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Fuzzy method for solving multi-objective assignment problem with interval cost

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Abstract- This paper concentrates on the solution procedure of the Multi-Objective Assignment Problem (MOAP) where the cost coefficients of the objective functions have been expressed as interval values by the decision maker. This problem has been transformed into a classical MOAP where the interval objective function is minimized. The order relations that represent the decision maker's preference between interval profits have been defined by the right limit, left limit, centre and half-width of an interval. Finally, the equivalent transformed problem has been solved by fuzzy programming techniques. Numerical example has been provided to illustrate the solution procedure.
Keywords: Multi-Objective Assignment problem; Interval analysis; linear membership function; Non-linear membership function; Fuzzy programming.

1. Introduction

The conventional single objective Assignment problem is a special type of linear programming problem and the constraints follow a particular mathematical structure. An assignment problem can be viewed as a balanced transportation problem in which all supplies and demands equal 1, and the number of rows and columns in the matrix are identical. The penalty (c_{ij}) that is, the co-efficient of the objective functions could represent assignment cost, execution time, and many others. Thus multiple penalty criteria may exist concurrently which lead to the research work on multi-objective assignment problem. Ravindran et al. [10] used the transportation simplex method for solving the assignment problem. Geetha et al. [5] first expressed the cost-time minimizing assignment as the multicriteria problem. Bit et al. [1] developed a procedure using fuzzy programming technique for solution of the multi-criteria decision-making transportation problems. Tsai et al. [12] provided a solution for balanced multi-objective decision making problem associated with cost, time and quality by fuzzy concept. Leberling [9] used a special type nonlinear (hyperbolic) membership function for the vector maximum linear programming problem. He showed that solutions obtained by fuzzy linear programming with this type of non-linear membership function are always efficient. Various effective algorithms were developed for solving assignment problems with the assumption that the co-efficients of the objective functions are specified in a precise way, namely, crisp. However, these conditions may not be satisfied always. For example, the unit assignment costs are rarely constant. To deal with ambiguous co-efficients in mathematical programming, inexact, fuzzy and interval programming techniques have been proposed. Chanas and kuchta [3] defined the transportation problem with fuzzy cost co-efficients and developed an algorithm for the solution. Tong et al.[11] has proposed linear programming models with interval objective functions. Ishibuchi and Tanaka [8] developed a concept for optimization of multi-objective programming problems with interval objective functions. A new treatment of the interval objective in linear programming problems was developed by Inuiguchi and Kume [6,7] by introducing the minimax regret criterion as used in decision theory. Chanas and Kuchta [2] have generalized the known concept of the solution of linear programming problem with interval co-efficients in the objective function based on preference relations between intervals. Das, Goswami and Alam [4] have proposed a method to solve the multiobjective transportation problem in which the co-efficients of the objective functions as well as the source and destination parameters are in the form of interval.
In this paper, we focus on the solution procedure of the multi-objective assignment problem (MOAP) in which cost coefficients of the objective function, have been expressed as interval values by the decision maker. This problem has been transformed into a classical A.P. where to minimize the interval objective function, the order relations that represent the decision maker's preference between interval profits have been defined by the right limit, left limit, center, and half- width of an interval. Finally, the equivalent transformed problem has been solved by fuzzy programming technique.

2. Multi-objective Interval Assignment Problem

$$\text{Minimize} \quad Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{Lij}^k, c_{Rij}^k]\, x_{ij} \quad \text{where} \quad k = 1,2,...,K \tag{1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1,2,...m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = 1, \qquad j = 1,2,...n \tag{3}$$

$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases} \tag{4}$$

Where $[c^k_{Lij}, c^k_{Rij}]$ is an interval representing the uncertain cost for the assignment problem. It can represent assignment cost, execution time etc.

Setting M = 1,2,…,m, N = 1,2,…,n and J = {(i, j) /i $\in$ M, j $\in$ N}, the problem may be restated as

$$\text{Minimize} \left\{ Z = \sum_{(i,j)\in J} c_{ij} x_{ij} \;\middle|\; \begin{array}{l} \sum_j x_{ij} = 1 \\ \sum_i x_{ij} = 1 \\ x_{ij} = 0,1; \forall i,j. \end{array} \right\} \tag{5}$$

Where, $z \in R^k$ and $c_{ij} = [c_{Lij}, c_{Rij}]$.

