks of comparable severity.

(6) Surface cracks: These are the cracks that open on the surface .They can normally be detected by dye-penetrates or visual inspection.

(7) Subsurface cracks: Cracks that do not show on the surface are called subsurface cracks . Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them.

1.5 Introduction to beam

A beam is generally considered to be any member subjected to principally to transverse gravity or vertical loading. The term transverse loading is taken to include end moments.

There are many types of beams that are classified according to their size, manner in which they are supported, and their location in any given structural system.

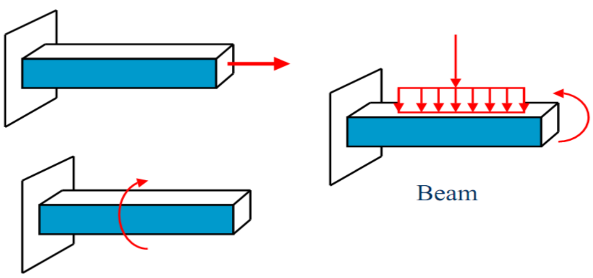


Fig. 1.5.1 Loading on Beams

Beams can be straight as shown in Figure 1.5.2.

• For example the straight member bde. Curved as shown in c.

• For example the curved member abc.

Beams are generally classified according to their geometry and the manner in which they are supported.

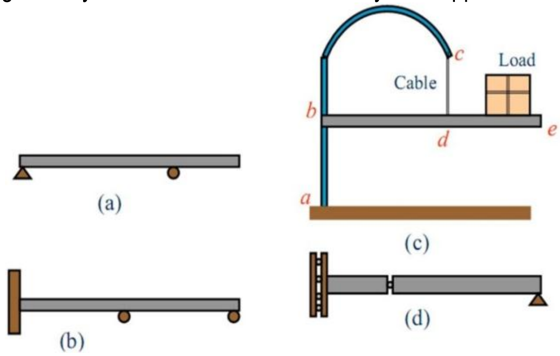


Fig.1.5.2 Loading on Beams

Geometrical classification includes such features as the shape of the cross section, whether the beam is Straight or – Curved Or whether the beam is Tapered, or – Has a constant cross section. Beams can also be classified according to the manner in which they are supported. Some types that occur in ordinary practice are shown in Figure 3, the names of some of these being fairly obvious from direct observation.

Note that the beams in (d), (e), and (f) are statically indeterminate.

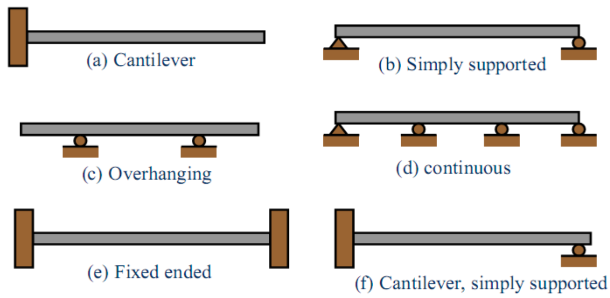


Fig. 1.5.3 Types of Beams Based on the Manner in Which They are supported.

A beam is a horizontal structural member used to support loads. Beams are used to support the roof and floors in buildings,

Fig.1.5.4. Beams to support the roof and floors in buildings

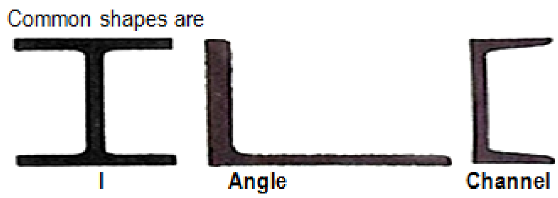


Fig.1.5.5 Shapes of Beam

Common materials are steel and wood. The parallel portions on an I-beam or H-beam are referred to as the *flanges*. The portion that connects the flanges is referred to as the web. Beams are supported in structures via different configurations Beams are designed to support various types of loads and forces.

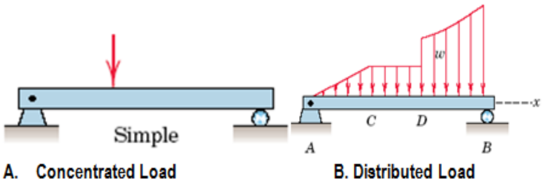


Fig.1.5.6 Load on Beam

Fig.1.5.6 Load on Beam

2. Literature Review

[1].Rao, Govardhana (2009) UK in Vibration Analysis of Beam analyze the vibration characteristics of beams. All real physical structures, when subjected to loads or displacements, behave dynamically. The additional inertia forces, from Newton's second law, are equal to the mass times the acceleration. If the loads or displacements are applied very slowly then the inertia forces can be neglected and a static load analysis can be justified. Hence, dynamic analysis is a simple extension of static analysis. Many developments have been carried out in order to try to quantify the effects produced by dynamic loading. Examples of structures where it is particularly important to consider dynamic loading effects are the construction of tall buildings, long bridges under wind-loading conditions and buildings in earth quake zones, etc.

Typical situations where it is necessary to consider more precisely the response produced by dynamic loading are vibrations due to equipment or machinery, impact load produced by traffic, snatch loading of cranes, impulsive load produced by blasts, earthquakes or explosions. So it is very important to study the dynamic nature of structures.

[2]. Sonam Lakra And Pradeep Guria , National Institute Of Technology, Rourkela (2011) make the analysis of cracked beamsin Vibration analysis of beam with multiple cracks. The present work deals with the free vibration analysis of a cracked beam with multiple transverse cracks using

finite element method. In this analysis, an overall additional flexibility matrix, instead of the local additional flexibility matrix is added to the flexibility matrix of the corresponding intact beam element to obtain the total flexibility matrix, and from there the result is compared with previous studies. The natural frequencies of free vibration of the beam with multiple cracks are computed. It is observed that with increase in number of cracks the natural frequencies decreases. The effect of cracks is more pronounced when the cracks are near to the fixed end than free end. The natural frequency decreases with increase in relative crack depth.

[3]. M. Behzad, a. Meghdari, a. Ebrahimi-Mechanical Engineering Department, Sharif University of Technology, Tehran, IRAN proposed new techniques for vibration analysis of a Cracked beam in A new approach for vibration analysis of a Cracked beam.

In this paper the equations of motion and corresponding boundary conditions for bending vibration of a beam with an open edge crack has been developed by implementing the Hamilton principle. A uniform Euler-Bernoulli beam has been used in this research. The natural frequencies of this beam have been calculated using the new developed model in conjunction with the Galerkin projection method. The crack has been modeled as a continuous disturbance function in displacement field which could be obtained from fracture mechanics. The results show that the natural frequencies of a cracked beam reduce by increasing crack depth. There is an excellent agreement between the theoretically calculated natural frequencies and those obtained using the finite element method.

[4]. Z.R. Lu, M. Huang and J.K. Liu Sun Yat-sen University P.R. China make the analysis by using new approach i.e. CME in Vibration Analysis of Beams with and without Cracks Using the Composite Element Model. Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges, tall buildings, and robot arms. The vibration of Euler–Bernoulli beams with one step change in cross-section has been well studied. The correctness and accuracy of the proposed method are verified by some examples in the existing literatures. The presence of cracks in the structural components, for instance, beams can have a significant influence on the dynamic responses of the whole structure; it can lead to the catastrophic failure of the structure. To predict the failure, vibration monitoring can be used to detect changes in the dynamic responses and/or dynamic characteristics of the structure. Knowledge of the effects of cracks on the vibration of the structure is of importance. Efficient techniques for the forward analysis of cracked beams are required. To this end, the composite element method is then extended for free and forced vibration analysis of cracked beams. The principal advantage of the proposed method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points and the whole beam can be treated as a uniform beam.

[5].MESUT ŞİMŞEK and TURGUT KOCATÜRK Department of Civil Engineering, Yildiz Technical University, Istanbul, Turkey make the vibration analysis of beams by using third order shear deformation theory in free vibration analysis of beams by using a third-order Shear deformation theory In this study, free vibration of beams with different boundary conditions is analyzed within the framework of the third-order shear deformation theory. The boundary conditions of beams are satisfied using Lagrange multipliers. To apply the Lagrange's equations, trial functions denoting the deflections and the rotations of the cross-section of the beam are expressed in polynomial form. Using Lagrange's equations, the problem is reduced to the solution of a system of algebraic equations. The first six eigenvalues of the considered beams are calculated for different thickness-to-length ratios. The results are compared with the previous results based on Timoshenko and Euler–Bernoulli beam theories.

3. Problem Classification

3.1 Introduction

According to S. H. Krandall (1956), engineering problems can be classified into three categories:

(i) Equilibrium problems
(ii) Eigen value problems
(iii)Propagation problems

(i)Equilibrium problems- are characterized by the structural or mechanical deformations due to quasi-static or repetitive loadings. In other words, in structural and mechanical systems the solution of equilibrium problems is a stress or deformation state under a given load. The modeling and analysis tasks are thus to obtain the system stiffness or flexibility so that the stresses or displacements *computed* accurately match the observed ones.

(ii)Eigen value problems can be considered as extensions of equilibrium problems in that their solutions are dictated by the same equilibrium states. There is an additional distinct feature in eigenvalue problems: their solutions are characterized by a unique set of system configurations such as resonance and buckling.

(iii)Propagation problems are to predict the subsequent stresses or deformation states of a system under the time-varying loading and deformation states. It is called initial value problems in mathematics or disturbance transmissions in wave propagation.

Modal testing is perhaps the most widely accepted words for activities involving the characterization of mechanical and structural vibrations through testing and measurements. It is primarily concerned with the determination of mode shapes (eigenvectors) and modes (eigenvalues), and to the extent possible the damping ratios of a vibrating system. Therefore, modal testing can be viewed as experimental solutions of eigenvalue problems. There is one important distinction between eigenvalue analysis and modal testing. Eigenvalue analysis is to obtain the eignvalues and eigenvectors from the analytically constructed governing equations or from a given set of mass and stiffness properties. There is no disturbance or excitation in the problem description. On the other hand, modal testing is to seek after the same eigenvalues and eigenvectors by injecting disturbances into the system and by measuring the system response. However, modal testing in earlier days tried to measure the so-called free-decay responses to mimic the steady-state responses of equilibrium problems.

3.2 Comparison of Engineering Analysis and System Identification

Parameters	Engineering analysis	System identification
Equilibrium	Construct the model first, and then obtain deformations under any given load.	Measure the dynamic input/output first, and then obtain the flexibility.
Eigen value	Construct the model first, and then obtain eigenvalues without any specified load.	Measure the dynamic input/output first, then obtain eigenvalues that corresponds to the specific excitation
Propagation	Construct the model first, and then obtain responses for time varying loads.	Measure the dynamic input/output first, and then obtain the model corresponds to the specific load.

Observe from the above Table that the models are first constructed in engineering analysis. In system identification the models are constructed only after the appropriate input and output are measured. Nevertheless, for both engineering analysis and system identification, *modeling* is a central activity. Observe also that, in engineering analysis, once the model is constructed it can be used for all of the three problems. On the other hand, the models obtained by system identification are usually valid only under the specific set of input and output pairs. The extent to which a model obtained through system identification can be applicable to dynamic loading and transient response measurements depends greatly upon the input characteristics and the measurement setup and accuracy.

3.3 Structural Modeling by System Identification

As noted in the previous section, modeling constitutes a key activity in engineering analysis. For example, the finite element method is a discrete structural modeling methodology. Structural system identification is thus a complementary endeavor to discrete modeling techniques. A comprehensive modeling of structural systems is shown in Fig. 1. The entire process of structural modeling is thus made of seven blocks and seven information transmission arrows (except the feedback loop). Testing consists of the first two blocks, Structures and Signal Conditioning along

With three actions, the application of disturbances as input to the structures, the collection of sensor output, and the processing of the sensor output via filtering for noise and aliasing treatment. **FFT** and **Wavelets Transforms** are software interface with the signal conditioniners. From the viewpoint of system identification, its primary role is to produce as accurately as possible impulse response functions either in frequency domain or in time domain variables. It is perhaps the most important software task because all the subsequent system realizations and the determination of structural model parameters *do* depend on the extracted impulse response data. About a fourth of this course will be devoted to learn methods and techniques for extracting the impulse response functions.
System realization performs the following task:
For the model problem of plant: $\dot{x} = Ax + Bu$
Given measurements of output: $y = Cx + Du$
Input: u,
Determine system characteristics: *A, B C* and *D*
Structural modeling block is to extract physical structural quantities from the system characteristics or realization parameters *(A, B, C, D)*. This is because realization characteristics still consist of abstract mathematical models, not necessarily in terms of the desired structural quantities.