Here, $c_{Lij} = (c^1_{Lij}, c^2_{Lij}, ..., c^k_{Lij})$ and $c_{Rij} = (c^1_{Rij}, c^2_{Rij}, ..., c^k_{Rij})$ represent, respectively, the left bound and right bound of c_{ij}. The set of all feasible solutions of the problem will be denoted by S.

3. Interval arithmetic

Throughout this paper lower case letters denote real numbers and upper case letters denote closed intervals. The set of all real numbers is denoted by R. An interval is defined by an ordered pair brackets as

$$A = [a_L, a_R] = \{a : a_L \le a \le a_R, a \in R\}, \tag{6}$$

Where a_L is left limit and a_R is the right limit of A. The interval is also denoted by its center and width as,

$$A = \langle a_c, a_w \rangle = \{a : a_c - a_w \le a \le a_c + a_w, a \in R\}, \tag{7}$$

where a_c is the center and a_w is the width of A. From (6) and (7) the center and width of an interval may be calculated as

$$a_c = \frac{1}{2}(a_R + a_L) \tag{8}$$

$$a_w = \frac{1}{2}(a_R - a_L) \tag{9}$$

The generalization of ordinary arithmetic to closed intervals is known as interval arithmetic. The basic definition is as follows:

Definition

Let $* \in \{+, -, \times, \div\}$ be a binary operation on the set of real numbers. If A and B are closed intervals, then

$$A * B = \{a * b : a \in A, b \in B\} \tag{10}$$

defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. The operation on intervals used in this paper may be explicitly calculated from above definitions as:

$$A+B = [a_L, a_R]+[b_L, b_R] \\ = [a_L+b_L, a_R+b_R], \tag{11}$$

$$A+B = \langle a_c, a_w\rangle+\langle b_c, b_w\rangle \\ = \langle a_c+b_c, a_w+b_w\rangle \tag{12}$$

$$kA = k[a_L, a_R] = \begin{cases} [ka_L, \quad ka_R] & \text{for } k \geq 0, \\ [ka_R, \quad ka_L] & \text{for } k < 0, \end{cases} \tag{13}$$

$$kA \quad = k\langle a_c, a_w\rangle = \langle ka_c, |k|\, a_w\rangle, \tag{14}$$

Where, k is a real number.

4. Definition of order relations between intervals

In this section, the order relations which represent the decision maker's preference between interval costs are defined for minimization problem. Let the uncertain costs from two alternatives be represented by intervals A and B respectively. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

Definition (4a)

The order relation $\leq_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \leq_{LR} B \quad \text{iff } a_L \leq b_L \text{ and } a_R \leq b_R,$$

$$A <_{LR} B \quad \text{iff } A \leq_{LR} B \text{ and } A \neq B$$

(15)

This order relation $\leq_{LR}$ represents the decision maker's preference for the alternative with lower minimum cost and maximum cost, that is, if $A \leq_{LR} B$, then A is preferred to B.

Definition (4b)

The order relation $\leq_{CW}$ between $A = \langle a_c, a_w\rangle$ and $B = \langle b_c, b_w\rangle$ is defined as

$$A \leq_{CW} B \quad \text{iff } a_C \leq b_C \text{ and } a_W \leq b_W \\ A <_{CW} B \quad \text{iff } A \leq_{CW} B \text{ and } A \neq B \tag{16}$$

The order relation $\leq_{CW}$ represents the decision maker's preference for the alternative with lower expected cost and less uncertainty, that is, if $A \leq_{CW} B,$ then A is preferred to B.

5. Formulation of the crisp objective function

In this section, we show that the formulation of the original interval objective function as a crisp one.

Definition (5a). $x^0 \in S$ is a solution of the problem (1-4) if and only if there is no other solution $x \in S$ which satisfies

$$Z(x) \leq_{LR} Z(x^0) \text{ or } Z(x) <_{CW} Z(x^0).$$

Proposition (5a). It can be easily proved that

$$A \leq_{RC} B \quad \text{iff } A \leq_{LR} B \text{ or } A \leq_{CW} B \\ A <_{RC} B \quad \text{iff } A <_{LR} B \text{ or } A <_{CW} B \tag{17}$$

Where the order relation $\leq_{RC}$ is defined as

$$A \leq_{RC} B \quad \text{iff } a_R \leq b_R \text{ and } a_C \leq b_C,$$

$$A \leq_{RC} B \quad \text{iff } A \leq_{RC} B \text{ and } A \neq B.$$

Using proposition (5a) and definition (5a) may be simplified as follows.