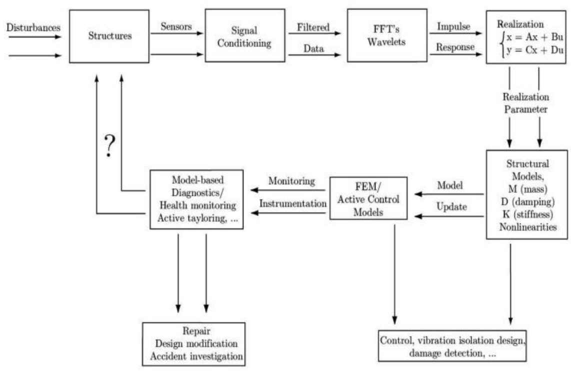


Fig.3.3.1. Chart for Analysis using CME

Specifically, one obtains,
Given Realization parameters: *A, B, C,* and *D*
Determine either Modal quantity: modes *(ω)* and mode shapes *(φ)*
Or physical matrices: mass *(***M***)*, stiffness *(***K***)* and damping *(***D***)*
Finite element model updating, active controls and health monitoring are the Beneficiaries of the preceding four activities. Hence, we will try to touch upon these topics, perhaps as term projects, depending on how this course progresses itself before the Thanksgiving recess. Finally, if necessary, one may have to repeat testing, hopefully this time utilizing the experience gained from the first set of activities. Even experienced experimentalists often must repeat testing. A good experimentalist rarely believes his/her initial results whereas a typical analyst almost always thinks his/her initial results are valid.

3.4 Analytical solution of vibrating structures

This section is a starting points of a guided tour, though an incomplete one at best, of modeling, analysis and structural system identification. To this end, we introduce a reference problem, which for our case is an analytically known model so that when we are astray from the tour path, we can all look up the map and hopefully steer ourselves back to the reference point and continue our tour.

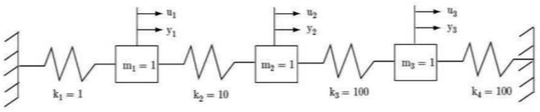


Fig.3.4.1. Three DOF Spring- Mass Systems

This study dealt with domain decomposition in the recently proposed generalized differential quadrature rule. In detail, the authors concentrated on the free vibration of multispan and stepped Euler beams, and beams carrying an intermediate or end concentrated mass. Since compatibility conditions should be implemented in a strong form at the junction of the sub domains concerned, the FEM techniques used for internal moments and shear forces must not be used. Compatibility conditions and their differential quadrature expressions were explicitly formulated. A peculiar phenomenon was found in deferential quadrature applications that equal length sub domains gave more accurate results than unequal length ones using the same number of sub domain grids. Various examples were presented and very accurate results have been obtained.

4. Methods of Vibration Analysis

4.1 Finite element method

4.1.1 Introduction

The field of Mechanics can be subdivided into three major areas:
--- Theoretical
Mechanics --- applied
--- Computational

Theoretical mechanics deals with fundamental laws and principles of mechanics studied for their intrinsic scientific value. Applied mechanics transfers this theoretical knowledge to scientific and engineering applications, especially as regards the construction of mathematical models of physical phenomena. Computational mechanics solves specific problems by simulation through numerical methods implemented on digital computers.

4.1.2 Computational Mechanics

Several branches of computational mechanics can be distinguished according to the physical scale of the focus of attention:

Computational Mechanics {
- Nanomechanics
- and micromechanics
- Continuum Mechanics
- System

Nano mechanics deals with phenomena at the molecular and atomic levels of matter. As such it is closely linked to particle physics and chemistry. A micromechanics looks primarily at the crystallographic and granular levels of matter. Its main technological application is the design and fabrication of materials and micro devices. Continuum mechanics studies bodies at the macroscopic level, using continuum models in which the microstructure is homogenized by phenomenological averages.

The two traditional areas of application are solid and fluid mechanics. The former includes structures which, for obvious reasons, are fabricated with solids. Computational solid mechanics takes an applied sciences approach, whereas computational structural mechanics emphasizes technological applications to the analysis And design of structures. Computational fluid mechanics deals with problems that involve the equilibrium and motion of liquid and gases. Well-developed subsidiaries are hydrodynamics, aerodynamics, acoustics, atmospheric physics, shock, combustion and propulsion.

A system is studied by decomposition: its behavior is that of its components plus the interaction between components. Components are broken down into subcomponents and so on. As this hierarchical breakdown process continues, individual components become simple enough to be treated by individual disciplines, but component interactions get more complex.

4.1.3 Statics vs. Dynamics

Continuum mechanics problems may be subdivided according to whether inertial effects are taken into account or not:

Continuum Mechanics {
- *Statics*
- *Dynamics*

In dynamics actual time dependence must be explicitly considered, because the calculation of inertial (and/or damping) forces requires derivatives respect to actual time to be taken. Problems in statics may also be time dependent but with inertial forces ignored or neglected. Accordingly static problems may be classed into strictly static and quasistatic. For the former time need not be considered explicitly; any historical time-like response ordering parameter, if one is needed, will do. In quasi-static problems such as foundation settlement, metal creep, rate-dependent plasticity or fatigue cycling, realistic measure of time is required but inertial forces are still neglected.

4.1.4 Discretization methods

A final classification of CSM static analysis is based on the discretization method by which the continuum mathematical model is discretized in space, i.e., converted to a discrete model with a finite number of degrees of freedom:

Spatial Discretization Method {
- *Finite Element (FEM)*
- *Boundry Element (BEM)*
- *Finite Difference (FDM)*
- *Finite Volume (FVM)*
- *Spectral*
- *Meshfree*

In CSM linear problems finite element methods currently dominate the scene as regards space discretization. Boundary element methods post a strong second choice in specific application areas. For nonlinear problems the dominance of finite element methods is overwhelming.

4.1.5 FEM Variants

The term Finite Element Method actually identifies a broad spectrum of techniques that share common features outlined in above sections. Two sub classifications that fit well applications to structural mechanics are

FEM Formulation / FEM Solution {
- *Displacement*
- *Equilibrium*
- *Mixed*
- *Hybrid*

of the variants listed above, emphasis is placed on the displacement formulation and stiffness solution. This combination is called the Direct Stiffness Method or DSM.

4.1.6 The Finite Element Method (FEM)

A promising approach for developing a solution for structural vibration problems is provided by an advanced numerical discretization scheme, such as; finite element method (FEM).The finite element method (FEM) is the dominant discretization technique in structural mechanics. The basic concept in the physical FEM is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called finite elements or elements for short. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points.

The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements. A straight beam element with uniform cross section is shown in Figure.1. The Euler-Bernoulli beam theory is used for constituting the finite element matrices. The longitudinal axis of the element lies along the x axis. The element has a constant moment of inertia I, modulus of elasticity E, density r and length l . Two degrees of freedom per node, translation along y-axis (y_1, y_2) and rotation about z-axis (y_1^l,y_2^l) are considered.

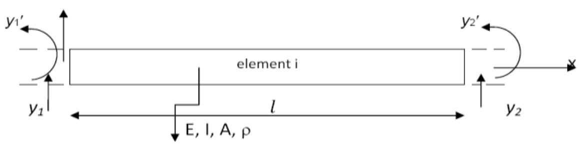


Fig.4.1.1 Straight Beam Element

Equations of Motion of the Beam

The equation of motion for a multiple degree of freedom undamped structural system is represented as follows

$[M]\{\ddot{y}\} + [K]\{y\}=\{F(t)\}$

Where $\{\ddot{y}\}$ and y are the respective acceleration and displacement vectors for the whole structure and {F (t)} is the external force vector. Under free vibration, the natural frequencies and the mode shapes of a multiple degree of freedom system are the solutions of the eigenvalue problem.

$[[K]-\omega^2[M]]\{\phi\}=0$

Where ω is the angular natural frequency and ϕ is the mode shape of the structure for the corresponding natural frequency.

4.2 Composite Element Method (CEM)

The composite element is a relatively new tool for finite element modeling. This method is basically a combination of the conventional finite element method (FEM) and the highly precise classical theory (CT). In the composite element method, the displacement field is expressed as the sum of the finite element displacement and the shape functions from the classical theory. The displacement field of the CEM can be written as

$$u_{CEM}(x,t) = u_{FEM}(x,t) + u_{CT}(x,t) \quad (1)$$

Where u_{FEM} *(x,t)* and u_{CT} *(x,t)* are the individual displacement fields from the FEM and CT,respectively. Taking a planar beam element as an example, the first term of the CEM displacement field can be expressed as the product of the shape function vector of the conventional finite element method *N*(*x*) and the nodal displacement vector

$$u_{FEM}(x, t) = N(x)q(t) \quad (2)$$

Where $q(t)=[v_1(t),\theta_1(t),v_2(t),\theta_2(t)]^T$ and ' v ' and θ represent the transverse and rotational displacements, respectively.

The second term uCT(x,t) is obtained by the multiplication of the analytical mode shapes with a vector of N coefficients c (also called the c degrees-of-freedom or c-coordinates).

$$uCT(x,t) = \sum_{i=0}^{N} \phi_i(x)\, C_i(t)\, x \quad (3)$$

Where $\phi_i(i=1,2,\ldots.N)$ is the analytical shape function of the beam. Different analytical shape functions are used according to the boundary conditions of the beam. Like the FEM, the CEM can be refined using the *h*-refinement technique by increasing the number of finite elements. Moreover, it can also be refined through the *c*-refinement method, by increasing the number of shape functions. Here, we apply the *c*-refinement from the CEM, where the beam needs only to be discretized into one element. This will reduce the total number of degrees-of-freedom in the FEM. The displacement field of the CEM for the Euler-Bernoulli beam element can be written from Equations (1) to (3) as

$$u_{CEM}(x, t) = S(x)Q(t) \quad (4)$$

$$S(x) = [N_1(x), N2(x), N3(x), N4(x), C_1(x), \phi_2(x),.., \phi_N(x)]$$

is the generalized shape function of the CEM,

$Q(t)=[v_1(t),\theta_1(t),v_2(t),\theta_2(t,c_1(t),c_2(t),\ldots.,c_N(t))]^T$

is the vector of generalized displacements, and *N* is the number of shape functions used from the classical theory.

The composite element method is proposed for both free and forced vibration analyses of beams with multiple steps. As the composite beam element is of a one-element-one-member configuration, modeling with this type of element would not need to take into account the discontinuity between different parts of the beam. The accuracy of this new composite element has been compared satisfactorily with existing results. One advantage of the method proposed is that it can be extended easily to deal with beams consisting of an arbitrary number of non-uniform segments. Regarding the free and forced vibration analysis for cracked beam using composite element, modeling with this type of element would allow the automatic inclusion of interaction effect between adjacent local damages in the finite element model. The accuracy of the present method has been compared satisfactory with existing model and experimental results.

4.3 Differential Quadrature Method (DQM)

The differential quadrature method (DQM) was first advanced by Bellman and his associates in the early 1970s aiming towards offering an efficient numerical method for solving non-linear partial differential equations. The method has since been applied successfully to various problems. When applied to problems with globally smooth solutions, the DQM can yield highly accurate approximations with relatively few grid points. This has made the DQM a favorable choice in comparison to standard "finite difference" and "finite element methods". In recent years, the DQM has become increasingly popular in solving differential equations and is gradually emerging as a distinct numerical solution technique.

An updating of the state of the art on the DQM and a comprehensive survey of its applications are available from two recent review papers. Bellomo focused his attention on the conventional DQM, which dealt with deferential equations of no more than second order. Bert and Malik have cited seven examples to explain its applications, six of which still belong to the conventional DQM. Only the third of the seven examples dealt with the high order deferential equation of Euler beam, whose governing equation is a fourth order one with double boundary conditions at each boundary. The main difficulty for such high order problems as Euler beams is that there are multiple boundary conditions but only one variable (function value) at each boundary.

To apply the double conditions, a d-point approximation approach of the sampling points was "first proposed by Jang et al. in 1989 and discussed thoroughly by Bert and Malik. The crux of the d-point technique is that an inner point near the boundary point is approximately regarded as a boundary point. The introduction of the d-point technique to multiple boundary condition problems indicated a major development in the application of the DQM to high order differential equations in solid mechanics. However, this breakthrough was also accompanied by a major disadvantage, an arbitrary choice of the d-value. Shu and Chen made a new endeavor to improve the distribution of the sampling points still using the d-point technique. In order to develop a better alternative to the d-point technique in solving fourth order differential equations for beam and plate problems, a new method was proposed in references, where the boundary points' rotation angles of beam and plate structures were employed as independent variables. Therefore, the shortcomings corresponding to the d-point technique have been overcome successfully. Wang and Gul also mentioned a generalization of their method to sixth and eighth order equations, and Bellomo also tried to generalize the conventional DQM to more than third order equations. However, they did not give the details of the implementation, and no paper related to the just-mentioned generalizations has appeared until the present time to the authors' knowledge.

The generalization of the DQM to any high order differential equations is apparently an urgent need in the present DQM research. The generalized differential quadrature rule (GDQR) has been proposed recently by the present authors and detailed formulations have been presented to implement any high order differential equations. The GDQR has been applied for the "first time to sixth and eighth order problems and to third order problems without using the d-point technique. Moreover, the GDQR has been extended to high order initial value differential equations of second to fourth orders, while no one has mentioned this generalization. In this paper the GDQR was still applied to the Euler beam problem. It is seemingly unnecessary for this kind of simple problem to be dealt with again, since it has been solved in many papers either using the d-point technique or not.