Definition (5b). $x^0 \in S$ is an optimal solution of problem (1-4) iff there is no other solution $x \in S$ which satisfies Z(x) $<_{RC}$ $Z(x^0)$.

The right limit of the interval objective function $Z_R^k(x)$ in the problem (1-4) may be calculated from (12) and (14) as

$$Z_R^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij}^k x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Wij}^k |x_{ij}| \tag{18}$$

where C_{Cij}^k is the center and C_{wij}^k is the half width of the co-efficient C_{ij}^k of $Z^k(x)$. In the case when $x_{ij} \geq 0$, i = 1,2,…,m, j = 1,2,…,n, $Z_R^k(x)$ can be modified as

$$Z_R^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij}^k x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Wij}^k x_{ij} \tag{19}$$

The center of the objective function $Z_C^k(x)$ can be elicited as

$$Z_C^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{Cij}^k x_{ij} \tag{20}$$

The solution set of equation (1) defined by definition (5b) can be obtained as the Pareto optimal solution of the following interval objective problem.

$$\text{Minimize } \{Z_R^k,\ Z_C^k\}, k=1,2,...,K \tag{21}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n \tag{22}$$

$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

Where $Z_R^k(x)$ and $Z_C^k(x)$ are stated in equations (19) and (20).

6. Fuzzy programming technique to MOAP

Algorithm

Step 1: Solve the interval assignment problem as a single objective assignment problem using, each time, only one objective and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From Step 2 we may find, each objective, the worst (U_k) and the best (L_k) values corresponding to the set of solutions. The initial fuzzy model can then be stated in terms the aspiration levels of each objective, as follows.

Find { x_{ij}, i =1, 2, …,m; j = 1, 2, …,n; }

So as to satisfy $Z_k \lesssim L_k$ and

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n$$

$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

$\lesssim$ (fuzzification symbol)) indicates nearly less than equal to

Step 4: Define membership functions for the k^{th} objective function as follows:

Case (i): A linear membership function for the k^{th} objective function is defined by

$$\mu_k Z^k_{[R,C]}(x)=\begin{cases} 1, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} \le L_k \\ \dfrac{U_k - \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij}}{U_k - L_k}, & \text{if } L_k < \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} < U_k \\ 0, & \text{if} \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} \ge U_k \end{cases} \tag{23}$$

Case (ii): An hyperbolic membership function for the k[th] objective function is defined by

$$\mu_k^H Z^k_{[R,C]}(x)=\begin{cases} 1, & \text{if} \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} \le L_k \\ \dfrac{1}{2}\tanh\left(\left(\dfrac{U_k+L_k}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij}\right)\alpha_k\right)+\dfrac{1}{2}, & \text{if } L_k < \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} < U_k \\ 0, & \text{if} \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} \ge U_k \end{cases} \tag{24}$$

Where α_k is a parameter given by $\alpha_k = \dfrac{3}{(U_k - L_k)/2} = \dfrac{6}{(U_k - L_k)}$

Step 5: From step 4, we can find an equivalent crisp model for the initial fuzzy model as follows:

If we will use the linear membership function as defined in (23) then an equivalent crisp model for the fuzzy model can be formulated as:

$$\text{Maximize } \lambda$$

$$\text{subject to}$$

$$\lambda \le \frac{U_k - \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij}}{(U_k - L_k)}, k=1,2,...,K$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n; \quad \lambda \ge 0$$

$$x_{ij}=\begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

The above problem can be further simplified as:

$$\text{Maximize } \lambda$$

$$\text{subject to}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k,\ Z_C^k\ ]x_{ij} + \lambda\,(U_k - L_k) \le U_k$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n; \quad \lambda \ge 0$$

$$x_{ij}=\begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

If we use a hyperbolic membership function as defined in (24) then an equivalent crisp model for the fuzzy model can be formulated as:

$$\text{Maximize} \quad \lambda$$
$$\text{subject to}$$
$$\lambda \le \frac{1}{2}\tanh\left(\left(\frac{U_k+L_k}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k, Z_C^k]x_{ij}\right)\alpha_k\right) + \frac{1}{2},$$
$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n; \quad \lambda \ge 0$$
$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