Consider a circular annular plate with an intermediate circular support, was studied in reference. The fundamental frequencies obtained using the DQM differed by about 10% from exact values, and the DQM did not provide satisfactory accuracy for some cases. It is apparent that an error must have occurred in these simple problems. Domain decomposition should have been employed at the intermediate supports but failed to be applied in references, because a shear force discontinuity exists there and the continuously differentiable trial functions are employed in the DQM. Domain decomposition has been used and very accurate results have been obtained by the present authors. The above analysis indicates that the study in this paper is necessary for a correct and thorough understanding of the DQ techniques. Moreover, the solution accuracy produced by the method itself can be substantiated quantitatively, since many examples have analytic solutions and the disturbance caused by d-point values is exempted.

4.3.1 Applications

The free vibration of Euler beams is governed by fourth order differential equations. The fourth order differential equations for various problems have been solved using the GDQR in papers, and the GDQR expression for a fourth order boundary value differential equation has been used as follows:

$$w^{(r)}(x_i) = \frac{d^r w(x_i)}{dx^r} = \sum_{j=1}^{N+2} E_{ij}^{(r)} U_j \quad (i = 1, 2, \ldots, N), \quad (1)$$

$\{U_1, U_2, \ldots, U_{N+2}\} = \{w_1^{(1)}, w_1, w_2, \ldots, w_{N-1}, w_N^{(1)}, w_N\}$ is employed for the Convenience of the notation. *wj* is the function of value at point x_i, $w_1^{(1)}$ and $w_N^{(1)}$ are the "first order derivatives of the displacement function, i.e.; rotation angles, at the "rst and *N*th points. *E*(*r*)*ij* are the *r*th order weighting coefficients at point *xi* . The GDQR explicit weighting coe$cients have been derived in references [12, 17] and were used directly in this paper. The cosine}type sampling points in normalized interval [0, 1] will be employed in this work. Their advantage has been discussed in paper [4]

$$x_i = \frac{1 - \cos[i-1)\pi/(N-1)]}{2} \quad (i = 1, 2, \ldots, N). \quad (2)$$

Example 1: Stepped Beams

Consider the free vibration of a straight Euler beam having stepped cross-section only at one place, as shown in Figure 3. These two sections have uniform cross-sections individually. They have different flexural rigidity (EI_1 and EI_2) and different cross-section area (A_1 and A_2). Here the GDQR's solutions are compared with those analytical solutions in paper, which considered a stepped beam with circular-section and with

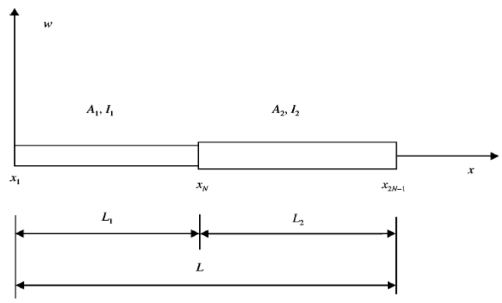


Fig.4.3.1. the stepped-beam geometry

$L_1 = L_2 = L/2$ and $\beta = I_2/I_1$ Then the "rst and second section's governing diferential equations are written, respectively, as follows:

$$EI_1 \frac{d^4 w}{dx^4} = \omega^2 \rho A_1 w, \quad x \subset [0, L_1], \quad (3)$$

$$EI_1 \frac{d^4 w}{dx^4} = \omega^2 \rho A_2 w, \quad x \subset [L_1, L], \quad (4)$$

Where 0 is the density, u the circular frequency, and ¸ the total length of the beam. Through normalization manipulation, equations (3) and (4) are written, respectively, as

$$\left(\frac{L}{L_1}\right)^4 \frac{d^4 w}{d\zeta^4} = \lambda^4 w, \quad \zeta \subset [0, 1], \quad (5)$$

$$\sqrt{\beta}\left(\frac{L}{L_2}\right)^4 \frac{d^4 w}{d\zeta^4} = \lambda^4 w, \quad \zeta \subset [0, 1], \quad (6)$$

is dimensionless frequency parameter, and f normalized local co-ordinate. Usually, the same number *N* of sampling points of sub domains is used.

$$w = \frac{d^2w}{dx^2} = 0, \quad w = \frac{dw}{dx} = 0,$$
$$\frac{d^2w}{dx^2} = \frac{d^3w}{dx^3} = 0, \quad \frac{dw}{dx} = \frac{d^3w}{dx^3} = 0. \qquad (7)$$

There are two boundary conditions at each end, and totally four boundary conditions, which have four GDQR analogues from a proper combination of equations.

4.4 Third-Order Shear Deformation Theory

In this study, free vibration of beams with different boundary conditions is analyzed within the framework of the third-order shear deformation theory. The boundary conditions of beams are satisfied using Lagrange multipliers. To apply the Lagrange's equations, trial functions denoting the deflections and the rotations of the cross-section of the beam are expressed in polynomial form. Using Lagrange's equations, the problem is reduced to the solution of a system of algebraic equations. The first six eigenvalues of the considered beams are calculated for different thickness-to-length ratios. The results are compared with the previous results based on Timoshenko and Euler–Bernoulli beam theories.

4.4.1. Introduction

There are many studies on the theory and analysis of beam-type structures in the literature. The oldest and the well-known beam theory is the Euler–Bernoulli beam theory (or classical beam theory—CBT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight and perpendicular to the mid-plane after bending. As a result of this assumption, transverse shear strain is neglected. Although this theory is useful for slender beams and plates, it does not give accurate solutions for thick beams and plates. The next theory is the Timoshenko beam theory (the first order shear deformation theory—FSDT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight, but no longer remain perpendicular to the mid-plane after bending.

In FSDT, the distribution of the transverse shear stress with respect to the thickness coordinate is assumed constant. Thus, a shear correction factor is required to compensate for the error because of this assumption in FSDT. The third-order shear deformation theory (TSDT) which assumed parabolic distribution of the transverse shear stress and strain with respect to the thickness coordinate was proposed for beams with rectangular cross-sections (Wang *et al* 2000). Also, zero transverse shear stress condition of the upper and lower fibres of the cross-section is satisfied without a shear correction factor in TSDT.

There are many studies related with the problem of free vibration of beams based on CBT and FSDT (Timoshenko & Young 1955; Hurty & Rubinstein 1967; Farghaly 1994; Banerjee 1998; Nallim & Grossi 1999; Kim & Kim 2001; Lee *et al* 2003; Auciello & Ercolano 2004; Zhou 2001; Lee & Schultz 2004; S¸ims¸ek 2005a, b; Kocat¨urk & S¸ims¸ek 2005a, b). The relationship between the bending solution of TSDT and those of CBT and FSDT was presented (Wang *et al* 2000). The exact stiffness matrix was derived from the solutions of differential equations according to TSDT for isotropic beams (Eisenberger 2003). Frequency equations and characteristic functions of homogeneous orthotropic beams having different boundary conditions were obtained, and the first six natural frequency parameter was tabulated for different values of stiffness ratios and values of thickness-to-length ratios (Soldatos & Sophocleous 2001). Static deflections of the laminated composite beams subjected to uniformly distributed load were studied using the classical, the first-order, the second-order and the third-order beam theories (Khdeir & Reddy 1997).

In the present study, free vibration of beams with different boundary conditions is analysed based on the third-order shear deformation theory (TSDT). Frequency equations of the beams are derived using Lagrange's equations. The boundary conditions of the beams are considered using Lagrange multipliers. The trial functions for the deflections and rotations of the cross-section of the beam are selected in polynomial form. The first six eigenvalues of the considered beams are calculated for different thickness-to-length ratios. The obtained results are compared with earlier results based on CBT and FSDT.

4.4.2. Theory and formulations

A straight uniform beam of length *L*, width *b*, depth *h*, having rectangular cross-section is shown in figure 1. A Cartesian coordinate system *(x, y, z)* is defined on the central axis of the beam, where the *x* axis is taken along the central axis, the *y* axis in the width direction and the *z* axis in the depth direction. Also, the origin of the coordinate system is chosen at the mid-point of the total length of the beam. The third-order shear deformation theory (TSDT) is based on the following displacement fields (Wang *et al* 2000);

$$u_x(x,z,t) = z\O(x,t) - \alpha z^3[\O(x,t) + \omega_x(x,t)] \qquad (1)$$
$$u_z(x,z,t) = \omega(x,t)$$

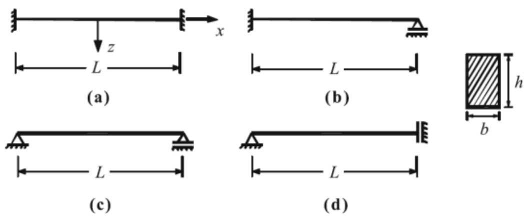


Fig. 4.4.1 (a) Clamped-clamped, (b) clamped-pinned, (c) pinned-pinned, (d) pinned-guided straight uniform beams with rectangular cross-section

Where *ux* and *uz* are displacements in *x* and *z* directions at any material point in the *(x, z)* plane, $\alpha = 4/(3h2)$,*w* is the transverse displacements, and *φ* represents the slope *∂ux/∂z* at *z* = 0 of the deformed line which was straight in the undeformed beam. In this case *φ(x, t)* and *α* together define the third-order nature of the deformed line. The symbol *(),x* indicates the derivative with respect to *x*.

By choosing the appropriate boundary conditions given by Equations, the constraint conditions of the beams are given as follows:

i. For the Clamped-Clamped Beam

$$w(x_A,t)=0, w_{,x}(x_A,t)=0, \phi(x_A,t)=0, w(x_B,t)=0, w_{,x}(x_B,t)=0, \phi(x_B,t)=0. \quad (2)$$

ii. For the Clamped-Pinned Beam

$$w(x_A,t)=0, w_{,x}(x_A,t)=0, \phi(x_A,t)=0, w(x_B,t)=0. \quad (3)$$

iii. For the Pinned-guided Beam

$$w(x_A,t)=0, w_{,x}(x_B,t)=0, \phi(x_B,t)=0. \quad (4)$$

iv. For the Pinned-Pinned Beam

$$w(x_A,t)=0, w(x_B,t)=0. \quad (5)$$

x_A and x_B denote the location of left and right supports of the beam respectively. By introducing
The Lagrange multipliers formulation, the Lagrangian functional of the problem is obtained.
The free vibrations of the beams have been investigated for different thickness-to-length ratios according to TSDT. The eigenvalues of the beams obtained with various boundary conditions are compared with the previously available results of CBT and FSDT. Using Lagranges equations with the trial functions in the polynomial form and satisfying the constraint conditions by the use of Lagrange multipliers is a nice way for studying the free vibration characteristics of the beams.

4.5 Fourier Series

4.5.1.Theory and Application

The Fourier series method with separation of variables is suitable to be used for the solution of free vibration of beams. As the method is trigonometric (sine and cosine), then the deflection modes are of the same shape for different types of loads.
The partial differential equation (p.d.e) for free undamped transverse vibration of beams is

$$\frac{\partial^2 y}{\partial t^2}+c^2\frac{\partial^2 y}{\partial x^2}=0 \quad (1)$$

Where c2 = $\frac{EI}{m}$
One method of solving this equation is by the separation of variables;
it assumes that:

$$Y(x,t)=\Phi(x)Y(t) \quad (2)$$

Where Φ(x) is a function of distance along the beam defining its deflection shape
When it vibrates and Y(t) defines the amplitude of vibration with time.
Substituting equation (2) for equation (1) yields:

$$\Phi(x)\frac{\partial^2 y}{\partial t^2}+c^2 Y(t)\frac{\partial^4 \Phi(x)}{\partial x^4}=0 \quad (3)$$

Since each of the variables x and t are independent variables, then each side of
equation is equal to a constant, say ω^2
The general solution is given by:

$$\Phi(x)= c_1 \sin\alpha x + c_2 \cos\alpha x + c_3 \sinh\alpha x + c_4 \cosh\alpha x \quad (4)$$

Where C_1, C_2, C_3 and C_4 are constants (5)

$$Y(t) = A\cos\omega t + B\sin\omega t \quad (6)$$

Substitution

$$Y(x,t) = (A\cos\omega t + B\sin\omega t)(c_1\sin\alpha x + c_2\cos\alpha x + c_3 \sinh\alpha x + c_4\cosh\alpha x) \quad (7)$$

The complete solution for a particular structure requires expressions for the displacement, slope, moment and shear at the supports which must be substituted for (7). This procedure will yield three coefficients in terms of the forth and will also yield a frequency equation from which may be evaluated. The final coefficient expression is a magnitude of vibration that would require
Acknowledging of the initial conditions of motions.
For the simply supported beams the boundary conditions are:

$$y(0,t)=0 \quad \text{and} \quad EI\frac{\partial^2}{\partial x^2}(0,t)=0 \quad (8)$$

$$y(L,t)=0 \quad \text{and} \quad EI\frac{\partial^2}{\partial x^2}(L,t)=0 \quad (9)$$

4.5.2. Intermediate Concentrated Load

If P is the concentrated load acting at distance 1 x from the left side of the
Beam as shown in Fig. (1).