This problem can be further simplified as:

$$\text{Maximize} \quad X_{mn+1}$$
$$\text{subject to}$$
$$\alpha_k \sum_{i=1}^{m}\sum_{j=1}^{n}[Z_R^k, Z_C^k]x_{ij} + X_{mn+1} \le (U_k + L_k)/2$$
$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n; \quad X_{mn+1} \ge 0$$
$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$
$$\text{where} \quad X_{mn+1} = \tanh^{-1}(2\lambda - 1)$$

Step 6: Solve the crisp model by an appropriate mathematical programming algorithm.

The solution obtained in step 6 will be the optimal compromise solution of the Multi-objective assignment problem.

7. Numerical Example

$$\text{Minimize} \quad Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{3}[c_{Lij}^1, c_{Rij}^1]x_{ij}$$
$$\text{Minimize} \quad Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{3}[c_{Lij}^2, c_{Rij}^2]x_{ij}$$
$$\text{Subject to}$$
$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,...m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,...n;$$
$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

where

$$C^1 = \begin{bmatrix} [1,3] & [5,9] & [4,8] \\ [7,10] & [2,6] & [3,5] \\ [7,11] & [3,5] & [5,7] \end{bmatrix}, \quad C^2 = \begin{bmatrix} [3,5] & [2,4] & [1,5] \\ [4,6] & [7,10] & [9,11] \\ [4,8] & [3,6] & [1,2] \end{bmatrix}$$

The equivalent deterministic problem becomes:

$$\text{Minimize } Z_R^1(x) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Rij}^1 x_{ij}, \quad \text{Minimize } Z_R^2(x) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Rij}^2 x_{ij}$$

$$\text{Minimize } Z_C^1(x) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Cij}^1 x_{ij}, \quad \text{Minimize } Z_C^2(x) = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{Cij}^2 x_{ij}$$

where

$$C_{Rij}^1 = \begin{bmatrix} 3 & 9 & 8 \\ 10 & 6 & 5 \\ 11 & 5 & 7 \end{bmatrix}, \qquad C_{Rij}^2 = \begin{bmatrix} 5 & 4 & 5 \\ 6 & 10 & 11 \\ 8 & 6 & 2 \end{bmatrix}$$

$$C_{Cij}^1 = \begin{bmatrix} 2 & 7 & 6 \\ 8.5 & 4 & 4 \\ 9 & 4 & 6 \end{bmatrix}, \qquad C_{Cij}^2 = \begin{bmatrix} 4 & 3 & 3 \\ 5 & 8.5 & 10 \\ 6 & 4.5 & 1.5 \end{bmatrix}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,\ldots m; \quad \sum_{i=1}^{m} x_{ij} = 1, \quad j=1,2,\ldots n;$$

$$x_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{cases}$$

Step 1 and step 2:

Optimal solution which minimizes objective $Z_R^1, Z_R^2, Z_C^1, Z_C^2$ subject to constraints (2-5) is as follows:

Pay-off matrix is

$$\begin{array}{c} \\ X_R^{(1)} \\ X_R^{(2)} \\ X_C^{(1)} \\ X_C^{(2)} \end{array} \begin{array}{c} \begin{array}{cccc} Z_R^1 & Z_R^2 & Z_C^1 & Z_C^2 \end{array} \\ \begin{bmatrix} 13 & 22 & 10 & 18.5 \\ 26 & 12 & 21.5 & 9.5 \\ 13 & 22 & 10 & 18.5 \\ 26 & 12 & 21.5 & 9.5 \end{bmatrix} \end{array}$$