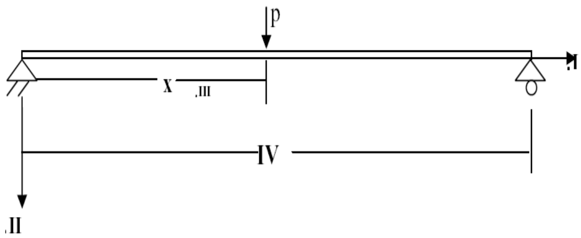


Fig.4.5.1. Intermediate Concentrated Load

In order to get the deflection due to the static load,it was assumed that the deflected shape represented by half range fourier series:

$$y=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{L}x \quad (10)$$

Which satisfies the boundary conditions of simply supported beams.

5. Case Study

5.1 Experimental Set up

The PULSE software analysis was used to measure the frequency ranges to which the foundations of various machines are subjected to when the machine is running with no load and full load. This will help us in designing the foundations of various machines in such a way that they are able to resist the vibration caused in them. Below we present the analysis of frequency measurements for a few cantilever beams measured in structural Engineering lab in N.I.T. Rourkela.

5.2. Equipments Required

1. Model hammer.
2. Accelerometer.
3. Portable pulse.
4. Connectors – Model no: AO 0087D
5. Specimen.
6. Display Unit.

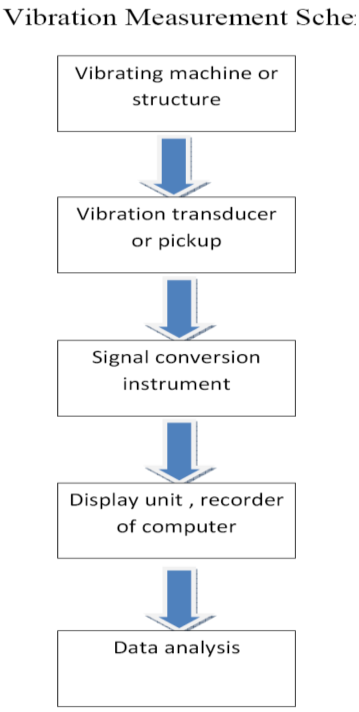


Fig.5.1 Vibration measurement

5.3. Equipments Description

(1) Deltatron Accelerometer:- Deltatron accelerometer combines high sensitivity, low and small physical dimensions making them ideally suited for model analysis. The slits in the oscilrometer housing make it simple to mount with bee box that one easily fitted to the plate.

Fig.5.2. Deltatron Accelerometer

(2) Model hammer:- The model hammer exits the structure with a constant force over a frequency range of interest. Three interchange tips are provided which determine the width of the input pulse and thus the band width the hammer structure is acceleration compensated to avoid glitches in the spectrum due to hammer structure resonance.

Fig.5.3. Model hammer

(3) Portable pulse T- type (3560C)

Bruel and kjaer pulse analyzer system type – 3560. The software analysis was used to measure the frequency ranges to which the foundation various machines are subjected to when the machine is running with no load and full load. This will help us in designing the foundations of various machines on such a way that they are able to resist the vibration caused in them.

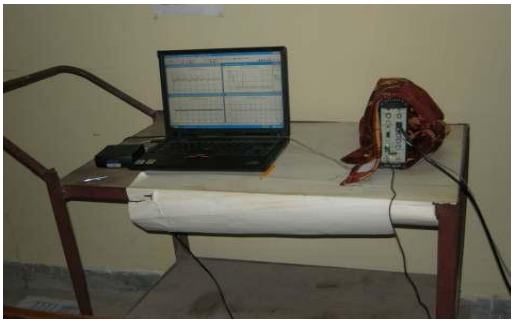


Fig 5.4 Pulse Analyser and Display Unit

(4) Display unit: - This is mainly in the form of PC(Laptop) when the excitation occurs to the structure the signals transferred to the portable PULSE and after conversion comes in graphical form through the software. Mainly the data includes graphs of force Vs time, frequency Vs time resonance frequency data etc.

5.4. Experimental Program

Equipments Brüel & Kjær PULSE™, Multi-analyzer System Type 3560 was used to measure the frequency ranges of a cantilever beam.

Setup and Procedure (FFT analyzer)

1. Aluminum beam of required length 40cm was cut from a bulk available beam.
2. By the use of screw gauge the depth and width of beam section were measured.
3. 10 cm length of beam was properly inserted to the concrete inside the mould and compacted using vibrator.
4. After seven days of curing the specimen was taken out.
5. Now the length of the cantilever beam from fixed end to the free end was found out.
6. The connections of the FFT analyzer, laptop, transducers, and model hammer along with the requisite power connections were made.

7. The accelerometer -4507 type was fixed by beeswax to the cantilever beam at one of the nodal points.
8. The 2302-5 modal hammers were kept ready to strike the beam at the singular points.
9. Then at each point the modal hammer was struck once and the amplitude Vs frequency graph was obtained from graphical user interface.
10. The FFT analyzer and the accelerometer are the interface to convert the time domain response to frequency domain. Hence the frequency response spectrumH1 (response, force) was obtained.
11. By moving the cursor to the peaks of the FFT graph (m/s2/N),the cursor values and the resonant frequencies were recorded.
12. At the time of the striking with modal hammer to the singular point precautions were taken whether the striking should have been perpendicular to the aluminium beam surface.
13. The above procedure is repeated for all the nodal points.
14. The values (i.e.,natural frequencies and resonant frequencies) obtained from the FRF spectrums were compared with respect to the FEM analysis.

5.6 Results and discussion

Beam specification

Software used	FFT analyzer and accessories, pulse lab shop version 9.0
Parameter	Frequency
Length of cantilever	20cm
Section dimentions	$0.0095X0.0095m^2$
Boundary conditions	One end fixed and another free
Material	Aluminum
Mass density	$2659kgm^{-3}$
Elastic modulus	$68.0E09Nm^{-2}$
Poison's ratio	0.205

Natural frequency of the beam was theoretically computed using the FORTRAN program.