The membership function $\mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x)$ for the objectives $(Z_R^1), (Z_R^2), (Z_C^1)$ and (Z_C^2) respectively, are as follows:

$$\mu_1(x) = \begin{cases} 1, & \text{if } Z_R^1(x) \le 13 \\ \dfrac{26 - Z_R^1(x)}{13}, & \text{if } 13 < Z_R^1(x) < 26 \\ 0, & \text{if } Z_R^1(x) \ge 26 \end{cases}; \quad \mu_2(x) = \begin{cases} 1, & \text{if } Z_R^2(x) \le 12 \\ \dfrac{22 - Z_R^2(x)}{10}, & \text{if } 12 < Z_R^2(x) < 22 \\ 0, & \text{if } Z_R^2(x) \ge 22 \end{cases}$$

$$\mu_3(x) = \begin{cases} 1, & \text{if } Z_C^1(x) \le 10 \\ \dfrac{21.5 - Z_C^1(x)}{11.5}, & \text{if } 10 < Z_C^1(x) < 21.5 \\ 0, & \text{if } Z_C^1(x) \ge 21.5 \end{cases}; \quad \mu_4(x) = \begin{cases} 1, & \text{if } Z_C^2(x) \le 9.5 \\ \dfrac{18.5 - Z_C^2(x)}{9}, & \text{if } 9.5 < Z_C^2(x) < 18.5 \\ 0, & \text{if } Z_C^2(x) \ge 18.5 \end{cases}$$

The problem was solved by the Linear Interactive and Discrete Optimization (LINDO) Software. The optimal Compromise solution is presented as follows:

$$X^{*}=\{x_{11}=x_{22}=x_{33}=1\}$$

$$Z_R^{1*}=16,\ Z_R^{2*}=17,\ Z_C^{1*}=12,\ Z_C^{2*}=14$$

$$\lambda=0.606060$$

Therefore, $Z_1=[12,16]$ and $Z_2=[14,17]$

The membership function $\mu_1^H(X)$, $\mu_2^H(X)$, $\mu_3^H(X)$ and $\mu_4^H(X)$ for the objectives (Z_R^1), (Z_R^2), (Z_C^1) and (Z_C^2) respectively are as follows:

$$\mu_1^H(X)=\begin{cases} 1, & \text{if } Z_R^1(x)\le 13 \\ \frac{1}{2}\tanh\left(\left(19.5-Z_R^1(x)\right)6/13\right)+\frac{1}{2}, & \text{if } 13\le Z_R^1(x)\le 26 \\ 0, & \text{if } Z_R^1(x)\ge 26 \end{cases}$$

$$\mu_2^H(X)=\begin{cases} 1, & \text{if } Z_R^2(x)\le 12 \\ \frac{1}{2}\tanh\left(\left(17-Z_R^2(x)\right)6/10\right)+\frac{1}{2}, & \text{if } 12\le Z_R^2(x)\le 22 \\ 0, & \text{if } Z_R^2(x)\ge 22 \end{cases}$$

$$\mu_3^H(X)=\begin{cases} 1, & \text{if } Z_C^1(x)\le 10 \\ \frac{1}{2}\tanh\left(\left(15.75-Z_C^1(x)\right)6/11.5\right)+\frac{1}{2}, & \text{if } 10\le Z_C^1(x)\le 21.5 \\ 0, & \text{if } Z_C^1(x)\ge 21.5 \end{cases}$$

$$\mu_4^H(X)=\begin{cases} 1, & \text{if } Z_C^2(x)\le 9.5 \\ \frac{1}{2}\tanh\left(\left(14-Z_C^2(x)\right)6/9\right)+\frac{1}{2}, & \text{if } 9.5\le Z_C^2(x)\le 18.5 \\ 0, & \text{if } Z_C^2(x)\ge 18.5 \end{cases}$$

The problem was solved by the Linear Interactive and Discrete Optimization (LINDO) Software The optimal compromise solution is presented as follows:

$$X_{10}=0.636364 \text{ and } X^{**} = \{X_{11}=X_{22}=X_{33}=1\}$$

$$Z_R^{1*}=16, Z_R^{2*}=17, Z_C^{1*}=12, Z_C^{2*}=14$$

$$\lambda=0.50555$$

Conclusion

The present paper proposes a solution procedure of the interval assignment problem, where the co-efficient of the objective functions has been considered as interval. Initially, the problem has been converted into a classical assignment problem where the objectives, which are right limit and center of the interval objective functions, are minimized. These objective functions can be considered as the minimization of the worst case and the average case. To obtain the solution of the transformed classical assignment problem, the fuzzy linear and non-linear programming techniques has been used. If we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function.

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