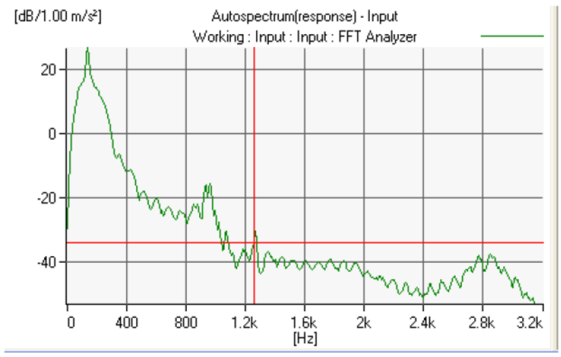
Experimental results for uncracked beam: Aluminum beam (fixed-free condition)

Mode	Frequency (by theoretical method)	Frequency (by practical method)	Percentage of error
First	197.42 Hz	187.00 Hz	5.27%
Second	1227.66 Hz	1180.00 Hz	3.88%
Third	3393.65 Hz	3308.00 Hz	2.52%
forth	6547.61 Hz	6425.00 Hz	1.87%

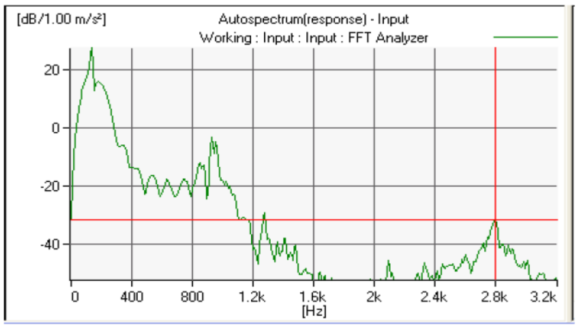
Experimental results for single crack: Aluminum beam (fixed-free condition)

Sr no		Crack depth	1st mode	2nd mode	3rd mode	4th mode
1.	Cracks at center	2mm	144	928	2088	2792
2.		6mm	136	887	2064	2744
3.		8mm	48	560	1448	2744
4.	Crack at 0.25L	2mm	128	960	1936	2832
5.		6mm	112	876	1720	2456
6.		8mm	88	448	856	1736
7.	No crack	nil	1180	756	3308	6425

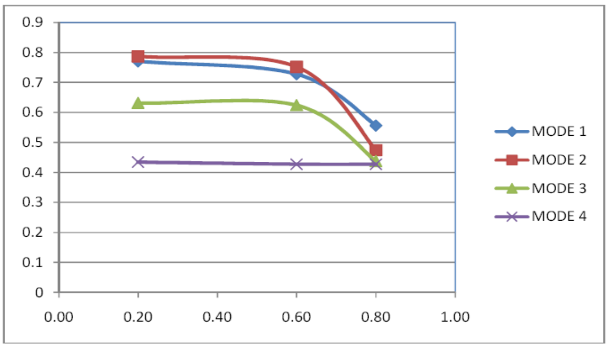
Graph for 0.25L 2mm crack::



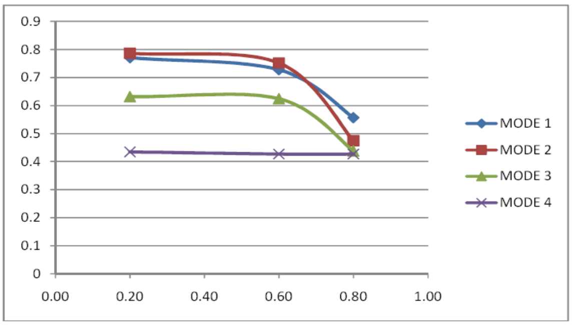
Graph for 0. 5L 2mm crack::



Crack @0.25L (single)



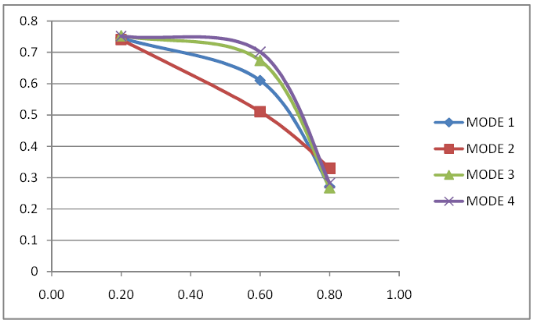
Crack @0.5L (single)



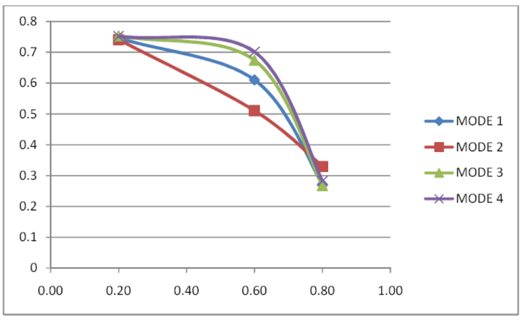
Experimental results for Multi crack Beam:
Beam Specification:
Length of the steel beam for free-free condition = 25 cm
Breadth = 9.2mm
Height = 9.2mm

Sr no	Location	Crack depth	1st mode	2nd mode	3rd mode	4th mode
1.	Crack at 10mm, 5mm	2mm	144	928	2088	2792
2.		6mm	136	887	2064	2744
3.		8mm	48	560	1448	2744
4.	Crack at 7.5mm, 2.5mm	2mm	128	960	1936	2832
5.		6mm	112	878	1720	2456
6.		8mm	88	448	856	1736
7.	No crack	nil	118	756	2125	4165

Cracks @ 0.4L , 5mm



Cracks @0.3L,2.5mm



6. Conclusion and Scope for Future Work

The vibration analysis of a structure holds a lot of significance in its designing and performance over a period of time. The verification of the analytical approach with a considerable amount of experimental data and with the results of calculations showed that the analytical approach enables one to obtain well-founded relationships between different dynamic characteristics and crack parameters and to solve the inverse problem of damage diagnostics with sufficient accuracy for practical purposes.

Using Lagranges equations with the trial functions in the polynomial form and satisfying the constraint conditions by the use of Lagrange multipliers is a nice way for studying the free vibration characteristics of the beams.

In this study, the system equations are solved by Differential Quadratic Method, which is a succeeding and easy transformation technique. By solving the algebraic functions set, which are the transforms of differential equations, natural frequencies are obtained. The results are tabulated and compared with the former studies and a great accuracy to exact results is obtained.

In case of cracks the frequencies of vibration of cracked beams decrease with increase of crack depth for crack at any particular location due to reduction of stiffness. The effect of crack is more pronounced near the fixed end than at far free end. The first natural frequency of free vibration decreases with increase in number of cracks. The natural frequency decreases with increase in relative crack depth.

The results obtained are accurate and are expected to be useful to other researchers for comparison. The study in this work is necessary for a correct and thorough understanding of the Vibration analysis techniques.

6.1 Future Work

- The cracked cantilever can be analyzed under the influence of external forces.
- The dynamic response of the cracked beams can be analyzed for different crack orientations.
- Stability study of the cracked beams can be done.
- Use hybrid neuro genetic technique for crack detection.

7. References

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